This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

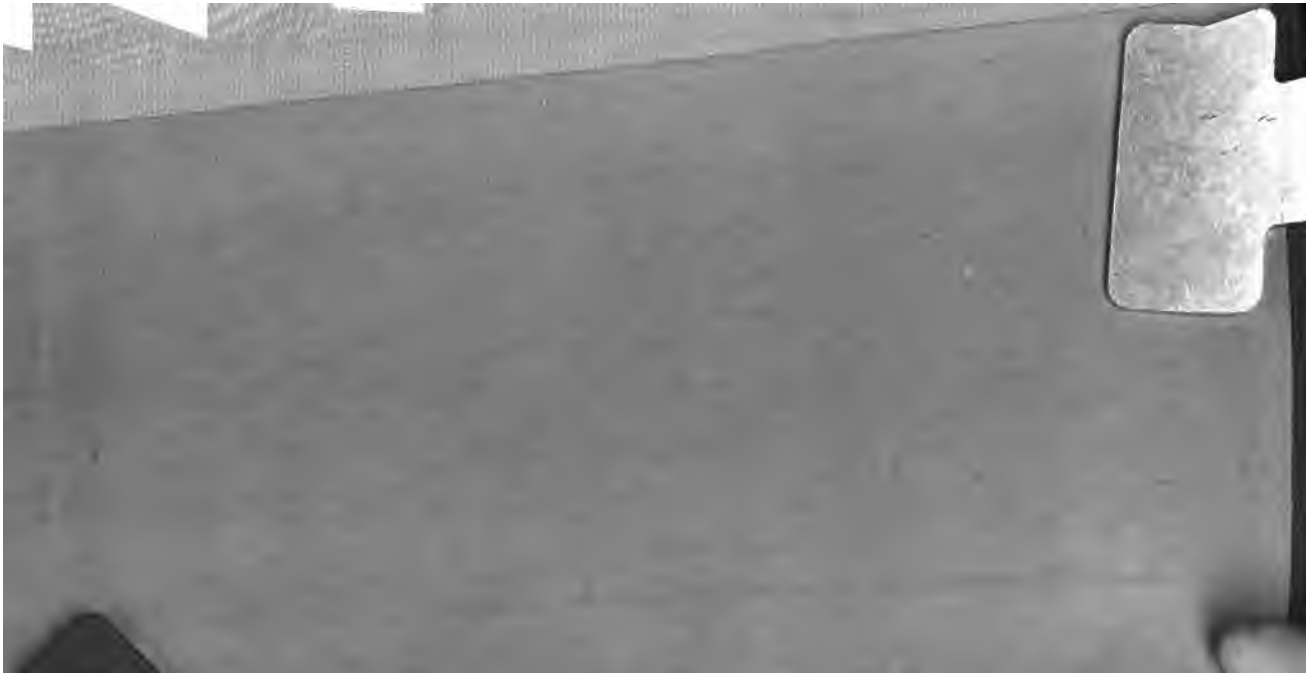
+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

NYPL RESEARCH LIBRARIES
3433 06907552 5



JH

Mathematics
OEA

THE

MATHEMATICAL

REPOSITORY.

BY T. LEYBOURN.

VOL. II.

The Mathematics are an universal science, which connects all the rest, and displays them in their happiest relations.

GANGANELLI.

LONDON:

Printed and sold by W. GLENDINNING, No. 25, Hatton-Garden:—Sold also by WEST and HUGHES, and SYMONDS, London; GREGORY, Cambridge; BULMER, Sunderland; and BRANSBY, Ipswich.

1801.

THE NEW YORK
PUBLIC LIBRARY
ASTOR, LENOX AND
TILDEN FOUNDATIONS.
R 1906 L

TO THE

Rev. NEVIL MASKELYNE, D.D. F.R.S.

ASTRONOMER ROYAL,

WHO,

BY A LONG AND

Indefatigable Application

OF HIS

Eminent Talents,

HAS DONE

EQUAL HONOUR AND SERVICE

TO HIS COUNTRY

AND

TO SCIENCE,

THIS VOLUME

OF THE

MATHEMATICAL REPOSITORY

IS

RESPECTFULLY DEDICATED

BY

THE EDITOR.

PREFACE.

THE Editor will not attempt to write an eulogium upon the Mathematical Sciences; because he is conscious that in their present advanced state, this would be, as it were,

——"To add another hue unto the Rainbow;
"or with taper light to seek the beauteous Eye of Heav'n to
"garnish."

That the greatest and wisest philosophers of which ancient or modern times can boast, have also been eminent Mathematicians, is a circumstance, which of itself is enough to induce all who aim at the enlargement of human knowledge and the consequent amelioration of society, to walk in that path, which has been trodden successfully from the days of THALES and PYTHAGORAS to those of BARROW and NEWTON; and which is still persevered in with honour and advantage. For, not only are the direct and immediate benefits of Mathematics obvious; but their incidental and collateral advantages are also very manifest; as has been often observed, and is expressed with peculiar beauty by the great Lord BACON, in his *Advancement of Learning*: "Pure Mathematics," saith this noble precursor of Newton, "do "remedy and cure many defects in the wit, and faculties "intellectual. For if the wit be dull, they sharpen it; if "too wandering, they fix it; if too inherent in the sense, "they abstract it. So that as tennis is a game of no use in "itself, but of great use in respect it maketh a quick eye, "and a body ready to put itself in all positions; so, in the "Mathematics, that use which is collateral and intervenient, "is no less worthy than that which is principal and in-"tended."

It is, therefore, not without sufficient reason that mathematical studies have been pursued with great avidity, in almost all ages; and it reflects considerable honour upon the present age, that such avidity is not diminishing, but evidently increasing.

With respect to periodical publications like the present, the experience of the eighteenth century has forcibly demonstrated their utility: indeed the Editor has reason to think there are scarcely any mathematicians of eminence in this country, who will not readily acknowledge, that in one part or other of their scientific course, they have been assisted by such

such works. Under this view of their beneficial tendency, the Editor trusts it will not be thought presumptuous in him to say that they are to the British a kind of *Gymnasium*, in some respects more honourable than the Gymnasium of the Grecians; for the exercises among the Grecians frequently inflamed the boisterous passions and stimulated to works of destruction; but in our Gymnasium the exercises tend only to promote a generous emulation who shall most expand the sphere of intellect, and who shall gain the deepest insight into the Laws of Nature, and soothing arts of peace.

The Editor of the Mathematical Repository, hopes that the publication he has had the honour of issuing into the world, will be found to have yielded its share of amusement and instruction: the nature of his plan must be now thoroughly understood,—for the Contents of each Number, together with the brief remarks which have accompanied, have undoubtedly rendered this clear. The ability and zeal of many of his Correspondents, for the advancement of science, deserves his warmest thanks. And he ventures to hope that his exertions, assisted by their talents and genius, will together make the Repository a *depot* of valuable matter; such as may be had recourse to with pleasure and advantage, by every friend to knowledge, and every promoter of science.

May, 1801.

CONTENTS

CONTENTS

OF THE

SECOND VOLUME.

ART. PAGE

I. LANDEN on the Ellipſis and Hyperbola, concluded 1

II. Geometrical Propoſitions, with XXXVII. Problems for Exerciſe. By Mr. *John Lowry* . . 6

III. XIV. XXV. XXXVII. XLV. and LII. ſeveral Props. from *Lawſon* on the Ancient Analyſis; to be demonſtrated 15, 78, 165, 302, 378, and 465

IV. and XV. Several Props. from Dr. *Stewart's* General Theorems . . . . 16 and 79

V. On the reſolution of Indeterminate Problems. By *John Leſlie*, A. M. concluded 17

VI. XXXI. Uſeful Propoſitions in Geometry. By Mr. *M. A. Harriſon* . . . . . 23 and 234

VII. Demonſtrations of Mr. *Keith's* method of conſtructing an Azimuth 26

VIII. Demonſtrations of Mr. *Emerſon's* Forms of Fluents. By Rev. *L. Evans* 27

IX. Anſwers to Mathematical Queſtions, 69 to 88, propoſed in Art. XLIII. Vol. I. 34

X. New Queſtions, 110 to 130, anſwered in Art. XXVII. 57

XI. Mr. *Navara's* Defence of his Geometry, in Reply to Art. LX. Vol. I. 61

XII. A Letter from Mr. *Frend*, on the Negative Sine . . 65

XIII. XXIII. XLII. Demonſtrations to *Lawſon's* Propoſitions 71, 158, and 316

XVI. On the Deſcription of Parabolic Trajectories. By Mr. *William Wallace* 80

XVII. Solutions to Queſtions, 89 to 109, propoſed in Art. LVIII. Vol. I. 83

XVIII. New Queſtions, 131 to 156, anſwered in Art. XXXIII. 106

XIX. Mathematical Lucubrations. By Mr. *Wm. Wallace* 111

XX. Extracts from a Paper on the Trigonometrical Tables of the Brahmins. By *Profeſſor Playfair*, F.R.S. Edin. 119

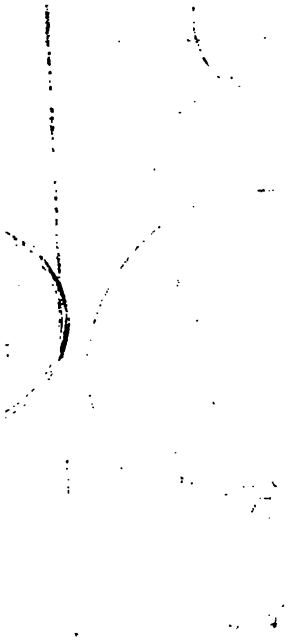
XXI. XXXII. and XLIX. Inveſtigations for determining the times of Vibration of Watch Balances, by *George Atwood*, Eſq. F.R.S. . . . 125, 237, and 401

XXII. Demonſtrations to Dr. *Stewart's* Propoſitions . 136

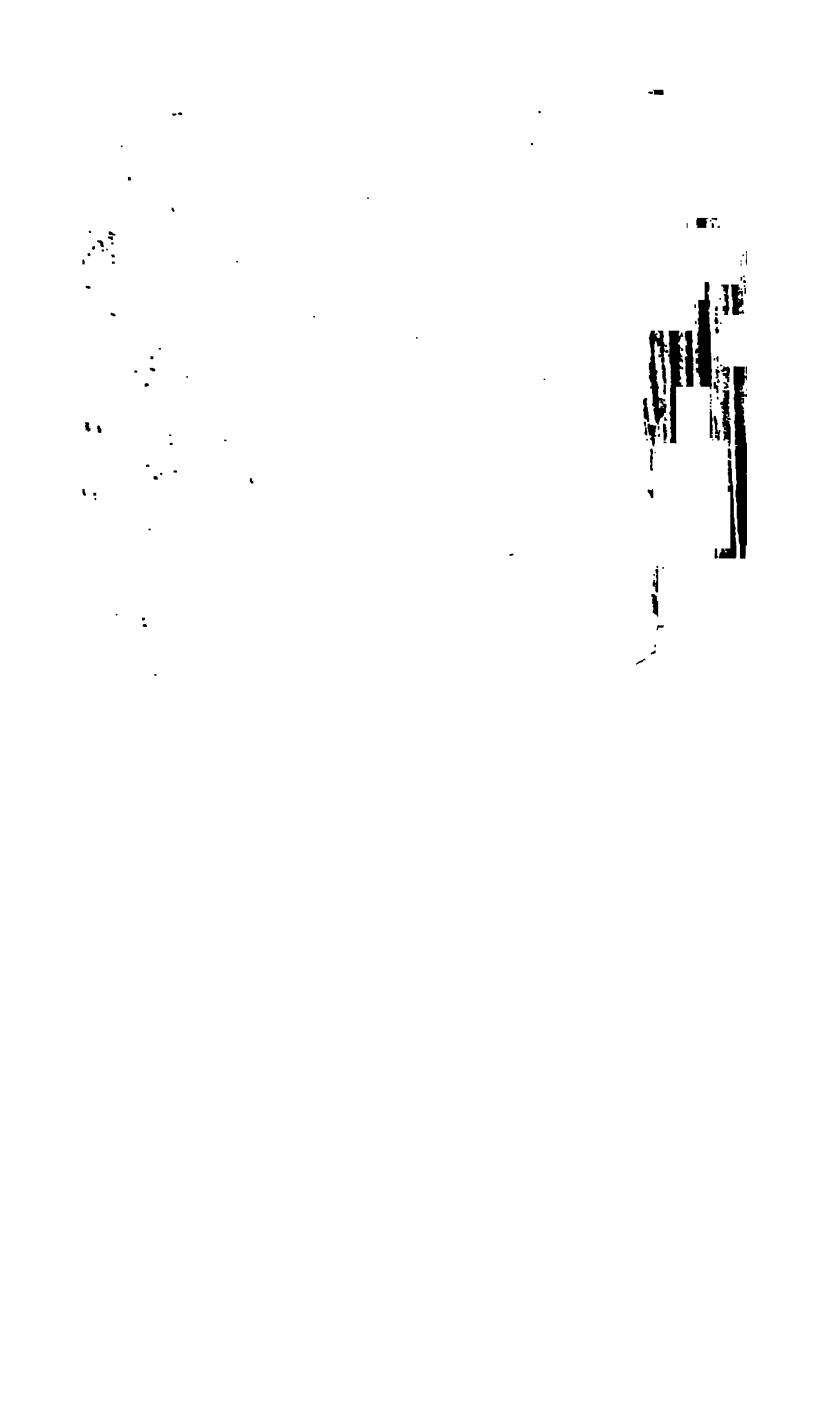
XXIV. A. B. on Fluents, with an inveſtigation . . 160

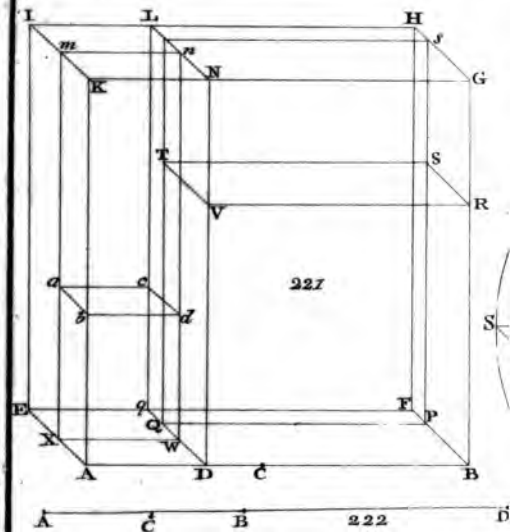
XXVI.

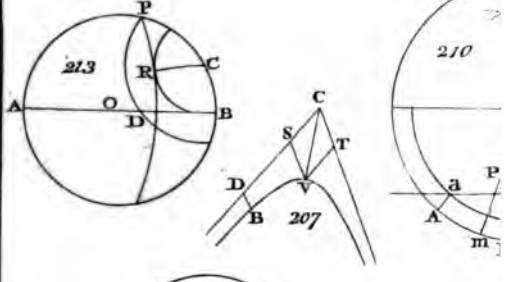
ART. P.
XXVI. An old Problem on Twilight, with a new Solut
By Mr. *John Surtees* . . .
XXVII. Anſwers to Queſtions, 110 to 130, propoſed in
X. . . . . .
XXVIII. New Queſtions, 157 to 182, anſwered in Art. XI
XXIX. Hints relative to Friction in Mechanics. By Mr. *Re*
Burrow. . . . .
XXX. and XLVI. Four Propoſitions on the Sun's Diſtance f
the Earth, by Mr. *Dawſon* . 220 and
XXXIII. Solutions to Queſtions, 131 to 156, propoſed in
XVIII. . . . . .
XXXIV. New Queſtions, 183 to 210, anſwered in .
XXXV. Method of diſcovering the number of negative and
poſſible Roots in any equation. By Mr. *Frend* .
XXXVI. Remarks on Mr. Landen's Correction of Emer
ſolution of the Tides. By Mr. *Benj. Gompertz* .
XXXVII. Lowry on Preſſure . . .
XXXIX. A Problem, in addition to the Lemmas of Sir I
Newton. By Mr. *Robert Wallace.* .
XL. Correction of an error in Art. XXXV. By Mr. *Frend*
XLI. Geometrical Sections. By Mr. *Lowry.* .
XLIII. Solutions to Queſtions, 157 to 182, propoſed in .
XXVIII. . . . .
XLIV. New Queſtions, 211 to 240, anſwered in Vol. III.
XLVII. Improved Solution to Queſtion 109. .
XLVIII. Mr. Cunliffe's new method of finding Fluents
L. Solutions to Queſtions, 183 to 210, propoſed in Art XXX
LI. New Queſtions, 241 to 270, anſwered in Vol. III.

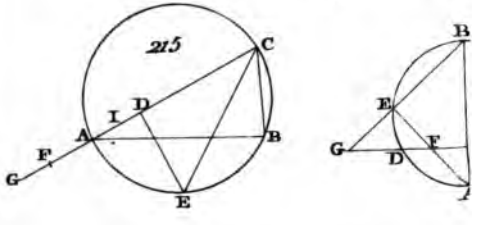


w
d
r
T
y
i
S
P
C
G
T
I
E
197
D
A
B
F
R
Q
K




I
L
H
m
n
s
K
N
G
T
S
V
R
a
c
227
b
d
S
E
q
F
Q
P
X
W
A
D
C
B
A
C
B
222
D


P
213
R
C
A
O
D
B
C
S
T
D
V
B
207
210
a
P
A
m


215
C
D
I
A
B
F
G
E
B
E
G
F
D

MATHEMATICAL REPOSITORY.

ARTICLE I.

Landen, *on the Ellipſis and Hyperbola.*

(Continued from p. 173, vol. 1.)

12. BY ſubſtituting $a-b$, $2\sqrt{ab}$, and $(a-b)^2-t$ for m, n and mz reſpectively, in art. 1, i appears that, if (in the hyperbola, fig. 34, plate 11, the ſemi-tranſverſe axis AC be $=a-b$, the ſemi-con jugate $=2\sqrt{ab}$, and the perpendicular CP$=\sqrt{(a-b)^2-t^2}$; the difference (DP—AD) betwee the tangent DP and the arch AD will be equal to th fluent of $\dot{t}\sqrt{((a-b)^2-t^2)\div((a+b)^2-t^2)}$.

13. It is well known, that, in any ellipſis whoſ ſemi-tranſverſe axis is h, and ſemi-conjugate n, i x be the abſciſſa, meaſured from the centre upo the ſemi-tranſverſe axis, and Q the arch betwee the conjugate axis and the ordinate correſpondin to x, $\dot{x}\sqrt{(h^2-gx^2)\div(h^2-x^2)}$ will be $=\dot{Q}$, g be ing $=(h^2-n^2)\div h^2$.

Hence, by ſubſtituting $a+b$, $2\sqrt{ab}$, and $t(a+b\div(a-b)$ for h, n and x reſpectively; it appears that, in the ellipſis aed (fig. 26, pl. 1.) whoſe ſemi tranſverſe axis cd is $=a+b$, ſemi-conjugate ca$=2\sqrt{ab}$, and abſciſſa cb (correſponding to the ordi

From whence, by taking the fluents, according art. 12 and 13, we find $R = ae$ (fig. 27, pl. 1.) $= \frac{1}{2}t + \frac{1}{4}$ (DP—AD) (fig. 34, pl. 2.) $+ \frac{1}{4}Q$. Consequently the hyperbolic arc AD is $= DP + Q - 4R + 2t$. Thus, beyond my expectation, I find that the *hyperbola* may in general be rectified by means of two *ellipses*!

If $p'e'$, $p''e''$ be equal tangents to the ellipsis $ae'e''d$; the arc ae' (denoted by R) will (by art. 9.) be equal to the arc $de'' +$ the tangent $p''e''$ or $p'e'$ (denoted by t). Therefore, substituting for t its value found by this last equation, it appears that AD is $= DP + Q - 2R - 2 \times de'' = DP + Q + 2 \times e'e'' - 2E''$, and $DP - AD = 2E'' - 2 \times e'e'' - Q$; E'' being put for the quadrantal arc, ad. It is observable that when t is $= a - b$, e'' (by art. 5.) coincides with e', and $e'e''$ is $= 0$; Q (by art. 13.) $=$ the quadrantal arc (ad); and (by art. 12.) DP and AD both become infinite.

Consequently writing E for that quadrantal arc (ad), and L the *limit* of the difference DP—AD, whilst the point of contact (D) is supposed to be carried to an infinite distance from the vertex A of the hyperbola, we find $2E'' - E = L$.

15. Exterminating a, b, and t, by means of the equations $a - b = m$, $2\sqrt{ab} = n$ and $(a-b)^2 - t^2 = mz$; and writing f for $(m^2 - n^2) \div 2m$, as in art. 1. it appears that (V) the fluent of $\frac{1}{2}m^{\frac{1}{2}}z^{\frac{1}{2}}\dot{z} \div \sqrt{n^2 \div 2fz - z^2}$ is $= 2 \times e'e''$—de generated whilst z increases from o. Moreover the fluent of $\frac{1}{2}m^{\frac{1}{2}}z^{-\frac{1}{2}}\dot{z} \sqrt{(n^2 \div m + z) \div (m - z)}$, or its equal $(\frac{1}{2}m^{-\frac{1}{2}}n^2z^{-\frac{1}{2}}\dot{z} + \frac{1}{2}m^{\frac{1}{2}}z^{\frac{1}{2}}\dot{z}) \div \sqrt{n^2 + 2fz - z^2}$ being $=$ de, as observed in art. 7, we find by subtraction (W) the

 fluent

fluent of $\frac{1}{2}m^{-\frac{1}{2}}n^2z^{-\frac{1}{2}}z \div \sqrt{n^2+2fz-z^2} = 2 \times (de-e'e''$.

The ſemitranſverſe axis of the ellipſis aed being no$= \sqrt{m^2+n^2}$, the ſemi-conjugate $= n$; the abſciſſa cb$= \sqrt{(m^2+n^2) \div m} \times \sqrt{m-z}$, and the ordinate be$= n\sqrt{z \div m}$. The ſemi-tranſverſe axis (a) of the ellipſis $ae'e''d = \sqrt{(m^2+n^2) \div 2} + m \div 2$, the ſemi-conjugate ($b$) $= \sqrt{(m^2+n^2) \div 2} - m \div 2$; the tangents $e'p'$, $e''p''$, intercepted by perpendiculars (cp', cp'') drawn thereto from the centre c, each $= \sqrt{n} \times \sqrt{m-z}$; and the abſciſſa ($cb'$ or cb'') on cd, correſponding to the point e' or e'' of the curve is determined by the expreſſion $a\sqrt{(m^2+n^2)^{\frac{1}{2}} + m - z} \mp (z^2+n^2z \div m)^{\frac{1}{2}} \div 2^{\frac{1}{2}} \times (m^2+n^2)^{\frac{1}{4}}$.

From what is done above, many other new Theorems for the calculation of fluents are deducible: the moſt remarkable of which I intend to inſert in the appendix, to be annexed to theſe Memoirs.

16. Taking m and n each equal to 1, we find by the above theorem (marked W) the whole fluent of $\frac{1}{2}z^{-\frac{1}{2}}z \div \sqrt{1-z^2} = 2 \times (e-d)$, e denoting one-fourth of the periphery of an ellipſis whoſe ſemi-tranſverſe and ſemi-conjugate axes are $\sqrt{2}$ and 1; and d one-fourth of the periphery of another ellipſis whoſe ſemi-tranſverſe and ſemi-conjugate axes are $1 \div \sqrt{2} + \frac{1}{2}$ and $1 \div \sqrt{2} - \frac{1}{2}$. But by art. 11, the ſame fluent is found $= \frac{1}{2}e + \frac{1}{2}\sqrt{e^2-2c}$, c denoting $\frac{1}{4}$ of the periphery of the circle whoſe radius

is

is 1. Consequently $2 \times (e-d)$ being $= \frac{1}{2}e + \frac{1}{4}\sqrt{e^2-2c}$, we have, from that equation, $d = \frac{1}{4}e - \frac{1}{4}\sqrt{e^2-2c}$; or $e = \frac{3}{2}d + \frac{1}{4}\sqrt{d^2-2c}$. Thus it appears that the periphery of one certain ellipſis may be found by means of the periphery of another ellipſis and the periphery of a circle!

Before Mr. MACLAURIN publiſhed his excellent Treatiſe of Fluxions, ſome very eminent Mathematicians imagined, that the *elaſtic curve* could not be conſtructed by the quadrature or rectification of the conic ſections. But that gentleman has ſhewn, in that Treatiſe, that the ſaid curve may in every caſe be conſtructed by the rectification of the hyperbola and ellipſis; and he has obſerved, that, by the ſame means, we may conſtruct the curve along which if a heavy body deſcended, it would recede equally in equal times from a given point. Which laſt mentioned curve, Mr. JAMES BERNOUILLI, conſtructed by the rectification of the elaſtic curve, and Mr. LEIBNITZ and Mr. JOHN BERNOUILLI by the rectification of a geometrical curve of a higher kind than the conic ſections. It is obſervable, that Mr. MACLAURIN's method of conſtruction juſt now adverted to, though very elegant, is not without a defect. The difference between the hyperbolic arch and its tangent being neceſſary to be taken, the method (for the reaſon mentioned at the beginning of this Memoir) always fails when ſome principal point in the figure is to be determined; the ſaid arc and its tangent then both becoming infinite, though their difference be at the ſame time finite. The contents of this Memoir, properly applied, will evince, that both the *elaſtic curve*, and the *curve of equable receſs from a given point*, (with many others) may be conſtructed by the rectification of the ellipſis only, without failure in any point.

ARTICLE II.

GEOMETRICAL PROPOSITIONS.

By Mr. John Lowry. (*Fig.* 201, *Pl.* 14.)

Let ABC be a triangle inſcribed in the circle ABC, whoſe centre is R, and let HMN be another circle inſcribed in the ſaid triangle, whoſe centre is O; alſo let QL, XqY, xq'y, be circles, each touching one ſide of the triangle, and the continuations of the other two, K, W, w, being their reſpective centres. Draw OK, OW, Ow, which will biſect the angles AB, AC, BL, in L, l, d, and meet the ſides of the triangle in the angular points C, B, A, reſpectively; alſo demit the perpendiculars KI, KL, KQ, OH, OM, ON, WX, WY, Wq, wx, wy, wq, IT, &c. &c. CP, Bp, Ap'; join LB, LA, BR, AO, BO, RF, BK, AK, AW, CW, Bw, Cw, and draw FRE, leRf, dhRk, which will be reſpectively parallel to AB, AC, BC, and will alſo biſect them in the points F, e, h.

Now, if AB be the baſe of the triangle ABC, the following properties are well known, viz.

1. CL = CI = BX = BY = Ax = Ay = ½ the perimeter.

2. CM = CN = ½(AC + BC — AB) = half the difference between the ſum of the ſides and the baſe.

3. CT = half the ſum of the ſides AC, CB.

4. BT = OE = EH = half their difference.

5. AB = IM = I'N.

6. 2Bt' = AB + BC, and 2At' = AB ~ BC = qe = eM.

7. 2At

7. 2At=AB+AC, and 2Bt=AB~AC=q'h=hN.

8. Xy=AC+BC; Ix=AB+BC; and I'Y=AB+AC.

9. AX=By=CM.

10. AI=Cx=BH=AQ=Mm=BN.

11. BI'=Cy=AH=BQ=Im=AM.

12. Hy=Mx=BC, and HX=NY=AC.

PROP. I.

A circle deſcribed about the centre L, with the diſtance BL, or AL, will paſs through the points O and K.

For, the ∠ LAB = ∠ ACL, and ∠ BAO = ∠OAC; therefore ∠LAO=∠LOA, and conſequently LA=LO; therefore the circle paſſes through the point O.

Again, the △IKA=△QKA; therefore ∠KAI=∠KAB; hence, the ∠KAO=∠KAI+∠OAC= a right angle; conſequently the circle paſſes through the point K.

Cor. 1. A circle deſcribed about the centre l, with the diſtance lA, or lC, will paſs through the points O and W.

Cor. 2. A circle deſcribed about the centre d, with the diſtance dB, or dC, will paſs through the points O and w.

Cor. 3. The right lines joining the points KW, Ww, and Kw, will paſs through the points A, k; F, C; and B, f, reſpectively.

Cor. 4. NT=TI'= half the baſe AB.

Cor. 5. TU = TB = half the difference of the ſides.

Cor. 6. The rect. ACB = rect. UCB = rect. OCK.

Cor. 7. The rect. BAC = rect. OAw.

Cor. 8. The rect. ABC = rect. OBW.

PR(

PROP. II.

As half the baſe of the triangle ABC, is to the ſegment of the diameter of the circumſcribing circle, EF, ſo is the radius of the inſcribed circle, to half the difference between the ſum of the ſides and the baſe; ſo is the radius of the circle touching the baſe and the continuation of the two ſides, to half the perimeter.

For, the triangles COM, CKI, BEF, are ſimilar;
therefore BE : EF :: OM : CM;
and BE : EF :: KI : IC.

Cor. 1. CM : OM :: BE : EL :: IC : IK.
Cor. 2. $2EL = KI - OM$.
Cor. 3. BE : EL :: AC + BC : IK + OM.
Cor. 4. $2BE^2 = (IK - OM) \cdot EF$.
Cor. 5. $BE \cdot (AC + BC) = EF \cdot (IK + OM) = 2BE^2 + 2EF \cdot OM = 2EF \cdot IK - 2BE^2$.
Cor. 6. $BE \cdot (AC + CB + AB) = 4BE^2 + 2EF \cdot OM = 2EF \cdot IK$.
Cor. 7. BH : OM :: OM + 2EL : AH.
Cor. 8. $(AC - CB)^2 = AB^2 - 4OM \cdot (OM + 2EL)$,
Cor. 9. BH : IK :: IK − 2EL : AH.
Cor. 10. $(AC - CB)^2 = AB^2 - 4IK \cdot (IK - 2EL)$.

PROP. III.

As the ſegment EL, is to the diameter FL, ſo is the ſquare of the radius of the inſcribed circle, to the ſquare of the diſtance of the centre of that circle from the vertex of the triangle; ſo is the ſquare of the radius of the circle touching the baſe and the continuation of the other two ſides,

to

to the ſquare of the diſtance of its centre from the vertex of the triangle.

For, by ſim. $\triangle$'s $BE : EF :: OM : CM$,
and $EL : BE :: OM : CM$;
therefore $EL : EF :: OM^2 : CM^2$;
and theref. $EL : FL :: OM^2 : OM^2 + CM^2 = OC^2$.

In the ſame way it is ſhewn that
$EL : FL :: IK^2 : KC^2$.

Cor. 1. As $OC \cdot CK : OM \cdot IK :: FL : EL$.

Cor. 2. As $OC \cdot CK : OC^2 :: CI : CM$.

Cor. 3. Join II′, XY, xy, and draw Mn perpendicular to CK. Then $OC \cdot IZ = OB \cdot XZ' = OA \cdot xZ'' = IK \cdot CM = WX \cdot BH = wy \cdot AH = KC \cdot Mn = OM \cdot IC =$ the area of the $\triangle ABC$.

Cor. 4. If the perpendiculars IK, WX, and wy, be given, the ratio of AK, AH, and BH, will be given. And if the diſtances OC, OB, OA, be given, the ratio of IZ, XZ′, and xZ″, that is, the ratio of the ſines of half the angles of the triangle will be given.

Cor. 5. The rect. $BH \cdot HA =$ rect. $IK \cdot OM$.

Cor. 6. As $BE^2 : OM \cdot IK :: EF : PC$.

Cor. 7. As $BE^2 : OM \cdot IK :: EFL : OCK = ACB$.

Cor. 8. As $BE : EF :: IK \cdot OM$: area $\triangle ABC$.

PROP. IV.

As the baſe is to half the difference between the ſum of the ſides and the baſe, ſo is the diameter of the inſcribed circle, to the difference between the perpendicular and the diameter of the ſaid circle.

For, $AB + CM : AB :: PC : 2OM$,
hence $AB : CM :: 2OM :: PC - 2OM$.

Cor. 1. As $AB : IC :: 2IK : PC + 2IK$.

Cor. 2. As $AB^2 : 4OM^2 :: FE : PC - 2OM$.

Cor. 3. As $AB^2 : 4IK^2 :: FE : PC + 2IK$.

PROP.

PROP. V.

The diſtance between the centres of the inſcribed and circumſcribing circles, is a mean proportional between the radius of the circumſcribing circle, and the difference between the ſaid radius and the diameter of the inſcribed circle.

Let Rb be drawn $\perp$ to CL, and then $Lb = Cb$;
therefore $LR^2 - RO^2 = Lb^2 - Ob^2 = LOC$:
but the $\triangle$s COM, LFA, are ſimilar;
therefore $LA \cdot OC$ or $LOC = LF \cdot OM$;
wherefore $LR^2 - RO^2 = LF \cdot OM$;
therefore $RO^2 = LR \cdot (LR - 2OM)$.

Cor. 1. $KR^2 = LR \cdot (LR + 2IK)$.
Cor. 2. $WR^2 = LR \cdot (LR + 2WX)$.
Cor. 3. $wR^2 = LR \cdot (LR + 2wy)$.
Cor. 4. $KR^2 - OR^2 = 2LR \cdot (IK + OM) = 2LR \cdot EL \cdot (AC + CB) \div BE$.

The preceding Propoſitions, though ſimple in appearance, are nevertheleſs very extenſive in their application. They enable us to give *ſimple* and *elegant Conſtructions* to a great variety of curious and difficult Problems in plain Geometry; a few of which I have ſelected as exerciſes for young Students in Geometry. Their ſolutions will reward the trouble of obtaining them, by the number of new Propoſitions which they will unavoidably ſuggeſt, and which I have omitted to exhibit above, as I did not wiſh to deprive the young Geometrician of the pleaſure of inveſtigating them himſelf.

Prob. I. Given the baſe, the vertical angle, and either the ſum, difference, or rectangle of the ſum

ium of the ſides and radius of the inſcribed circle, to determine the triangle.

Prob. II. Given the vertical angle, the perimeter, and either the rectangle of the ſides, or the area, to conſtruct the triangle.

Prob. III. Given the perpendicular, and the radii of the circumſcribing and inſcribed circles, to deſcribe the triangle.

Prob. IV. Given the baſe, the vertical angle, and the difference between the perpendicular and the diameter of the inſcribed circle, to determine the triangle.

Prob. V. Given the vertical angle, the ſum or difference of the ſides, and the ſum or difference of the baſe and radius of the inſcribed circle, to determine the triangle.

Prob. VI. Given the vertical angle, the rectangle of the ſides, and the rectangle of the radii of the circumſcribing and inſcribed circles, to conſtruct the triangle.

Prob. VII. Given the vertical angle, the rectangle of the ſides, and the ſum, difference, or rectangle of the baſe and radius of the inſcribed circle, to conſtruct the triangle.

Prob. VIII. Given the vertical angle, the perimeter, and the difference of the ſides, to conſtruct the triangle.

Prob. IX. Given the vertical angle, the rectangle under the ſum of the ſides and the baſe, and the ſum of the ſquares of the baſe, and radius of the inſcribed circle, to conſtruct the triangle.

Prob. X. Given two ſides, and the radius of the inſcribed circle, to conſtruct the triangle.

Prob. XI. Given two ſides, and the diſtance of their interſection from the centre of the inſcribed circle, to deſcribe the triangle.

Prob. XII. Given the radius of the inſcribed circle'

circle, the rectangle of the segments of the base made by its contact with the said circle, and the difference between the base and the diameter of the circumscribing circle, to determine the triangle.

Prob. XIII. Given the vertical angle, the rectangle of the sides, and the rectangle of the base and sum of the sides, to construct the triangle.

Prob. XIV. Given the difference of the angles at the base, the sum of the squares of the sides, and the distance between the centre of the inscribed circle and the vertex, to construct the triangle.

Prob. XV. Given the vertical angle, the radius of the inscribed circle, and the rectangle under the base and sum of the sides, to construct the triangle.

Prob. XVI. Given the vertical angle, either the sum of the sides, or the radius of the inscribed circle, and the rectangle of the segments of the base made by the point of contact with the inscribed circle, to construct the triangle.

Prob. XVII. Given the radii of the circumscribing and inscribed circles, and the nearest distance of the periphery of the latter from the vertex, to describe the triangle.

Prob. XVIII. Given the sum of the sides, and the radii of two circles, each touching a side of the triangle, and the continuation of the base and the other side, to construct the triangle.

Prob. XIX. Given the base, the line bisecting the vertical angle, and the difference between the radius of the inscribed circle and the radius of a circle touching the base, and the continuation of the other two sides, to determine the triangle.

Prob. XX. Given the perimeter, and the radii of two circles, each touching a side of the triangle and

and the continuation of the other two ſides, to determine the triangle.

Prob. XXI. Given the ſum of the ſides, the ſum of the radii of the inſcribed circle and the circle touching the baſe, and the continuation of the other two ſides, and the line biſecting the baſe, to determine the triangle.

Prob. XXII. Given the three diſtances from the centre of the inſcribed circle to the angles of the triangle, to deſcribe it.

Prob. XXIII. Given the baſe, the vertical angle, and the radius of a circle touching the baſe and the continuation of the other two ſides, to deſcribe the triangle.

Prob. XXIV. Given the baſe and the radii of two circles, each touching a ſide and the continuation of the other ſide and baſe, to conſtruct the triangle.

Prob. XXV. Given the area, the rectangle of the ſegments of the baſe made by the point of contact of the inſcribed circle, and the line biſecting the vertical angle, to conſtruct the triangle.

Prob. XXVI. Given the vertical angle, the ſum of the ſides, and the diſtance between the centres of the circumſcribing and inſcribed circles, to conſtruct the triangle.

Prob. XXVII. Given the vertical angle, the difference between the perpendicular and the diameter of the inſcribed circle, and the diſtance between the centres of the circumſcribing and inſcribed circles, to conſtruct the triangle.

Prob. XXVIII. Given the perimeter, the radius of the circumſcribing circle, and the radius of a circle touching the baſe and the continuation of the other two ſides, to conſtruct the triangle.

Prob. XXIX. Given the vertical angle, and

 the

the radii of two circles, each touching a ſide of the triangle and the continuation of the other ſide and the baſe, to deſcribe it.

Prob. XXX. Given the vertical angle, the diſtance between the centres of the inſcribed and circumſcribing circles, and the rectangle under the baſe and radius of the inſcribed circle, to conſtruct the triangle.

Prob. XXXI. Given the vertical angle, the perimeter, and the ſolid whoſe baſe is the rectangle of the ſides, and altitude the baſe of the triangle equal to a given cube, to determine the triangle.

Prob. XXXII. Given the vertical angle, and the perimeter of a plane triangle, to conſtruct it, when the pyramid, whoſe baſe is the triangle, and altitude the radius of the circumſcribing circle, is equal to a given cube.

Prob. XXXIII. Given the difference of the ſides, and the radii of the inſcribed circle and a circle touching the baſe and the continuation of the other two ſides, to conſtruct the triangle.

Prob. XXXIV. Given the perimeter of a triangle to conſtruct it, when the ſolid contained under the radii of three circles, each touching a ſide of the triangle and the continuation of the other two, is a maximum.

Prob. XXXV. Given the radius of the inſcribed circle, and the radii of two circles, each touching a ſide of the triangle and the continuation of the other two ſides, to conſtruct the triangle.

Prob. XXXVI. Given the ratio of the lines from the angles to the centre of the inſcribed circle, and the perimeter, to conſtruct the triangle.

Prob. XXXVII. Given the radii of three circles,

ircles, each touching a ſide of the triangle and he continuation of the other two ſides, to determine the triangle.

ARTICLE III.

Three Propoſitions from *Lawſon.*

(To be anſwered in Number VIII.)

PROP. XXVIII.

IN AB, the diameter of a circle produced, let be taken the point C, and CD be perpendicular to AB, and therein be taken two points E and F, on different ſides of C, ſuch that the rectangle ECF may be equal to the rectangle ACB, and from the points E and F let EG, FG, be inflected to any point G in the circle, meeting the ſame in H and K, and let HK, when drawn, meet the diameter AB in L; then I ſay that

AL : LB :: AC : CB.

PROP. XXIX.

Let AB touch a circle in B, and any line AE be drawn equal to AB, and likewiſe from A let any line be drawn to cut the circle in C and D, and let EC, ED, be drawn meeting the circle again in F and G; then FG being drawn, will be parallel to AE.

PROP. XXX.

Let AB touch a circle in B, and therein be taken two points E and F on the ſame ſide of A,

ſuch that the reſtangle EAF may be equal to the ſquare of AB, and from A let any line be drawn meeting the circle in C and D, and EC, FD, be drawn, meeting the circle again in G and H; then GH being drawn, will be parallel to AB.

ARTICLE IV.

Two Propoſitions from *Stewart's* Theorems.

(To be anſwered in Number VIII.)

PROP. XXV. THEO. XXII.

LET there be any figure given by poſition of a greater number of ſides than three, and let *a*, *b*, *c*, &c. be given magnitudes, as many in number as there are ſides in the figure; four right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn perpendiculars to the ſides of the figure, and likewiſe there be drawn perpendiculars to the four lines found, the cube of the perpendicular drawn to one of the ſides of the figure, together with the ſolid to which the cube of the perpendicular drawn to another of the ſides of the figure has the ſame ratio that *a* has to *b*, together with the ſolid to which the cube of the perpendicular drawn to another of the ſides of the figure has the ſame ratio that *a* has to *c*, and ſo on, will be equal to the ſolid to which the ſum of the cubes of the perpendiculars drawn to the four lines found has the ſame ratio that ſome *a* has to the ſum of *a*, *b*, *c*, &c.

PRO

PROP. XXVI. THEO. XXIII.

Let there be any regular figure inſcribed in a circle, and from all the angles of the figure let there be drawn right lines to any point in the circumference of the circle; the ſum of the fourth powers of the chords will be equal to ſix times the multiple of the fourth power of the ſemi-diameter of the circle by the number of the ſides of the figure.

ARTICLE V.

On the Reſolution of Indeterminate Problems.

By John Leſlie, A. M.

(Continued from p. 367, vol. 1.)

PROBLEM VIII.

LET c and d be known values of x and y in the expreſſion, $ax^2+b=y^2$, and from theſe let it be required to diſcover others.

Since $ax^2+b=y^2$, and $ac^2+b=d^2$, ſubtracting theſe equations, we ſhall obtain $ax^2-ac^2=y^2-d^2$, and by reſolution, $(ax-ac)(x+c)=(y+d)(y-d)$; whence $ax-ac=my-md$, and $x+c=(y+d)\div m$. From the firſt of theſe equations, $x=(my-md+ac)\div a$, and from the ſecond, $x=(y+d-mc)\div m$; whence $(my-md+ac)\div a=(y+d-mc)\div m$, and $y=(m^2d+ad-2mac)\div(m^2-a)$, or $((m^2+a)d-2mac)\div(m^2-a)$. But $x=(y+d)\div m-c$; therefore $x=\frac{2md}{}$

[illegible] To resolve these [illegible] and [illegible]. [illegible] I. [illegible] the [illegible] will be [illegible]. [illegible] and [illegible]. [illegible] [illegible] [illegible]

But from the first equation, $y=a-x$; wherefore $a-x=(m^2x-x)\div 2m$, and $x=2am\div(m^2+2m-1)$; consequently, $y=a(m^2-1)\div(m^2+2m-1)$, and $z=a(m^2+1)\div(m^2+2m-1)$.

Suppose $a=23$, and $m=4$; then $x=8$, $y=15$, and $z=17$. For $8+15=23$, and $64+225=289=(17)^2$.

PROBLEM X.

To find two numbers, whose sum shall be a given number, and the product of the sums, formed by adding given numbers to them, a square.

By hypothesis, $x+y=a$, and $(x+b)(y+c)=z^2$. From the second equation, we obtain by assumption, $x+b=mz$, and $z=my+mc$; therefore, $z=(x+b)\div m=my+mc$; and so $x=m^2y+m^2c-b$. But from the first equation, $x=a-y$; consequently, $m^2y+m^2c-b=a-y$, and $y=(a+b-m^2c)\div(m^2+1)$; also $x=a-y=(am^2+m^2c-b)\div(m^2+1)$, and $z=(x+b)\div m=(am+mc+mb)\div(m^2+1)$.

Suppose $a=17$, $b=6$, $c=2$, and let $m=2$; then $y=(17+6-8)\div(4+1)=3$, $x=(68+8-6)\div(4+1)=14$, and $z=(34+4+12)\div(4+1)=10$. But $14+3=17$, and $(14+6)(3+2)=100=(10)^2$.

PROBLEM XI.

Let it be required to find two numbers, such that, if to each, their sum and difference, unit be added, the numbers resulting shall be squares.

The first condition will be observed, if the numbers be denoted by x^2-1 and y^2-1. The hypothesis will then require $x^2+y^2-1=z^2$ and $x^2-y^2+1=v^2$.

Transposing the first equation, $x^2-1=z^2-y^2$,

and

and reſolving into factors, $(x+1)(x-1)=(z+y)(z-y)$; whence, $x+1=mz-my$, and $z+y=mx-m$; therefore, $z=mx-m-y=(x+1+my)\div m$, from which we have $y=(m^2x-x-m^2-1)\div 2m$.

Again, tranſpoſing the ſecond equation, $x^2-y^2=v^2-1$, and reſolving, $(x+y)(x-y)=(v+1)(v-1)$ and by aſſumption, $x+y=pv-p$, and $v+1=px-py$, and therefore, $v=px-py-1=(x+y+p)\div p$. Hence $y=(p^2x-x-2p)\div(p^2+1)$.

But it was found, that $y=(m^2x-x-m^2-1)\div 2m$; wherefore $(p^2x-x-2p)\div(p^2+1)=(m^2x-x-m^2-1)\div 2m$; and by reduction, $x=(p^2m^2+p^2-4mp+m^2+1)\div(p^2m^2-2mp^2-p^2+m^2+2m-1)$, or $=((m^2+1)(p^2+1)-4mp)\div(p^2((m-1)^2-2)+(m+1)^2-2)$.

In the ſame manner, by finding the values of x in terms of y, &c. we obtain $y=(m^2p^2-2m^2p+p^2-m^2+2p-1)\div(p^2m^2-2mp^2-p^2+m^2+2m-1)$, or $=(m^2((p-1)^2-2)+(p+1)^2-2)\div p^2((m-1)^2-2)+(m+1)^2-2)$.

PROBLEM XII.

To find three numbers, the product of any two of which, increaſed by unit, ſhall be a ſquare.

By hypotheſis, $xy+1=v^2$, $xz+1=s^2$, and $yz+1=w^2$.

1. Tranſpoſing the firſt equation, $xy=v^2-1$, and reſolving $x\times y=(v+1)(v-1)$, whence $y=mv-m$, and $v+1=mx$; conſequently, $v=(m+y)\div m=mx-1$, and $x=(2m+y)\div m^2$.

2. Again, tranſpoſing the ſecond equation, $xz=s^2-1$, and reſolving, $x\times z=(s+1)(s-1)$; whence, $z=ps-p$, and $s+1=px$; conſequently, $s=(z+p)\div p=px-1$, and reducing, $x=(z+2p)\div p^2$. But

$x=$

$x=(2m+y)\div m^2$; wherefore $m^2z+2m^2p=2mp^2+p^2y$, and $y=(m^2z+2m^2p-2mp^2)\div p^2$.

3. Moreover, by the third equation, $yz=w^2-1$; whence, $y\times z=(w+1)(w-1)$, and $y=qw-q$, and $w+1=qz$; wherefore, $w=(y+q)\div q=qz-1$, and $y=q^2z-2q$. But $y=(m^2z+2m^2p-2mp^2)\div p^2$; conſequently $p^2q^2z-2p^2q=m^2z+2m^2p-2mp^2$, and $z=(2p^2q+2m^2p-2mp^2)\div(p^2q^2-m^2)$. Now, $y=q^2z-2q$; whence by ſubſtitution, $y=(2m^2pq^2-2mp^2q^2+2m^2q)\div(p^2q^2-m^2)$. And becauſe $x=(2m+y)\div m^2$, we have alſo $x=(2pq^2-2m+2q)\div(p^2q^2-m^2)$.

Cor. Let $m=1$, then the formulæ will be more ſimple; $x=(2pq^2+2q-2)\div(p^2q^2-1)$, $y=(2pq^2-2p^2q^2+2q)\div(p^2q^2-1)$, and $z=(2p^2q+2p-2p^2)\div(p^2q^2-1)$.

There is a remarkable caſe in which the above formulæ do not directly apply, the numerators and denominators vaniſhing at the ſame time. It is when $m=1$, $p=2$, and $q=\frac{1}{2}$. For, by art. 3, $y=(2m^2pq^2-2mp^2q^2+2m^2q)\div(p^2q^2-m^2)=(1-2+1)\div(1-1)=0\div 0$; wherefore the value of y may be expreſſed by any aſſumed number, n. But, by art. 1, $x=(2m+y)\div m^2=y+2$; whence, $x=n+2$. Alſo, by art. 2, $x=(z+2p)\div p^2=(z+4)\div 4$; therefore, $z+4=4n+8$, and $z=4n+4$. Thus, 2, 4, 12; for $2\times 4+1=9$, $2\times 12+1=25$, and $4\times 12+1=49$.

PROBLEM XIII.

To find a cube which ſhall be equal to the product of a ſquare by a given number.

By hypotheſis, $x^3=ay^2$, and reſolving, $x\times x^2=a\times y^2$; whence $x=ma$, and $y^2=mx^2$; but $x^2=(ma)^2$, conſequently, $y^2=m^3a^2$, and $y\times y=ma\times m^2a$; and by a ſecond aſſumption, $y=pma$, and $m^2a=p^2$.

meets the line (CE) bisecting the vertical angle, bisected by the perpendicular (PQ) drawn through (L) the middle of the base, upon the said bisecting line.

Demon. Since GT, AU, are perpendicular to CE, they are parallel to each other;
but PQ is perpendicular to CE;
hence, GT, PQ, AU, are parallel to each other;
consequently, CG : CT :: GP : TQ :: PA : QU;
but, GP=PA, therefore TQ=QU.

Q. E. D.

PROP. X. THEO.

If LD be bisected in n, and the perpendicular no demitted to meet PQ produced in o: then I say o will be the centre of a circle passing through the points L, T, D, U.

Demon. It has been demonstrated, that the points L, T, D, U, are in the periphery of a circle; and LD, TU, are chords.

But by hyp. Ln=nD, and no is ⊥ to LD, therefore, no passes through the centre.
Again, by Prop. IX. TQ=QU, and Qo is ⊥ to TU; therefore, Qo passes through the centre.

Q. E. D.

PROP. XI. THEO.

If LU be joined, it will be parallel to BC, and equal to half the difference of the sides.

Demon. By the last proposition, a circle described about o as a centre, with the distance oL, will pass through the points L, T, D, U;
Also by Prop. IX. TQ=QU, and QL is ⊥ to TU;
therefore the ∠s LUT, LTU, standing on=arches, are=:
but, by ‖s, ∠LTU=∠BCT;

hence,

hence, $\angle LUT = \angle BCT$;
therefore, LU, BC, are parallel.

And becauſe the $\perp$ of an iſoſceles Δ biſects the baſe, UL will be $=$ LT $=$ AP (PG) $=$ half the diff. of AC, CB.

Q. E. D.

PROP. XII.

If DT be joined, the $\angle$s DTB, DCB, will be equal to each other.

Demon. The $\angle$s LDT, LUT, (LTU) ſtanding on the ſame arch will evidently be equal. And

DBT + DTB = LDT = TCB = TCD + DCB:
but, TCD = DBT (Prop. II.);
therefore, DTB = DCB.

Q. E. D.

Cor. 1. The $\angle$ LDT $= \frac{1}{2} \angle$ ACB; alſo, $\angle$TDC $= \angle$TBC, and the points T, C, B, D, are in a circle.

Cor. 2. LD $\times$ TU $=$ (TD + DU) $\times$ TU(LU); (DU being joined).

Cor. 3. If IR be joined, it will be $\parallel$ to TD.

Cor. 4. If DU be drawn; DT, DU, will be parallel to AE, BE, reſpectively.

Cor. 5. The Δs ACB, UDT, are equiangular; as alſo are the Δs ALU, CTD; and DBT, UEL.

(To be Continued.)

ARTICLE VII.

Demonſtration of Article XXXVIII. *Vol.* I.

By Mr. Thomas Keith, *the Propoſer.*

LET the figure (202, pl. 15.) be conſtructed, as in page 283, vol. 1. Imagine IP and IZ to be joined by elliptical arches; then will the angle IZP, or its ſupplemental angle IZ*p*, be the azimuth; we have, therefore, only the angle IZ*p* to meaſure.

With Q as a centre, and radius O*a*, deſcribe the arch *ara*, and produce IR to *r*. Join O*r*, then the plane angle *r*OI is equal to the ſpherical angle IZ*p* (*Crakelt's Tranſlation of Mauduit's Trigonometry, page* 112.)

Now, C*m*=O*r*=O*a*, by conſtruction, and *x*C =OI, becauſe RI is parallel to ON, and IO parallel to *x*C; therefore the triangles *x*C*m* and IO*r* are equal to each other (*Simpſon's Geometry*, I. 17.): hence the angle *x*C*m*, meaſured by the arch S*n*, is equal to the angle IO*r*, which meaſures the ſpherical angle IZ*p*.

Q. E. D.

The ſame by Mr. Lowry.

This method of conſtructing an azimuth is an evident conſequence of the orthographic projection of the ſphere on the plane of the meridian. For, let Z be the zenith, P the pole, A the object, *a*A*a* the parallel of altitude, *p*A*p* the parallel of declination, SDL the horizon, EQF the equinoctial, and let the reſt of the lines and arches be drawn as in the figure. Then ZP is the colatitude, ZA, or Z*a*, the coaltitude, and PA,

PA, or Pp, the co-declination; and by the nature of the orthographic projection, the interſection I of the diameters *aa*, *pp*, of the leſſer circles *aAa*, *pAp*, will be the projection of the point A, and the ellipſis deſcribed through the points Z, I, and meeting SCL in B, will be the repreſentation of the arch ZAD upon the plane ZNRS.

Join B*n*, then (*Walker on the Sphere*, B. IV. Prop. IV. Cor. 1.) IO (C*x*) : O*a* :: BC : SC;
but, by *conſ.* C*m* = O*a*, and C*n* = CS;
therefore, C*x* : C*m* :: BC : CN;
wherefore *n*B is parallel to *x*R.

Now, ſince Z is the pole of LDS, the arch DL is the meaſure of the angle DZL, and, becauſe L is the pole of the great circle ZCN, and B*n* is parallel to CN, and B is the projection of the point D, it follows, that *n*L is equal to DL; therefore, the arch *n*L is the meaſure of the angle AZP, or the azimuth from the North, and, conſequently, the arch *n*S (the ſupplement of *n*L) is the azimuth from the South. Whence, Mr. *Keith*'s conſtruction is evident.

Q. E. D.

ARTICLE VIII.

Demonſtrations of Mr. EMERSON'S FORMS *of* FLUENTS, *by the Rev.* L. Evans, *Froxfield.*

THE finding a fluent from a given fluxion, is certainly the moſt difficult and moſt uſeful problem in the whole doctrine of fluxions. There are direct and general rules for finding the fluxion of any quantity, be it ever ſo compounded;

pounded;—but no direct and general rules can b given which will enable us, in all cases, to fin a fluent from a given fluxion, even in some o the the most simple cases. We are frequentl obliged to have recourse to infinite series, and ar as frequently unable to determine the fluent at all The few direct rules that we know, are only of us in finding the fluents of those fluxions whose form shew them to have been derived from that fluent, and, by the comparison of fluents, other forms are again derived. By these means several learned authors have been at the trouble of calculating a great variety of general forms of fluents, and of disposing them in tables for the purpose of facilitating the finding of the fluents of such fluxions as correspond with any of the forms in the tables. The most extensive and complete tables that are known to me, are Mr. *Landen's*, in his *Appendix* to the 1st Vol. of MEMOIRS (part of which have been published in this Repository), and Mr. *Emerson's*, in his Treatise on Fluxions. Most beginners in fluxions have exclaimed against the obscurity of several parts of Mr. Emerson's book, but more particularly against his undemonstrated formulæ for fluents. To remove this difficulty, I have presented the method of their respective investigation.

FORM I. *demonstrated.*

This is evident, for $z^{\frac{n}{\cdot}}\div n$, put into fluxions, is $=z^{n-1}\dot{z}=\dot{\varphi}$, therefore, $\varphi=z^{\frac{n}{\cdot}}\div n$.

It may also be proved thus:

Assume $\varphi=mz^n$, then $\dot{\varphi}=nmz^{n-1}\dot{z}$; hence

z^{n-1}

$z^{n-1}\dot{z} = nmz^{n+1}\dot{z}$, and $m = \frac{1}{n}$; therefore, $\varphi = mz^{\frac{1}{n}} = z^{n \div n}$.

If $n = 0$, the given fluxion becomes $z^{-1}\dot{z}$, but this fluxion cannot be generated by z^0, it being a conſtant quantity, hence its fluent cannot be found by this form.

Form II. *demonſtrated.*

1ſt. To prove $\varphi = L \times \log. z$.*

In fig. 207, pl. 15, let DC, CT, be aſymptotes to the hyperbola BV, and draw VT, VS, parallel to CD, CT, reſpectively. Put CS $= a$, VS $= b$, DS $= x$, and BD $= y$; then, by the property of the hyperbola, $ab \div (a + x) = y$; therefore the fluxion of the area SDBV is $= ab\dot{x} \div (a + x) = \dot{z} \div z = z^{-1}\dot{z}$, aſſuming a and b each $= 1$, and $a + x = z$. But the area SDBV (when CS $= 1$) is $=$ the hyperbolic log. of CD, or z; therefore, φ or the fluent of $z^{-1}\dot{z} =$ the hyp. log. $z = L \times \log. z$.

2d. To prove $\varphi =$ area $\div R^2$ of the hyperbola between the aſymptotes, whoſe inſcribed parallelogram is any ſpace R^2, abſciſſa (taken on the aſymptote) Rz or z.

In $ab\dot{x} \div (a + x)$, the expreſſion for the fluxion of the area SDBV, put R^2 for ab, and Rz (or z) for $a + x$, and we have $R^2 z^{-1}\dot{z}$ for the fluxion of

* L multiplied by the common log. of any number, is $=$ the hyperbolic logarithm of that number.

the area SDBV; therefore φ or the fluent of $z^{-1}\dot{z}$, is $=$ area $\div R^2$.

FORM III. *demonstrated.*

This is evident, for $(\alpha+\beta z^n)^{\mu+1} \div (\mu+1)\beta n$ put into fluxions, is $=(\alpha+\beta z^n)^{\mu} z^{n-1}\dot{z}$ the given fluxion. It may be proved otherwise, thus:—Put $\alpha+\beta z^n = x$, then $z^n = (x-\alpha)\div\beta$; this in fluxions gives $z^{n-1}\dot{z} = \dot{x}\div n\beta$; now substituting x, and $\dot{x}\div n\beta$ for their equals in the given fluxion, and it becomes $x^{\mu}\dot{x}\div n\beta$, and the fluent found by form 1, is $x^{\mu+1}\div(\mu+1)n\beta$, hence, by restoring the value of x, φ will be $=(\alpha+\beta z^n)^{\mu+1}\div(\mu+1)n\beta$. When $\mu=-1$, the fluxion becomes $x^{-1}\dot{x}\div n\beta$, whose fluent cannot be found by this form, as is evident from form 1.

FORM IV. *demonstrated.*

1st. To prove $\varphi = (L\div n\beta)\times$ log. of $(\alpha+\beta z^n)$. Put $\alpha+\beta z^n = x$, then $\dot{x} = n\beta z^{n-1}\dot{z}$, and $x^{-1}\dot{x} = n\beta z^{n-1}\dot{z}\div(\alpha+\beta z^n)$, therefore, by form 2, the fluent of $n\beta z^{-1}\dot{z}\div(\alpha+\beta z^n) = L\times$ log. of $(\alpha+\beta z^n)$; and therefore,

therefore, φ, or the fluent of $z^{n-1}\dot{z} \div (\alpha + \beta z^n) = (\mathrm{L} \div n\beta) \times$ log. of $(\alpha + \beta z^n)$.

2d. To prove $\varphi =$ area $\div n\beta \mathrm{R}^2$ of an hyperbola between the afymptotes, whofe inferibed parallelogram is R^2, abfciffa $\alpha \div \beta + z^n$.

Aſſume $\alpha \div \beta + z^n = x$, then $\dot{x} = nz^{n-1}\dot{z}$, and $x^{-1}\dot{x} = nz^{n-1} \div (\alpha \div \beta + z^n)$; therefore, by form 2, the fluent of $nz^{n-1}\dot{z} \div (\alpha \div \beta + z^n)$, or its equal, $\beta nz^{n-1}\dot{z} \div (\alpha + \beta z^n)$, is $=$ area $\div \mathrm{R}^2$ of an hyperbola, &c. therefore φ, or the fluent of $z^{n-1}\dot{z} \div (\alpha + \beta z^n)$ is $=$ area $\div n\beta \mathrm{R}^2$ of an hyperbola between the afymptotes, &c.

Form V. *demonſtrated.*

1ſt. To prove $\varphi = (2\mathrm{N} \div n\sqrt{\alpha\beta}) \times$ degrees in that arch of a circle whoſe rad. is 1 and nat. tangent $\sqrt{\beta z^n} \div \alpha$.

If r be the rad. and t the tangent of an arch, the fluxion of that arch will be $r^2\dot{t} \div (r^2 + t^2)$; and when $r^2 = \alpha$, and $t^2 = \beta z^n$, $\dot{t}$ will be $= \frac{1}{2} n \beta^{\frac{1}{2}} z^{\frac{1}{2}n-1}\dot{z}$; therefore, by ſubſtitution, the fluxion of the arch becomes $\frac{1}{2} n \beta^{\frac{1}{2}} z^{\frac{1}{2}n-1}\dot{z} \div (\alpha + \beta z^n)$, the fluent of which is evidently the arch whoſe rad. is $\sqrt{\alpha}$, and nat.

[illegible] ; therefore [illegible], as the [illegible] of $z^{\frac{1}{n}-1} \div (a+bz^n)$ is $= (a \div na\sqrt{\beta}) \times$ arch[illegible] whole rad. is $\sqrt{a}$, and tan. $= \sqrt{\beta z^n} = (\sqrt{\beta} \div a\sqrt{a\beta}) \times$ degrees in the arch whole radius is $\sqrt{a}$, and $\sqrt{\beta z^n} = (2N \div a\sqrt{a\beta}) \times$ degrees in the arch whole rad. is 1, and tan. $\sqrt{\beta z^n} \div a$.

2d. To prove $\varphi =$ arch $\div \frac{1}{2}aR\sqrt{a\beta}$ of a circle whose rad. is any line R, and tangent $R\sqrt{\beta z^n} \div a$. It has been shewn that $\varphi = (2N \div a\sqrt{a\beta}) \times$ deg. in the arch whose rad. is a, and tangent $\sqrt{\beta z^n} \div a$, and therefore $= (2NR \div aR\sqrt{a\beta}) \times$ degrees of (arch $\div \frac{1}{2}aR\sqrt{a\beta}$) of a circle whose rad. is a, and tan. $R\sqrt{\beta z^n} \div a$.*

3d. To prove $\varphi = (4$ fectors $\div aR^2\sqrt{a\beta})$ of the circle whose rad. is R, and tan. $R\sqrt{\beta z^n} \div a$.

Now, arch $\times R = 2$ fectors, or arch $= (4$ fectors $\div 2R)$, this fubftituted for the arch in the laft cafe, gives $\varphi = (4$ fectors $\div aR^2\sqrt{a\beta})$ of the circle whofe rad. is R, and tan. $R\sqrt{\beta z^n} \div a$.

Form VI. *demonftrated.*

1ft. To prove $\varphi = (1 \div a\sqrt{a\beta}) \times$ log. of $(\sqrt{a} + \sqrt{\beta z^n}) \div (\sqrt{a} - \sqrt{\beta z^n})$. Put $a = b^2$, and $\beta z^n = x^2$,

* The length of any arch of a circle whofe rad. is R, is $= N \times R \times$ degrees in that arch.

If $\sqrt{a} : \sqrt{\beta z^n} :: 1 : \sqrt{\beta z^n} \div a$.

If $1 : \sqrt{\beta z^n} \div a :: R : R\sqrt{\beta z^n} \div a$.

then

then $z^{\frac{1}{2}n} = x \div \beta^{\frac{1}{2}}$, and $\frac{1}{2}nz^{\frac{1}{2}n-1}\dot{z} = \dot{x} \div \beta^{\frac{1}{2}}$, therefore $\dot{\varphi}$ becomes $2\dot{x} \div n\beta^{\frac{1}{2}}(b^2 - x^2) = 2b\dot{x} \div bn\beta^{\frac{1}{2}}(b^2 - x^2) = (1 \div nb\beta^{\frac{1}{2}}) \times ((\dot{x} \div \overline{b+x}) - (-\dot{x} \div \overline{b-x}))$. Now the fluent of $\dot{x} \div (b+x)$ is $= L \times$ log. $(b+x)$; and the fluent of $-\dot{x} \div (b-x)$ is $= L \times$ log. $(b-x)$; therefore $\varphi = (L \div nb\beta^{\frac{1}{2}}) \times$ (log. of $\overline{b+x}$ − log. of $\overline{b-x}$) $= (L \div nb\beta^{\frac{1}{2}}) \times$ log. of $(\overline{b+x} \div \overline{b-x}) =$ (by restoring the values of b and x) $(L \div n\sqrt{\alpha\beta}) \times$ log. of $(\sqrt{\alpha} + \sqrt{\beta z^n}) \div (\sqrt{\alpha} - \sqrt{\beta z^n})$.

2d. φ is evidently $= (L \div n\sqrt{\alpha\beta}) \times$ log. of $(\sqrt{\beta z^n} + \sqrt{\alpha}) \div (\sqrt{\beta z^n} - \sqrt{\alpha})$, by changing the signs of the quantities of the preceding case.

3d. φ is proved $= (L \div n\sqrt{\alpha\beta}) \times$ log. of $(\alpha + \beta z^n + 2\sqrt{\alpha\beta z^n}) \div (\alpha - \beta z^n)$, by multiplying the numerator and denominator in the 1st case by $\sqrt{\alpha} + \sqrt{\beta z^n}$.

4th. In the same way this case is proved by multiplying the numerator and denominator in the second case by $\sqrt{\alpha} - \sqrt{\beta z^n}$.

5th. φ is evidently $=$ (area $\div nR^2\sqrt{\alpha\beta}$) of the hyperbola between the asymptotes whose inscribed parallelogram is R^2, and abscissas terminating this area $\sqrt{\alpha} + \sqrt{\beta z^n}$ and $\sqrt{\alpha}$ & $\sqrt{\beta z^n}$.

6th. To prove $\varphi =$ (4 sectors $\div nR^2\sqrt{\alpha\beta}$) of a right-angled hyperbola whose semi-transverse is R, and tangent at the vertex $R\sqrt{\beta z^n} \div \alpha$.

Put

Put a for the semi-transverse and semi-conjugate and t for the tangent at the vertex: then by the property of the hyperbola $\frac{1}{2}a^2 \times \dot{t} \div (a^2 + t^2) =$ the fluxion of the sector. Now, substituting and $R\sqrt{\beta z^n} \div \alpha$ for a and t, then $\dot{t}$ becomes $\frac{1}{2}n R \sqrt{\beta \div \alpha} \times z^{\frac{1}{2}n-1}\dot{z}$, and the fluxion of the sector $= \frac{1}{4}R^2 \times (\frac{1}{2}n R\sqrt{\beta \div \alpha} \times z^{\frac{1}{2}n-1}\dot{z}) \div (R^2 + R^2 \times \beta z^n \div \alpha) = (\frac{1}{8}nR^2\sqrt{\beta \div \alpha} \times z^{\frac{1}{2}n-1}\dot{z}) \div 4(\alpha + \beta z^n)$; therefore the fluent of $z^{\frac{1}{2}n-1}\dot{z} \div (\alpha + \beta z^n) = (\frac{1}{4}$ sector $\div \frac{1}{2}nR\sqrt{\alpha\beta})$ of a right-angled hyperbola, &c.

ARTICLE IX.

Answers to the Mathematical Questions proposed in ARTICLE XLIII. No. IV. *Vol.* I.

I. QUESTION 69, *answered by* Tamerlane.

LET EABCD in fig. 203, pl. 15, represent the bastion, in which BC = BA (the faces of the bastion) is given = 60 yards, and the angles (of the shoulders) EAB, BCD, and the saliant angle ABC each = 110°, to find (the flanks) CD = AE when (the gorge) ED = 80 yards.

Here the $\angle BCA = BAC = \frac{1}{2}(180° - 110°) = 35°$; hence, s.BAC : BC :: s.ABC : AC = 98·29825 yds. Then the $\angle dCD = 110° - 35° = 75°$, $CDd = 90° - 75° = 15°$, and $DC = \frac{1}{2}(AC - ED) = 9·149125$; therefore s.$dDC$: dC :: s.DdC (rad.) : DC = AE = 35·3495 yards the length of each flank as required.

Other ingenious answers were received from Messrs. Gregory, Simpson, Swale *and* Thornoby.

II.

ESTION 70, answered by Mr. R. Simpſon, *Croxdale, Durham.*

. 47, pl. 3, take DC = 30 feet, (not 50 as and make DE perpendicular thereto and et; join EC and draw DB, EB making the B, CEB = 40° 30′ and 50° 30′ reſpectively. , the interſection of DB, EB, demit, on DC d, the ⊥ BA; then AC will be the breadth iver and AB the height of the ſpire.

lation. In the right ∠d △ CDE there is CD, DE to find the ∠ DEC, which added ∠ BEC gives the ∠ DEB. Again the ∠ aken from a right ∠ leaves the ∠ BDE. n the △ DEB there is given all the ∠s and DE, to find DB. Whence in the right ABD we have given the ſide BD and the to find AB = 707·622 feet the height of e, and AC (= DA—DC) = 792·6647 feet adth of the river.

eſtion was alſo ingeniouſly anſwered by Meſſrs. Lowry, Swale, Tamerlane *and* Thornoby.

QUESTION 71, anſwered by Mr. Simpſon.

half the given area be repreſented by *a*, ciſs by *x*, and the ordinate by *y*. Then by 53 (2d. Ed.) of Dr. *Hutton's* Menſuration, $+y^2)$ = the length of the ſingle curve; there- the queſtion $\sqrt{(\frac{4}{3}x^2+y^2)}+y$ (= half the ter) muſt be a minimum, and its fluxion = 0, $(\frac{4}{3}x\dot{x}+y\dot{y})\div(\sqrt{(\frac{4}{3}x^2+y^2)})+\dot{y}=0$. But parabola is = $\frac{2}{3}$ of its circumſcribing paral- m, therefore $x=3a\div 2y, x^2=9a^2\div 4y^2$ and $3a\dot{y}\div 2y^2$; hence, ſubſtituting for x, x^2 and reſpective equals, and dividing by $\dot{y}$, we get $\div y^3\sqrt{(3a^2\div y^2+y^2)}+y\div\sqrt{(3a^2\div y^2+y^2)}$ 0. This equation, by proper reduction be- $y=\sqrt{a}=49·1934$ yards. Hence $x=73$ ·7802;

·7802; the length of the single curve = 98·37694. consequently, the whole perimeter (= the double curve + the double ordinate) = 295·1406 yards, which at 4*d*. per yard, comes to 4*l*. 18*s*. 4½*d*. And from the above dimenſions the parabolic field may be eaſily deſcribed.

Other anſwers were alſo given by Meſſrs. Lowry, Swale, Tamerlane *and* Thornoby.

IV. QUESTION 72, anſwered by Mr. Johnſton, *the Propoſer.*

Let x, y and z be the numbers required; x being the greateſt.

Aſſume 1. $x - y = m^2$,
2. $x - z = n^2$,
then 3. $y - z = n^2 - m^2$.

By harmonic proportion we have $x : z :: x - y : y - z$; or, $xz = xy - xz + zy$.

Again, by multiplying the firſt and third equations together, $xy - xz + yz = m^2n^2 - m^4 + y^2$; therefore $xz = m^2n^2 - m^4 + y^2$; add four times this equation to the ſquare of the ſecond, and we have $\overline{x+z}^2 = 4m^2n^2 - 4m^4 + n^4 + 4y^2$. Hence $2x = \sqrt{(4m^2n^2 - 4m^4 + n^4 + 4y^2)} + n^2$; but $2x = 2m^2 + 2y$; therefore $2m^2 + 2y = \sqrt{(4m^2n^2 - 4m^4 + n^4 + 4y^2)} + n^2$. Whence $y = 2m^2(n^2 - m^2) \div (2m^2 - n^2)$, where $n^2 - m^2$ muſt be a ſquare number, and $2m^2$ greater than n^2.

Suppoſe $n = 5$, and $m = 4$, then $x = 400 \div 7$, $y = 288 \div 7$ and $z = 225 \div 7$, or multiplying by 7, the numbers become 2800, 2016, and 1575.

Meſſrs. Gregory *and* Lowry *ſolved it in very nearly the ſame manner, and brought out the ſame reſult.*

V. QUESTION 73, anſwered by Mr. M. A. Harriſon.

It is well known that the quantity of any fluid overflowing the containing veſſel when full, in conſequence of the immerſion of any ſolid, is univerſally equal

equal to the number of cubic inches in the part or whole of the ſolid ſo immerſed; and conſequently in the preſent caſe, the immerſed part of the cylinder muſt be a maximum. Now this being admitted, let ACDBEF (fig. 204, pl. 15.) repreſent a ſection of the ſphere, of which the given veſſel is one-half; draw the diameter AB repreſenting the water's ſurface, and parallel thereto apply the chord CD equal the given length of the cylinder; perpendicular to CD draw DE meeting AB in G and the periphery in E; ſo ſhall DE be the required diameter of the cylinder.

For, the cylinder whoſe length is CD and diameter DE, is evidently the greateſt that can be inſcribed in the ſame ſphere, and of the ſame altitude CD: but DG = GE; hence the part immerſed is equal to half the whole cylinder, and conſequently the greateſt poſſible; for it is well known that *halves* are as their *wholes*.

Calculation. By the circle $AG \cdot GB = GD^2$ ($= GE^2$); then $DE = 2\sqrt{AG \cdot GB} = 2\sqrt{40 \times 8} = 16\sqrt{5}$ = the diameter required.

The ſame by Mr. Johnſton, *Birmingham.*

Since the length of the leaden cylinder is given, it is obvious that the part immerſed will be a maximum (or the greater quantity of water will overflow) when the area of the end of that part of the cylinder immerſed in the water is the greateſt poſſible. This will evidently happen when half the cylinder is immerſed in the fluid, for then the edges of the ends of the cylinder touch the veſſel quite round. Now AB = 48, CD = 32; hence AI = 8 and IB = 40; therefore CI $= \sqrt{AI \cdot IB} = 8\sqrt{5}$ = the radius of the cylinder's baſe.

True ſolutions were alſo received from Meſſrs. Evans, Gregory, Lee, Lowry, Simpſon *and* Swale.

To VI. QUESTION 74, we have received no ſatisfactory anſwer. If any of our correſpondents will favour us with a ſolution it ſhall be inſerted in a future Number. The Propoſer's ſolution is alſo requeſted.

VII. QUESTION 75, answered by Mr. William Peacock, *Land Surveyor, at Birmingham.*

Put $g = 16\frac{1}{12}$ feet, then $2\sqrt{ga}$ = the velocity acquired by the ball in descending the height a, and were the ball perfectly elastic, it would ascend again to the same height. Let its ratio to perfect elasticity be as $1 : m$, then it is evident that the velocity on reflection will be diminished in the ratio of $1 : m$, and will be equal $2\sqrt{ga} \div m$; with this velocity the ball will ascend to a certain height, and descend again with the same velocity. This velocity will be again diminished in the same ratio of $1 : m$, and will be $= 2\sqrt{ga} \div m^2$. In the same way it appears that the velocity at the third reflection will be $= 2\sqrt{ga} \div m^3$. Now, the distance is = the square of the velocity divided by $4g$; therefore the distance is $= a \div m^6$; but, this is per question $= \frac{3}{4}a$, therefore $m^6 = \frac{4}{3}$, and $m = \sqrt[6]{\frac{4}{3}} = 1{\cdot}07$, &c.

Otherwise by Mr. James Lee, *London.*

Instead of the ball falling upon the plane, we will suppose it to impinge directly upon an equal and perfectly non-elastic ball, at the given distance a: since the result will not, by such a supposition, be any way affected. Now put $16\frac{1}{12} = c$, and the ratio to be determined as x to 1. Then, the velocity of the first impact is evidently $= 2\sqrt{ac}$; and from the laws of collision, the velocity of the percutient body after the third stroke or impact is $= (1 + 2x)(1 - x)^2 \sqrt{ac} \div 8(1 + x)$.

But this velocity is manifestly the same as would be acquired by a heavy body in descending a space $= \frac{3}{4}a$; therefore $(1+2x)(1-x)^2\sqrt{ac} = 16(1+x)\sqrt{\frac{2}{3}ac}$; from which equation we obtain $x = {\cdot}93$, and consequently the required ratio is that of 93 to 100.

Ingenious solutions to this Question were also given by Messrs. Lowry, Simpson, Swale *and* Thornoby.

VIII.

VIII. QUESTION 76, answered by Mr. O. G. Gregory, Teacher of the Mathematics, Cambridge.

Either the propoſer of this queſtion has been inattentive to the manner of wording it; or, through haſte in tranſcribing he has omitted ſomething; or there is a preſs errour; for as it now ſtands, it will admit of various anſwers, as perſons may take up various ſuppoſitions. The ſolid which is to be cemented to the fruſtum when ſuſpended, may be fixed either above it, below it, or at the leſs end, ſo that the axes of the two ſolids may form one right line;—perhaps the latter may be the meaning of the propoſer. Yet, even then, ſome will imagine that the part to be added is not a ſimilar ſolid, but the part neceſſary to complete the cone: whilſt others may ſuppoſe the part added muſt be a ſimilar fruſtum of a cone. I will give a ſolution on each ſuppoſition.

And firſt, let it be ſuppoſed that the part to be added is to complete the cone. In fig. 206, pl. 15, let VBC repreſent a ſection of the whole cone from the vertex to the diameter of its baſe; and ABCD the fruſtum given, in which AD $=\frac{1}{4}$BC. It is evident, that when the cone is completed VK $=$ KL: therefore as C the point in the axis to be ſuſpended by the chain is in the midway between K and L, VC $=\frac{1}{4}$VL. But it is known that the centre of gravity of a homogeneous cone is at the diſtance of $\frac{1}{4}$ths of its axis from the vertex, and of courſe, VAD, the part to be cemented, muſt be of the ſame ſpecific gravity as the given fruſtum.

In the ſecond place, let it be imagined that the part to be added is a ſimilar fruſtum; here a difficulty ariſes, as none of its dimenſions are given. I will ſuppoſe that its greater baſe is equal to the leſs baſe of the given fruſtum: then will AD $=$ 2, EF $=$ 1 and MK $=$ 4. Now the queſtion amounts to this: given the dimenſions of three conic fruſtums PBIC,

APDI, and EAFD, cemented together as per figure and the ſpecific gravities of the two larger equal to each other, to find the ſpecific gravity of the ſmaller, ſo that the whole compound body EBCF ſhall remain in equilibrio when that part of the axis is ſuſpended which is the centre of the circle that divides the two larger fruſtums.

Suppoſe f, g and G to be the centres of gravity of the three fruſtums, and let the weight of EADF be denoted by F, that of APID by w, and that of PBCI by W. Then, becauſe the weight of every body may be conſidered as acting in its centre of gravity, we ſhall, from the nature of the common centre of gravity of a ſyſtem of bodies, have $CG \times W = Cf \times F + Cg \times w$. The ſolidity of the three fruſtums are, by the rules of menſuration, found 7·3304, 19·8968, and 38·7464 inches reſpectively: and by means of the rule for determining the centre of gravity of bodies, we find $fC = 5\frac{4}{7}$, $gC = 1\frac{1}{19}\frac{4}{9}$, and $GC = 2\frac{7}{37}$. If the ſpecific gravities of the two larger fruſtums be denoted by 1, their weights will be expreſſed by their ſolidities. And as the weight F of the ſmaller fruſtum, is equal to the product of its ſolidity S into its ſpecific gravity D, we ſhall, by ſubſtituting SD for F in the above equation, after proper reduction, get $D = (W \times GC - w \times gC) \div S \times fC = (38{\cdot}7464 \times 2\frac{7}{37} - 19{\cdot}8968 \times 1\frac{1}{19}\frac{4}{9}) \div (7{\cdot}3304 \times 5\frac{4}{7}) = 1{\cdot}23076$ the ſpecific gravity of the part to be added, that of oak being 1. But when oak is compared with water as a ſtandard, its ſpecific gravity is 925, water being 1000, and then the ſpecific gravity of the piece to be cemented on is 1138, being nearly the ſame as a mixture of equal portions of roſin and pitch.

N. B. The rule which I make uſe of to find the centre of gravity of a conic fruſtum, is not a new one; but, as I believe it is not generally known, I think it may deſerve a place in the Repoſitory.

Put

Put D = greater diameter of the fruſtum, d = leſs diameter, h = half difference of the diameters, and L = length of the fruſtum: then $L \times (6h^2 + 6hd + 3d^2) \div (8h^2 + 12hd + 6d^2)$ = diſtance of the centre of gravity from the leſs baſe, meaſuring on the axis.

Other ingenious ſolutions were alſo received from Meſſrs. Harriſon, Lee, Lowry *and* Swale.

IX. QUESTION 77, anſwered by Mr. J. Harris, *Land-Surveyor and Teacher of the Mathematics, at Caermarthen.*

Put x for the nat. coſine of the declination, then will $\sqrt{1-x^2}$ = its ſine; $\sqrt{\frac{1}{2}(1-x)}$ = ſine of the alt. at 6. Whence, by ſpherics $\sqrt{1-x^2} : 1 :: \sqrt{\frac{1}{2}(1-x)} : \sqrt{1 \div (2+2x)}$ = ſine of latitude; and $\sqrt{1 - 1 \div (2+2x)}$ = its coſine. Now, the ſine of the ſun's meridian alt. = ſine of (the declination + the complement of the latitude) = coſ. dec. × coſ. lat. + ſine dec. × ſine latitude; and that of the midnight depreſſion = ſine of (the complement of the latitude — the dec.) = coſ. dec. × coſ. lat. — ſine dec. × ſine lat. and their difference is = 2 × ſine dec. × ſine lat. that is $= 2\sqrt{1-x^2} \times \sqrt{1 \div (2+2x)} = \sqrt{2-2x}$, which by the queſtion muſt be = ſine of half the latitude; hence this equation, $\sqrt{2-2x} = \sqrt{\frac{1}{2}\left(1 - \sqrt{1 - 1 \div (2+2x)}\right)}$, which being reduced gives $x^3 - \frac{1}{2}x^2 - x + \frac{7}{32} = 0$. Solved $x = .965879$ = nat. coſine of 15° 0′ 37″ = ſun's declination; and the latitude of the place is 30° 17′ 10″.

The ſame by the Rev. Mr. L. Evans.

Let $x =$ ſine alt. at 6, then will $\sqrt{1-x^2} =$ its coſine; $2x\sqrt{1-x^2} =$ ſine of declination, and $1-2x^2 =$ its coſine. Now by ſpherics $2x\sqrt{1-x^2} : 1 :: x : 1 \div 2\sqrt{1-x^2} =$ ſine of latitude: and $\sqrt{(3-4x^2) \div (4-4x^2)} =$ its coſine.

Then $(1-2x^2)\cdot\sqrt{\frac{3-4x^2}{4-4x^2}} + \frac{2x\sqrt{1-x^2}}{2\sqrt{1-x^2}} =$ ſine of meri. alt.

and $(1-2x^2)\cdot\sqrt{\frac{3-4x^2}{4-4x^2}} - \frac{2x\sqrt{1-x^2}}{2\sqrt{1-x^2}} =$ ſine of depreſſ.

The difference of theſe is $2x$, which by the queſtion muſt be $=$ to the ſine of half the latitude, that is $2x = \sqrt{\frac{1}{2}(1-\sqrt{(3-4x^2) \div (4-4x^2)})}$. This equation expanded, &c. $x^6 - \frac{5}{4}x^4 + \frac{1}{4}x^2 - \frac{1}{256} = 0$. Hence $x = \cdot 130655$ the nat. ſine of $7^\circ\ 30'\ 18''\cdot 5$; $2x\sqrt{1-x^2} = 15^\circ\ 0'\ 37'' =$ ſun's declination, the time anſwering to which is the 30th of April, or the 11th of Auguſt, and $1 \div 2\sqrt{1-x^2} = 30^\circ\ 17'\ 10''\cdot 5$ the latitude.

This Queſtion was alſo anſwered by Meſſrs. Lowry, Simpſon *and* Swale.

X. QUESTION 78, anſwered by Mr. Swale, Leeds.

Let AGP (fig. 208, pl. 15,) repreſent the cylindrical monument inclined to the horizon BP, as by the queſtion; take PG $= \frac{2}{3}$ PA, and with the centre P and radius PG deſcribe the quadrantal arch BGC: then from known principles, the time of the

the cylinder's deſcent will be equal that in which a heavy body would deſcribe the circular arch GB. To determine which, draw GH parallel to CP; put $5'' = a$; $3{\cdot}1416 = 4n$, $16\frac{1}{12} = e$, coſine of $70° = m$; and PG $= x$. Then the time in which a heavy body would gravitate thro' the diameter $= \sqrt{2x \div c}$, and it has been determined that the times of deſcribing the diameter and any arch GB whoſe coſine is $= m$, to radius 1; is as $\frac{1}{n}$ to $1 + \frac{1-m}{8} + \frac{(1-m)^2 \times 9}{64 \times 4} + \frac{(1-m)^3 \times 225}{6 \times 36 \times 8}$ &c. Hence, we ſhall have $\frac{1}{n} : \frac{1-m}{8} + \frac{(1-m)^2 \times 9}{64 \times 4}$ &c. $:: \sqrt{\frac{2x}{c}} : a$; from which $x = a^2c \div 2n^2 \times \left(1 + \frac{1-m}{8} + \frac{(1-m)^2 \times 9}{64 \times 4} + \text{&c.}\right)^2$ $= 277{\cdot}8$ feet, then AP $= \frac{3x}{2} = 416{\cdot}7$ feet $= 138{\cdot}9$ yards, the height of the monument.

This Queſtion was alſo anſwered by Mr. Lowry.

XI. QUESTION 79, anſwered by Mr. Louis Hill.

The lines being drawn as directed in the queſtion (fig. 209, pl. 15.) through the points A and B draw AP, BR, parallel to DE, meeting EF in P and R; draw CE, on which demit the ⊥s LG, FH, and SQ, and let SK be produced (if neceſſary) to meet CE in T. Draw IEO ⊥ to DE, and CMN to CE, meeting DE in M, and EF in N.

Put a = RB, c = AP, b = ER, m = EN, n = NC, r = EC, p = EM, q = MC, z = EF, x = CQ, and

y=SQ. Then by ſim. $\triangle$s, LG=$(q \div p)$ LE. And ſince the points A, B, are equidiſtant from DE, OE will be = IE, and PE=ER; therefore by ſim. $\triangle$s, LE : EF :: BR : FR=FE+ER=$z+b$, or LE=$za \div (z+b)$; therefore LG=$qza \div p(z+b)$, EG=$rza \div p(z+b)$, and CG=EG—CE =$rza \div p(z+b) - r = (rza - rp(z+b)) \div p(z+b.)$

Again, the $\triangle$s CLG, TSQ, being ſimilar, it will be as LG : CG :: SQ:QT=$(ryza - rpy(z+b)) \div qza$; therefore x=CT—QT=CT—$(ryza - rpy(z+b)) \div qza$.

But, by ſim. $\triangle$s LE : EF :: BR : FE+ER,
and EF : EK :: ER—EF : AP;
ther. com. LE:EK::BR×(ER-EF):AP×(FE+ER);
but LE : EK :: EC : ET;
theref. EC:ET::BR×(ER—EF):AP×(FE+ER);

hence, $ET = \frac{rc(b+z)}{a(b-z)}$ and $CT = r - \frac{rc(b+z)}{a(b-z)}$;

theref. $x = r - \frac{rc(b+z)}{a(b-z)} - \frac{ryza - ryp(b+z)}{qza}$.

Now FH=$(n \div m) \times$EF, EH=$(r \div m) \times$EF;
hence, CH=$r - (r \div m) \times EF = r(m-z) \div m$,
And by ſim. $\triangle$s, FH : HC :: SQ : QC,
or, $(n \div m)z : r(m-z) \div m :: y : x$;
therefore $z = ymr \div (nx+yr)$. This value of z being ſubſtituted in the value of x found above,

gives $x = r - \frac{rc}{a} \times \frac{bnx+byr+ymr}{bnx+byr-ymr} - \frac{ryam+p(bnx+byr+ymr)}{qm}$

the equation required.

And thus the anſwer is given by Meſſrs. Lowry, Swale, *and* Thornoby.

XII.

XII. QUESTION 80, *answered by Mr.* William Adamſon, *of Lower Belgrave Place, Pimlico, London, late Teacher of the Mathematics at Bridlington, in Yorkſhire.*

1. Having drawn a circle with the chord of 60°, make the arch AB (fig. 210, pl. 15,) equal to the time between the obſervations in degrees, and draw the diameter G*d*, parallel to a line paſſing through A B.

2. Make de = the ſun's declination; and with the coſine C*x*, and one foot in the centre C, draw the ſemicircle xba.—From the points A and B towards the centre C, draw Aa, Bb to the inner circle, and through theſe two points draw a right line.

3. With the chord of the firſt co-altitude and one foot in a, draw an arch, and with the chord of the ſecond co-altitude and one foot in b, croſs that arch in D—draw DEF perpendicular to the diameter, and ſet the ſine of the declination xe towards the centre C from G to H.

4. With the radius ED and one foot in E, draw an arch, and with radius FG and foot in H, croſs that arch in p, and draw pP parallel to the diameter. Then CPm being drawn, will repreſent the meridian of the place, and CP the coſine of the latitude.

Scholium. When the obſervations are one before, and the other afternoon; the arch Am turned into time, will ſhew how long before noon the firſt obſervation was taken; and, in like manner the arch Bm will ſhew the time of the ſecond obſervation afternoon; and conſequently, when both obſervations are before noon, the point m will fall on the contrary ſide of B, and when they are taken afternoon, on the contrary ſide of A.

Note alſo, that when the ſun's declination and latitude of the place are of contrary names, that is, one north and the other ſouth, the points p and H muſt fall on the ſame ſide of the line DF, as in this

this example; but when the latitude and declination are of different names, they muſt be taken on contrary ſides of DF.

The ſame by Mr. John Dawes, *Surgeon.*

Conceive a figure in which Z repreſents the zenith, P the pole, A and B the ſun's places at the times of obſervation, Then in the iſoſceles triangle ABP there is given AP = BP = the co-declination, and the angle APB = the elapſed time; therefore the triangle ABP may be projected by art. 211, Crakelt's Trigonometry; and then AB becomes known. Then in the triangle AZB the three ſides are given (AZ, BZ being = the co-altitudes) and the triangle AZB may be projected without drawing ellipſes by *art.* 209, ibid.

A true and ingenious ſolution was alſo given by Mr. Lowry.

XIII. QUESTION 81, *anſwered by Mr.* Gregor

In fig. 211, pl. 15, let A and B repreſent the ſituations of the tacks, placed on a line making an angle (BAH) of 45° with the horizontal line AH, and let C be the place of the ring at reſt. Then (ſuppoſing the ſtring without weight) by the nature of forces, the angles ACD and BCF are equal when the ring is at reſt; whence this pleaſing queſtion may be ſolved by the following algebraical proceſs. Put AC = y, DC = x, DF = AH = BH = AB × nat. ſine of BAH = $3\sqrt{\frac{1}{2}} = b$, and AC + CB = 5 (per queſtion) = c. Then by Trigonometry, as $y : 1 :: x : x \div y =$ ſin. DAC; and as $c - y : 1 :: b - x : (b - x) \div (c - y) =$ ſin. CBF but DAC and CBF are equal, becauſe their complents are; therefore $x \div y = (b - x) \div (c - y)$, hence $cx = by$, and $y : x :: c : b$ the ratio of AC : CD: therefore the nat. ſine of CAD = CBF = $b \div c$ and their coſine is $= \sqrt{1 - b^2 \div c^2}$. Again by Trig

metry, fine DAC : DC : fine ACD : AD
$b) \times \sqrt{1-b^2+c^2}$: but $(c \div b) \times \sqrt{1-b^2 \div c^2}$
374, let this be denoted by d, and then
:: alſo ſin. CBF : CB :: ſin. BCF : BF $=$
$(c \div b) \times \sqrt{1-b^2+c^2} = (b-x)d$. But it is
that BF—AD $=$ BH $=$ DF; hence this
, $(b-x)d - dx = b$, that is, $db - 2dx = b$, from
we readily get $x = (bd-b) \div 2d = \cdot 5637181$
and $b - x = 3\sqrt{\frac{1}{2}} - \cdot 5637181 = 1 \cdot 5576023$
And by ſim. Δs we have DC + CF :
B :: DC : AC :: CF : CB—hence AC and
1·327 and 3·673 reſpectively; thus the
of the ſtring is determined.

this queſtion may be anſwered without alge-
the following manner. The two tacks A
to which the ſtring is fixed will be the foci
ellipſe ICL which will be deſcribed by the
of the ring along the ſtring when kept tight.
g will reſt in equilibrio when it is at the loweſt
which it can arrive; which will be at the point
ne ellipſe, where the tangent GCF is parallel
horizon. Let E be the centre of the ellipſe,
lines EF and ED, be drawn from it to meet
ints F and D where, perpendiculars from the
and A, fall upon the tangent. Then by *Hut-
nits*, pages 25, 26, 27. ACD $=$ BCF, ED
$=$ EL the ſemitranſverſe; alſo ED parallel to
nd EF parallel to AC. Since AGD $=$ EA*a*
it is evident EAD $= 135°$: hence, as $\frac{1}{2}$(AC
$=$ EL $= 2 \cdot 5$, we have, ED$(2 \cdot 5)$: ſin. EAD
:: AE$(1 \cdot 5)$: ſin. ADE $= 25° 6' \frac{1}{4}$, whence
s found $= 19° 53' \frac{1}{4} =$ ABC alſo, as is obvious. If
parallel to GF, then *b*BC $=$ BCF : hence *b*BA
-ABC $= 64° 53' \frac{1}{4} =$ BCF $=$ ACD $=$ CA*a*;
ıre EA*a*$(45°)$ + CA*a* $= 109° 53' \frac{1}{4} =$ BAC.
$180° - (BAC + ABC) = 50° 12' \frac{1}{2} =$ ACB.
have now given each angle of the triangle
and the ſide AB $= 3$, to find the other two
ſides,

ſides, which are AC=1·328 and CB=3·672 : ag
ing nearly with the algebraical reſult, and would h
correſponded exactly, if the angle ADE, and th
depending upon it, had been determined with ſu
cient accuracy.

Or, when ADE was determined, we might h
proceeded in a different manner to arrive at the ſa
concluſion. Thus, ADE, and EAD being kno
AED is known alſo; theref. AD may be readily fou
AGD being =45°, AD=DG ; hence GA may
determined. As the triangles GAC and GEF
ſimilar, alſo GED and GBC, AC and BC may
eaſily diſcovered by the following proportio

GE : EF :: GA : AC;
GE : ED :: GB : BC*.

We cannot cloſe this ſolution without expreſ
our regret at the loſs of the ingenious propoſer of
queſtion, Mr. BUCHANAN; who has *paid the deb*
nature! By his death the Sciences are deprived
a ſteady advocate in their cauſe, and the Editor
various periodical publications, of an able cor
pondent.

The ſame anſwered by Mr. J. H. Swale, *Le*

Who, proceeding on the ſame principles, de
mines the point C in the following manner, prod
BC and AD 'till they meet in Q; then by reaſo
the =∠s, QC=AC, and ∠CAD=CQD. He
there is given AB=3, BQ=5, and ∠BAQ=1
to find ∠BQA=25°6′. Whence ∠BCA=50°1
CBA = 19° 54′, and BAC = 109° 54′. Co
quently BC = 3·67, and AC(CQ) = 1·33 f

Elegant Solutions to this Queſtion were alſo rece
from Meſſrs. Buchanan, Evans, Lee, Lowry
Simpſon.

* Or, ADE having been determined as before =HB*a*=25°
Then, ſince BH = 3√½, B*a* is found =2·346. But by re
of the equal angles and parallels, AC = *a*C; therefore 2A(
AC+CB—B*a* = 5—2·346 = 2·654, hence AC = 1·327
BC = 3·673. EDI

X

XIV. QUESTION 82, *anſwerd by Mr.* J. Lowry, *Officer of Exciſe, Birmingham.*

Conſ. Let O (fig. 212, pl. 15.) be the centre of the circle of which BMC is a ſegment; take AQ = 2AL—2AO, and thereon deſcribe the ſemicircle QRA. At C erect the ⊥CP, and let it meet the quadrant in P; in AD produced take AZ=CP, and draw ZR parallel to OAE meeting the ſemicircle QRA in R. Through R, parallel to AC, draw RSM to meet the ſegment in M, and through M draw the required radius AML.

Demon. Let OC, OM be joined.

Then by *Conſ.*(2AL—2AO—MN)·MN = (AQ —MN)·MN = QSA = RS^2 = CP^2 = AL^2—AC^2. Therefore AL^2—2AL·MN+MN^2 = AC^2—2OA·MN, or, $(AL - MN)^2 = AC^2 - 2OA \cdot MN$.

But by Simp. Geo. II. 10 $(AL-ML)^2 = AC^2 - 2OA \cdot MN$, that is, $AM^2 = AC^2 - 2OA \cdot MN$; therefore, $(AL - MN)^2 = (AL - ML)^2$, conſequently MN = ML. *Q. E. D.*

Remark 1. If the points C and D coincide, 2AL — 2AO = MN.

2. If C falls on the contrary ſide of D, the conſtruction will be effected in nearly the ſame manner.

The ſame anſwered by Mr. Swale.

Analyſis. Suppoſe the thing done, and O the centre of the circle of which the given ſegment is a part; in AO produced take AT=AE; join OM, OD, and demit upon BO, the ⊥ MS.

Then by Emerſon's Geo. II. 22. $OM^2 = OA^2 + AM^2 + 2OAS$: but OM=OB is given, and AM = AL — ML = AL—MN = AL—AS; hence, $OA^2 + AL^2 + AS^2 + 2OAS - 2LAS = OM^2$. Conſequently (2OT ∽ AS)·AS = OD^2 ∽ OM^2. Hence this conſtruction. Take any line TV=OT, and divide it in F, ſo that the rectangle TFV may

be $=OD^2 \sim OM^2$; then is FV$=$AS, and the point M is determined by drawing SM parallel to AD, as is evident from the analyſis.

XV. QUESTION 83, *anſwered by Mr.* Lowry.

Let APB (fig. 213, pl. 15.) be the primitive circle. Draw the right circle AOB, and lay off BC, BD, equal the given ſides reſpectively; about the point C as a pole, with diſtances CB, CD, deſcribe the leſſer circles BR, DP, the latter interſecting the primitive in P. Through P draw the great circle PR to touch the leſſer circle BR in R, and PCR will be the triangle to be projected.

For, the angle at R is a right one, CP$=$CD, and CR$=$CB, and therefore PR$=$DB.

Mr. *Robertſon, at page* 122 *of his Treatiſe on Mathematical Inſtruments*, has given a very ingenious and general conſtruction of this problem to the following effect.

On the right line *ed* (fig. 214, pl. 15.) touching the primitive circle at any point *b*, take *db*, *be* $=$ the tangents of the given ſides; make the $\angle$ *dba* $=$ the given one, and take *ba* $=$ *be*. From *d* and O, with diſtances *da*, O*e*, deſcribe arches croſſing in *x*; draw *d*DOF, *x*AOE, and from *d* and *x*, with diſtances *db* and *eb*, deſcribe arches croſſing in B. Through the points A, B; D, B deſcribe great circles, then is ADB the triangle required.

Since O*x* $=$ O*e*, therefore B*x* ($=$*be*) is a tangent to the circle ABE at B, alſo B*d* ($=$*bd*) is tangent to the circle at B; therefore BD and AB are equal the given ſides. Moreover the triangles B*dx*, *dba* having the three ſides equal, have alſo $\angle$ *d*B*x* $=$ *dba*; therefore $\angle$ DBA $=$ *dba* $=$ the given angle.

This queſtion was alſo anſwered by Mr. Swale.

XVI. QUESTION 84, *anſwered by Mr.* Harris.

Caſe 1. When the ſum of the greater ſide and the adjacent ſegment of the baſe is given.

From

From any point B (fig. 191, pl. 13.) in the indefinite right line AD, draw BE making the ∠CBE = comp. of ½ the given diff. of the ∠s at the baſe. Take BE = the given biſecting line, and BA = the given ſum. Join AE, and make the ∠OEA=OAE, and draw EO interſecting AD in O; then make the ∠BEC = BEO, and draw EC meeting AD in C; ſo ſhall OEC be the △ required.

For, by *Conſ.* the angle OEB = BEC. And (Prop. II. p.284) ∠EBC= comp. ½(∠B–∠A), alſo by *Conſ.* ∠OEA=∠A; theref. AO=OE; and theref. OB+OE=OB+AO=AB, the given ſum.

Caſe 2. Draw the line BE, &c. as in the 1ſt caſe, and take BD = the given difference. Join ED, and make the ∠DEO=ODE, and draw EO meeting AD in O; make the ∠BEC=BEO, and draw EC, ſo ſhall OEC be the △ required.

Elegant conſtructions were alſo received from Meſſrs. Lee, Lowry *and* Swale.

XVII. QUESTION 85, *anſwered by Mr.* Swale.

Conſ. Draw the great circles AB, BQ (fig. 73, pl. 4.) to contain the given vertical angle, and take BP, BQ each = half the given ſum of the ſides. Draw the perpendicular arches QD, PD, to interſect in D, and draw DC to make the ∠DCQ = the complement of half the given diff. of the ∠s at the baſe. Make AP = CQ, and through the points A and C deſcribe a great circle, and ABC will be the △ required, as is evident from Prop. XIV. (art. XI. Rep.) of *Lowry's Spherical Lucubrations*, and its corollaries.

Mr. *John Lowry*, after giving the ſolution exactly in the ſame way, ſolves the problem as follows, by conſtructing the ſupplemental △, in which there is given the baſe, the diff. of the ∠s at the baſe, and the difference of the ſides.

In fig. 73, pl. 4, let AC be taken = the given baſe, and the arches CD, AD be drawn to make the

∠s ACD, CAD each $=\frac{1}{2}$ the ſupplement of the given ſum of the ∠s at the baſe. Conſtruct the △ DCQ, ADP, right angled at Q and P, and having CQ, AP, each $=\frac{1}{2}$ the given diff. of the ſides; continue AP, CP 'till they meet in B, and ACB will be a triangle ſupplemental to the required one. For, Prop. XIV. art. XI. *Cor.* 1, ∠BAD+BCD=180° or, ∠BCA + BAC + 2ACD = 180°: but the given ſum of the ∠s = 180° — 2ACD; therefore the ∠BCA + BAC = the given ſum. Moreover, it is evident that PB is = BQ; therefore AB—BC=2QC=2AP= the given diff.

XVIII. QUESTION, 86, *anſwered by Mr.* Sanderſon, *London.*

If the △ ACB, fig. 215, pl. 15, be circumſcribed by the circle ACBE, and CE be the given line biſecting the given angle ACB; and from the point E, the line ED be drawn at right angles to AC: then $CD=\frac{CA+CB}{2}$, and $AD=\frac{AC-CB}{2}$; but CE, and the angle ACE, are given, therefore CD is given, and $\overline{CD+AD}^3$ $(CA^3)+\overline{CD-AD}^3$ (CB^3) $=c^3$, the given cube per queſtion. Or $2CD^3+6CD\times AD^2=c^3$; whence, $2CD : c :: c^2 : CD^2+3AD^2 :: 2CD\times c : c^2$; it remains to find AD. Make CF=c the ſide of the given cube; and CG a mean proportional between 2CD and c; CI a third proportional to CG and CF: then $CI^2=CD^2+3AD^2$, and $CI^2-CD^2=3AD^2$: or $\frac{CI+CD}{3}\times\overline{CI-CD}(ID)=AD^2$, whence half the difference of the ſides (AD) is a mean proportional between $\frac{CI+CD}{3}$ and ID.

Obſervation. The next queſtion may be conſtructed in the ſame manner, as it is reduceable to the ſum, and the ſum of the cubes as above.

The

The ſame anſwered by Mr. Swale.

Analyſis. Fig. 175, pl. 12. Suppoſe the problem ſolved, and ACB the required △, circumſcribed by the circle ACBE, the diameter of which biſects the baſe AB. Demit upon AC the ⊥ EP, and join EA, EB.

Since CE and the ∠ECP are given, CP will be given; hence AC+CB=2CP will be given.

Again, $AC^3+CB^3=(AC^2+CB^2-ACB)\times(AC+CB)$.

But $AC^2+CB^2=(AC+CB)^2-2ACB=4CP^2-2ACB$; therefore, $AC^3+CB^3=(4CP^2-3ACB)\times 2PC=M^3$, or, $4CP^2-3ACB=(M\div 2PC)\times M^2=Q^2$, taking $Q^2 : M^2 :: M : 2PC$. Therefore $3ACB=4PC^2-Q^2$; hence, the problem is reduced to this, viz. To divide the right line AB into two parts AC, CB, ſo that the rectangle ACB may be = to a given magnitude; which is a well-known problem.

This queſtion was alſo anſwered by Meſſrs. Lowry *and* Peletarius.

XIX. QUESTION 87, *anſwered by Mr.* Lowry.

Conſ. Let ABCD (fig. 216, pl. 15,) be the given ſquare, AC its diagonal, which biſect in O, and demit OQ ⊥ to AB; produce AB till $AR^2 :: M^2 :: M : OQ$ (M being = to the ſide of the given cube.) To CB produced apply AL=AR. On the diagonal take OD, ſo, that $6OD^2$ may be $=BL^3$, and D will be the point required.

Demon. Demit the perpendiculars DE, DF, DG, and DH.

By *Conſ.*	$6OD^2=BL^2=AL^2-AB^2$:
but	$AL^2=LR^2=M^3\div OQ$;
therefore	$6OD^2=M^3\div OQ-AB^2$:
but	AB is equal to 2OQ,
therefore	$6OD^2+4OQ^2=M^3\div OQ$,
or,	$4OQ^3+6OD^2\cdot OQ=M^3$.

But O is evidently the centre, and OQ the radius of a circle inſcribed in the given ſquare, therefore by Prop. XXIII. of Dr. *Stewart's General Theorems* (which will be demonſtrated in its proper place).

$DE^3+DF^3+DG^3+DH^3=4OQ^3+6OD^2\cdot OQ$,

therefore $DE^3+DF^3+DG^3+DH^3=M^3=$ the given cube.

The *locus* of the point D is evidently a circle, whoſe centre is O and radius OD; conſequently there will be four points in the diagonal of the given ſquare which will ſatisfy the conditions of the problem.

The ſum of the cubes will evidently be a maximum when D coincides with C, and a minimum when it coincides with O.

The ſame anſwered by Mr. Swale.

Analyſis. Let ABCD be the given ſquare, D the point required, and DE, DF, DG, DH, the perpendiculars.

Then $DF=BF=DG$, and $DH=CE=DE$;

theref. $DF^3+DE^3+DH^3+DG^3=2DF^3+2DE^3=2BE^3+2CE^3=M^3=$ a given cube.

But it appears, as in the laſt queſtion, that $(BC^2-3BEC)\times 2BC=2BE^3+2CE^3=M^3$;

theref. $BC^2-3BEC=(M\div 2BC)\times M^2=Q^2$,

taking $Q^2 : M^2 :: M : 2BC$;

hence $3BEC=BC^2-Q^2$;

therefore BC and the rectangle BEC are given to find BE, EC; the method of doing which is well known.

XX. Or PRIZE QUESTION 88, *anſwered by the propoſer, Mr.* William Wallace.

Let DC, DE, CE (fig. 217, pl. 15,) be three tangents to a parabola, a circle deſcribed about the triangle DCE ſhall paſs through F, the focus of the parabola.

Let

Let V be the vertex, and let FL be drawn perpendicular to LM the directrix; let FG be drawn perpendicular to DC any of the tangents, meeting the directrix in M, join VG.

From the nature of the parabola FG is equal to GM, and FV is equal to VL, therefore GV is parallel to ML, hence GV is a tangent to the curve at the vertex, and in like manner it will appear that if FH, FK be drawn perpendicular to the tangents DE, EC, the lines joining the vertex and the points H, K, alſo touch the curve at the vertex; hence a tangent at the vertex paſſes through the points G, H, K. Join FD, FC, FE. Becauſe FGC, FKC are right angles, the points F, G, C, K are in the circumference of a circle, hence the angle FCK is equal to FGK, that is, FCE is equal to FGH: but FGH is equal to FDH or FDE, for, the angles FHD, FGD being right angles, the points F, D, G, H are in the circumference of a circle, therefore FCE is equal to FDE; hence the points F, D, C, E are in the circumference of a circle, or if a circle be deſcribed about the triangle DCE, it will paſs through F, the focus of the parabola, as was to be demonſtrated.

Cor. 1. If CD, CE touch the curve at A and B, and if FA, FB be joined, the triangles AFC, CFB are ſimilar to the triangle DFE.

For if AM be joined; from the nature of the curve AM is parallel to FV, and the angle AFG is equal to AMG, or to GFV, now AGF, GVF are right angles, therefore the triangles AGF, GVF are ſimilar, and the angle FAG is equal to FGV, or FGH, that is, becauſe the points F, D, G, H are in a circle to FDG; now the angle FCG is equal to FEH, therefore the triangles FAC, FDE are ſimilar. In the ſame way it may be ſhewn that the triangle FCB is ſimilar to FDE.

Cor. 2. Let AC, BC, be two ſtraight lines given by poſition touching a given parabola, if any other tangent be drawn meeting them in D and E, and if FD, FE be joined, the triangle FDE is given in ſpecies. For it is ſimilar to each of the given triangles AFC, CFB.

Cor. 3. If AC, BC are two tangents to a parabola, and if a circle be deſcribed through the focus to touch BC, one of them at C, it ſhall paſs through A, the point where the other tangent meets the curve.

For it has been ſhewn that the angle FAC is equal to FCB.

Cor. 4. If AC, BC, DE are tangents to a parabola at the points A, B, N; theſe lines are ſimilarly divided at their interſections and the points of contact, that is, AD : DC = DN : NE = CE : EB.

For by *Cor.* 1, the triangles AFC, CFB, DFE are ſimilar, and if FN be joined, it will appear in like manner that the triangles AFD, DFN, CFE are ſimilar, and that NFE, EFB, DFC are ſimilar

similar, therefore AD : DF = DN : NF
and DF : DC = NF : NE,
hence AD : DC = DN : NE;
and in the same way it appears that DN : NE = CE : EB.

The proposition contained in this last corollary appears to h been first observed by Dr. *Halley*, in his Translation of the *Se Rationis* of *Apollonius*, and it may be found in most books on conic sections.

From the preceding proposition and its corollaries may be rived easy solutions to certain problems respecting the descript of parabolas that may pass through given points and touch strai lines given by position.

The same answered by Mr. Lowry.

Let AC, BC, DE, be three tangents to the parabola AVB, terfecting each other in C, D, and E, and let F be the focus, the vertex, and GVK a tangent to the parabola at V. Draw F FH, FK, which, by prop. 12, book III, *Emerson's* Conics, v be perpendicular to AC, DE, and BE respectively;

therefore the points F, H, E, K, are in a circle,
and so are also the points F, G, C, K;
therefore the ∠FCG or SCD=FKG=FEH or SEF;

but the ∠DSC=FSE; therefore the △s DSC, FSE, are e angular; therefore, the rect. DSE= rect. CSF; wherefore points C, D, E, F, are in a circle.

Mr. *Robert Wallace*, *Teacher of the Mathematics*, *at Newca upon-Tyne*, after giving a solution exactly the same as Mr. Low above, adds the following Coroliaries:

1. GY : YF :: CY : to YK, Y being the intersection of G CF.

2. GK·CF=GC·FK=GF·CK.

3. ∠KCW=KFG; and ∠FKC+FGC=GFK+GC

4. If there be given four tangents to a parabola, *viz.* DE, A BC, GK; to find the focus and describe the parabola.

It is evident, from the proposition, that the circles which circumscribe the △s DCE, GCK, will intersect each other in focus F; which being found, the parabola may be described the known methods.

5. If it be required to draw a tangent from a point A in a p bola; or from a point X without it.

Join FA, or FX, and draw GVK a tangent at the vert Upon FA, or FX, describe a semicircle intersecting GK in through which draw AGX, the tangent required.

6. If there be given three tangents AC, BC, GK, and ratio of GF : FK; to find the focus and describe the parabola

Because the points G, F, K, C, are in a circle, the ∠GFI

giv

given, and GK is given in magnitude, therefore the △GFK is given in magnitude and ſpecies; hence, the point F, which is the focus, becomes known, and the parabola may be deſcribed.

Ingenious ſolutions were received from Meſſrs. Lee, Sanderſon, *and* Swale.

The Medal for ſolving the Mathematical Prize Queſtion is decided in favour of Mr. William Wallace, *who will pleaſe to ſend for it to Mr. Glendinning's, by whom it will be delivered, free from expence, to any part in London.*

ARTICLE X.

MATHEMATICAL QUESTIONS,

(To be anſwered in Number VIII.)

I. QUESTION 110, *by Mr.* Newton Boſworth, *Teacher of the Mathematics at* Peterborough.

IF a cubic piece of fir ſink 6 inches in water, how far will it be immerſed in oil of turpentine, and what are its dimenſions, allowing the ſpecific gravities of water, oil of turpentine, and fir to be 1000,800, and 550, reſpectively?

II. QUESTION 111, *by Mr.* O. G. Gregory.

A neighbour of mine has a caſk in form of a conic fruſtum, the altitude of which is 28 inches, and the diameter of the baſes 18 and 14 inches reſpectively; in each end is a cork hole one inch in diameter. As this caſk is filled with excellent October, and my neighbour is fearful of loſing it by one of the corks flying out, he wiſhes to be informed which end of the veſſel ſhould be ſet downwards, that the liquor may run out moſt ſlowly if an accident ſhould happen; and in how much longer time the caſk would be emptied with one end at the bottom than with the other?

III. QUESTION 112, *by Mr.* Newton Boſworth.

There is a cylindric veſſel, kept conſtantly full of water, which ſtands upon a wall 9 feet high; and at two feet from the bottom of the veſſel there is a ſmall orifice through which the water ſpouts and falls into another veſſel on the level ground, at the diſtance of 14 feet from the wall. It is requeſted that ſome of the ingenious Correſpondents to the Mathematical Repoſitory will, from theſe data, determine the altitude of the veſſel?

IV.

XVI. QUESTION 125, *by Mr.* Francis, *Jun. of* Hungerford.

A globe 15 inches diameter being placed in water ſunk ſo deep as to conceal $\frac{2}{5}$ of its ſurface; what was its weight? and what will the diameter of a globe of copper, which being connected to it by a ſtring of no weight, ſunk it juſt beneath the ſurface of the water?

XVII. QUESTION 126, *by Mr.* Lowry.

Given the vertical angle, the radius of the inſcribed circle, and the ſum of the diſtances from the centre of that circle to the angular points of the triangle to conſtruct it.

XVIII. QUESTION 127, *by* Mydorgius.

If two polygons a, b, c, d, e, &c. and p, q, r, s, t, &c. be inſcribed in an ellipſis, and through the reſpective points a, b, c, d, e, &c. and p, q, r, s, t, &c. the tangents AB, BC, CD, DE, &c. and PQ, QR, RS, ST, &c. be drawn, and the $\angle abB=\angle cbC$, $\angle bcC=\angle dcC$; $\angle cdD=\angle edE$; &c. and $\angle pqQ=\angle rqR$, $\angle qrR=\angle srS$; $\angle rsS=\angle tsT$, &c. Then $ab+bc+cd+de+$&c. is $=pq+qr+rs+st+$&c. Required the demonſtration?

XIX. QUESTION 128, *by Mr.* Wm. Davis, London.

It is required to find to what height a building may be raiſed at any given latitude, before the ſtones are carried off its top, by reaſon of the centrifugal force overcoming the centripetal?

XX. QUESTION 129, *by Mr.* Lowry.

In a given circle to inſcribe a triangle, ſo that its ſides may paſs through three given points, not in the ſame ſtraight line.

XXI. PRIZE QUESTION 130, *by* Theodoſius.

If the ends of a thread of a given length be faſtened to two given points on the ſurface of a given ſphere; it is required to determine the nature of the curve made on its ſurface by moving a pencil round with the thread, keeping it always ſtretched cloſe to the ſurface of the ſphere?

ARTICLE

ARTICLE XI.

Reply to Mr. Lowry's *Animadverſions on Mr.* Howard's *Spherical Geometry.*

To the EDITOR *of the* REPOSITORY.

SIR,

RELYING on that impartiality with which the Mathematical Repoſitory has hitherto been conducted, I requeſt permiſſion to reply to Art. LX. Vol. I. of your ingenious work, entitled "Animadverſions on ſome remarks in the preface to my Book. By Mr. *John Lowry*."

To the firſt charge I had, previous to this attack, pleaded guilty, by publicly acknowledging the error, in an Appendix to my Book, ſent to Mr. *Longman*, the beginning of June laſt; and which alſo, I preſume, may be found in moſt of the latter iſſued copies.

Mr. *Lowry* next aſſerts that "the reaſoning in ſeveral other of my theorems will, on examination, be found equally fallacious and abſurd, and by dividing them into their particular caſes, numerous errors will be diſcovered, which are at preſent concealed in each common maſs."

To the former part of this remark, I can only reply, that aſſertions are not proofs, and, until Mr. *L.* brings forward ſomething more, that neither the world nor I can be juſtified in aſſenting to his mere *Ipſe dixit*. And with regard to the latter part, I did not know, until now, that the failure of a general theorem in a particular inſtance, came under the denomination of an error. Mr. *L.* here paſſes an eulogy upon his own conduct, which ſeems not eaſily reconcileable either with what precedes or

immediately follows it. This behaviour app
ſome what like that of a gueſt, at an ordinary,
affures the company that many of the diſhes or
table are unwholeſome, and yet declines to p
them out. I need not ſay what the conſequ
would be to the hoſt.

For the next charge, it will be ſufficient to c
pare the paragraph there hinted at by Mr. *L.*
his paraphraſe upon it, in order to ſhew his cor
neſs and candour in quotation, his clearneſs in
prehenſion, and elegance of compoſition.
expreſſion in particular " to whoſe confidenc
communicated his ſecrets" has a grace beyond
reach of art, not eaſily, I think to be comprehen
which might for ever have remained in dark
ſcurity, but for the ſublime conception of
Lowry.

To explain and confute miſrepreſentation
abſurdity, when thus jumbled together, were
eaſy taſk. Leaving Mr. *L.* therefore, to co
his own blunders, I ſhall explain, by a fir
ſtatement of facts, the purport of my note;
then, perhaps, he will find an anſwer to his qu
reſpecting what diſcoveries I lay claim to,
who it is that I blame for having betrayed
confidence.

In a Letter from Mr. *L.* dated *Jan.* 22, 1
after largely acknowledging favours received,
that the knowledge of Mathematics which he
ſeſſed he owed almoſt entirely to me, he adds,
am in great hopes of ſeeing the publication of
Spherics and other new matters. If a ſuffic
number of ſubſcribers cannot be procured, I w
recommend publiſhing them in Numbers,
Eſſays, Quæries, &c. I think there would be
doubt of their ſucceſs that away: ———'s pu
cations (works infinitely inferior to your's) a
proof that works of this kind meet with ſucce

V

Who that reads this would believe that the writer of it had already done, in his own name, what he had here so ſeemingly adviſed me to do? And yet (if there is faith to be put in dates) this was really the caſe. His packet containing three Spherical Problems, inſerted in No. 7, Scientific Receptacle, founded upon the principles contained in my Treatiſe *, though publiſhed the April following, muſt have been ſent to Mr. *Whiting*, at leaſt, in the December antecedent!

The complaint in ſuch a caſe, was juſtifiable, the world muſt allow; and that a confeſſion of error, and a diſcontinuation of the offence, muſt have been the reſult of ſuch complaint (by an honeſt man, at leaſt) muſt alſo be allowed. I received no ſuch ſatisfaction; 'twas in vain I remonſtrated againſt a behaviour ſo unprincipled, urged confidence betrayed, or pleaded the cauſe of injured fame. I did not even receive an anſwer to my Letter. A partial acknowledgement of hints communicated, in page 926, No. 9, of the Scientific Receptacle, was all the reparation made; though the offence has been uniformly repeated till all the ſtore became exhauſted.

The evil has, however, proved much leſs than might have been expected. From a barrenneſs of invention, he has (like a hound at fault) been continually retracing the ſame path; whilſt the clumſy and ungeometrical mode of his demonſtrations (evidently borrowed from Emerſon's Trig. latter Eds.) with the injudicious order in which he has arranged his propoſitions, ſo far from inciting emulation, have hitherto even prevented competition.

Mr. *L's* invidious miſinterpretation of the meaning of my note, which has given him ſo much

 ſcope

* See, in particular, Queſt. 142, which is C. 4, T. 7, B. 4 of my Treatiſe.

ſcope for animadverſion, has alſo given me mu
additional trouble in confuting him. Obli
thereby to follow this PROTEUS through all
doublings and windings, though to as little purp
as a man would ſift a buſhel of chaff for a ſin
grain of corn.

I now return back to where he ſays, "had I poin
out what diſcoveries I laid claim to, he ſhould h
been better able to give me an anſwer." It wo
hence appear that Mr. *L.* has not read my pref
(much leſs the book), or he muſt have found tl
independant of having digeſted a book of Eleme
there is not a ſection in the whole where I do
and I may add where I cannot, claim the merit
originality both in matter and manner.

"No," ſays Mr. *L.* "there are ſimilar propoſition
Euclid's Elements, Emerſon's Geometry, &c.
the application of ſimilar principles and Sphe
was evident, natural, and eaſy."

The anecdote of the Egg, related by Columb
was never more applicable than here. If the tr
ſition was ſo evident, &c. why were not th
props. diſcovered before, and why did not Mr.
who certainly has taken much trouble, bring th
forth to light ere this time? Why did he ſo
P. 18, Art. Sph. in the caſe of maximum only, wh
it might have been done generally and geome
cally? See P. 20, B. 1, of Con. Why did
propoſe Queſt. 190, Scientific Receptacle, o
partially, when Mr. *Armſtrong* proved it was t
in general *? Why too did he ſolve P. 21, A
Sph. (and that by the ungeometrical mode of ſi
and tangents) only partially, when it admitted c
general geometrical ſolution? See P. 21, B. 1

C

* Mr. *L.* has with his uſual facility at manufacturing the l
of others, wrought the ſubſtance of that Demon. into a prop. (
of his Lucubrations, but without being able to diſcern its impo
uſes.

Con. And why did he ſolve P. 23, Ibid. partially, when he (now) knows that it alſo admits of a general ſolution? See P. 19, B. 1, of Con. Laſtly, why did he not attempt P. 22, B. 1, of Con. at all? The truth is, he could find no hint in Simpſon's Geometry, (where the ſame props. in plano are to be found) to lead him to their ſolutions, and his new method of ſolving Geometrical Problems by Trigonometry, could not apply here.

In anſwer to Mr. *L's* wiſh for a compariſon between ſome Theorems in No. 3, Repoſitory, and B. 4, of my Treatiſe, I refer him to the fable of the Frog and the Ox, informing him, at the ſame time, that there are propoſitions, as well as modes of demonſtration, in that book not to be found in Simpſon's Geometry, nor in any other work that I know of.

I am, Sir, With much reſpect,
Your obliged humble Servant,
Oct. 18, 1798. JOHN HOWARD.

ARTICLE XII.

To the Editor of the Mathematical Repoſitory.

SIR,

IN page 400, Vol. I. of your valuable work, there is a perſonal alluſion to myſelf, on which I beg leave to trouble you with a few obſervations. The paragraph runs thus, " If the equation in the 30th queſtion, No. 2, had been $\frac{bx-ab}{c}=y$; the locus would have been the right line GH (fig. 237, pl. 16.) cutting the right line AE in the given angle ABH, where AB $=a$, AD $=x$, FD $=y$; and BD to DF in the given ratio of c to b; or when bx is leſs than ab; Bd to df in the given ratio of c to b.

But the author of a book lately publiſhed, has pronounced DF (y) impoſſible when bx is leſs than ab: his reaſon for this it is not difficult to perceive, as he has in ſeveral parts of the work, given abundant reaſon for preſuming that he knows of no other uſe of the negative ſign than that of ſubtracting a leſs quantity from a greater."

To the charge of knowing "no other uſe of the negative ſign than that of ſubtracting a leſs quantity from a greater," I anſwer. *Habes confitentem reum.* I know no other uſe of it, but I know an abuſe of it, namely, the applying of it ſo, that a term ſhall be underſtood to be leſs than nothing, which is nonſenſe. Indeed the writer of the paragraph, which I have quoted, is not accurate in his preſumption, ſince for ſeveral years I abuſed the ſign in the ſame way that he and many others do, talking conſequently of algebraical terms, by rote, like a parrot; and being incapable through the infatuation, in which I had been educated at the Univerſity of Cambridge, of underſtanding, for a conſiderable time, the beautiful ſimplicity pervading the conſtruction and theory of equations. It is difficult, I know, for perſons accuſtomed to the preſent unſcientific mode of reaſoning in Algebra, to come back to the pure principles from which they have deviated; but having had conſiderable experience in the modern, and not a little already in my own ſyſtem, I have no the leaſt doubt, that in the next generation impoſſible and negative roots will ſhare the ſame fate with the vortices of Deſcartes, and the ultimately equal of Sir Iſaac Newton.

I pronounce DF to be impoſſible, when bx is less than ab, for a very plain reaſon: namely, when x is leſs than ab, the term $bx—ab$ gives no number at all. Conſequently DF cannot exiſt in ſpace, for it is impoſſible to aſſign any part or parts of a foot

mil

mile, or any other meaſure, for its length. This is plain to any one. $\frac{bx-ab}{c}=y$, put into words, is to be thus read, y is equal to a number of feet denoted by $\frac{bx-ab}{c}$. To find this number, take away ab from bx, and divide the remainder by c; but, this is impoſſible, for I cannot take away ab from bx, when ab is greater than bx. The locus of the equation $\frac{bx-ab}{c}=y$, when $AD=x$, and $AB=a$, I pronounce to be BG, and the locus of the equation $\frac{ab-bx}{c}=y$, may be BH. Algebra cannot determine the poſition of the line BH, whether it ſhould be above or below the line AB, and if the locus of this equation $\frac{bx \infty ab}{c}=y$ were required, the true anſwer is, that it will be either HG or IK, or IBG or HBK, according as the firſt ordinate is taken above or below the line; as you permit the ordinate to be taken on either ſide, or require that it ſhould be always on the ſame ſide of the line AE.

I might excuſe myſelf from ſaying any thing on the expreſſion in Simpſon's Fluxions $x-\frac{x+n^2\,a^2\,x^{2\,n-1}}{n-1}=y$, given in page 399, by the ſame correſpondent, becauſe the remarks on it are intended for two anonymous writers in a magazine, and I muſt expoſe myſelf to the epithet of a dabbler in the ſcience I pretend to teach. I am content, however, to be called a dabbler, and leave the

the " real Mathematicians" to overthrow, if they can, my examination of the proof given, that, when $x=0$, and $n=\frac{1}{2}$, y will be equal to $\frac{a^2}{2}$. The proof is thus given in page 399. " Multiply the equation by x, and we have $x^2-\frac{x^2+n^2a^2\times x^{2n}}{n-1}=xy$; make $n=\frac{1}{2}$, and it will be $x^2-\frac{x^2+\frac{1}{4}a^2\times x}{-\frac{1}{2}}=xy$, or $y=x-\frac{x+\frac{1}{4}a^2\times 1}{-\frac{1}{2}}$, equal, when $x=0$, to $-\frac{\frac{1}{4}a^2\times 1}{-\frac{1}{2}}=\frac{a^2}{2}$ as before." When n is made equal to $\frac{1}{2}$, the equation becomes $x^2-\frac{x^2+\frac{1}{4}a^2\times x}{\frac{1}{2}-1}=xy$. Now $\frac{1}{2}$ is leſs than one; therefore I cannot take one from $\frac{1}{2}$, conſequently the next equation $x^2-\frac{x^2+\frac{1}{4}a^2\times x}{-\frac{1}{2}}=xy$ is nonſenſe. Let us, however, purſue the reaſoning, and to do it more eaſily, we will call the term $\frac{x^2+\frac{1}{4}a^2\times x}{-\frac{1}{2}}$ *hocus pocus*, and then the equation becomes x^2- *hocus pocus* $=xy$. The next ſtep is to divide by x, and conſequently $x-\frac{\textit{hocus pocus}}{x}=y$, which, when $x=0$, becomes nothing $-\frac{\textit{hocus pocus}}{\text{nothing}}$ is equal to y; but how nothing $-\frac{\textit{hocus pocus}}{\text{nothing}}$ can make $\frac{a^2}{2}$ the " real Mathematicians" muſt determine. The real fact is, and a very obvious one it is, that, when $x=0$, the expreſſion

preſſion $x-\frac{x+n^2a^2x^{2n-1}}{n-1}$ becomes nothing, take away nothing, add nothing, that is nothing at all; and if a radius of curvature is equal to ſuch an expreſſion in ſuch a ſtate of x, that radius will alſo be nothing at all. It may be aſked then, whether I diſallow of theſe expreſſions found, as it is termed, by fluxions. By no means. I am not ſo zealous an admirer of the ancients, as to inſiſt ſeverely on that rigid mode of demonſtration, for which ſo many of the moderns ſeem to have loſt the taſte.

Fingere cinctutis non exaudita Cethegis
Continget dabiturque licentia ſumpta pudenter.

Let *licentia* be taken *pudenter*, but ſcience muſt not be degraded by *hocus pocus*.

The equation, then, $x-\frac{x+n^2n^2x^{2n-1}}{n-1}=y$, I allow to be true, for what is called the radius of curvature, though I do not admit any thing to be true about Curves, which is not conſiſtent with the ſixteenth propoſition of the third book of Euclid. But this expreſſion is not true in all poſſible values of n, for the leaſt acquaintance with the principles of Algebra, will enable any one to ſee that it is not true unleſs n is greater than one, and that when n is leſs than one the equation is $\frac{x+n^2a^2x^{2n-1}}{1-n}-x=y$, conſequently when $n=\frac{1}{2}$, the equation is $\frac{x+\frac{1}{4}a^2}{\frac{1}{2}}-x=y$, or $2x+\frac{1}{2}a^2-x=y$, or $x+\frac{1}{2}a^2=y$. Now if I could permit myſelf to uſe the language of the "real mathematicians" and ſay, let $x=0$, and $y=\frac{1}{4}a^2$, then the radius at the ſuppoſed point would

would be one-half of a^2, but Algebra allows not such an abuſe of language. I cannot ſay that x is equal to nothing at the concluſion, any more than at the beginning, of a queſtion, and if x could be called nothing at any period, then the radius of curvature of all curves, when x is equal to nothing, would alſo be nothing. Thus, ſubſtitute nothing for x in the equations $x - \frac{x+n^2a^2x^{2\,n-1}}{n-1} = y$ or $\frac{x+n^2a^2x^{2\,n-1}}{1-n} - x = y$, and whether n is equal to, greater, or leſs than one, y muſt be nothing.

From the above your Correſpondent will ſee that I am not to be frightened by his diſtinctions between dabblers and real mathematicians, and as the firſt part * of my Principles of Algebra does not pleaſe him I do not expect his approbation of the ſecond part now in the preſs. For his beloved negative ſign is throughout uſed only in one ſenſe, that of taking away; the theory of equations is laid down in a direct manner; and the poſition, that an equation has as many roots as it has dimenſions, is rejected, becauſe it is built upon error. A ſmall ſpecimen of my plan I gave a ſhort time ago, in a letter † to the Vice-Chancellor of Cambridge, when I was candidate for the Lucaſian Profeſſorſhip of Mathematics, and from that ſpecimen ſome of your readers may perhaps be induced to labour with

* Principles of Algebra, by William Frend. Sold by Robinſon, Paternoſter-row, and White, Fleet-ſtreet.

† A Letter to the Vice-Chancellor of the Univerſity of Cambridge, by William Frend, price 6d. White, Fleet-ſtreet.

ſame career, and, inſtead of troubling with negative and impoſſible roots, aſſiſt Algebra to its true principles.

I remain, Sir,

Your's, &c.

WILLIAM FREND.

Hare-court, Temple,
Dec. 1798.

ARTICLE XIII.

itions to Lawſon's *Propoſitions propoſed in ICLES XL. and LIII. Vol. I. PROP. . Fig.* 218, *Pl.* 15, *and Fig.* 223, 5.

ſtrated by Mr. Colin Campbell, Kendal.

G, and let it meet the circle again in P, and l perpendicular to AB, meeting DG in Q, in N.

ſe of the parallels NC, LK,
as LK : LD :: NC : NQ;
ntly LKD : LDK :: NCQ : NQC,
ndo DK^2 : LDK :: CQ^2 : NQC,
ando NQC : LDK :: CQ^2 : DK^2 :: GQ^2 : DG^2.
p. II. DP : DG :: GP : GQ,
: PDG : DG^2 :: PGQ : GQ^2
p. XXI. PGQ = NQC; and PDG = ADB;
: ADB : DG^2 :: NQC : GQ^2
ndo ADB : NQC :: DG^2 : GQ^2;
do NQC : ADB :: GQ^2 : DG^2.
: NQC : ADB :: NQC : LDK;
: the rect. ADB = the rect. LDK.

Q. E. D.

The

The ſame by Meſſrs. Harris, Lowry, *and* Swale.

ANALYSIS, by Meſſrs. Harris *and* Swale.

Draw HDM to meet the circle in M, and join GM.

By hypotheſis the rect. LDK = the rect. ADB, and, by Prop. XI. ADB + DK^2 = GKH; therefore LDK + DK^2, ie, LKD = GKH; therefore LK : GK :: KH : KD; wherefore the △'s LGK, HDK, are equiangular and therefore the ∠ HDK = ∠LGK = ∠GMH; Hence, GM is parallel to DE, that is, perpendicular to AB, conſequently by Prop. VII. AC : CB :: AD : DB.

Q. Q. V.

SYNTHESIS, by Mr. Lowry.

Since AC : CB :: AD : DB, by conv. Prop. VII. GM is perpendicular to AB, and therefore parallel to DE. Hence, the ∠ HDK = GMH = LGK; therefore the △s LGK, HDK, are equiangular; wherefore KD : KH :: GK : LK; therefore LKD, ie, LDK + DK^2 = GKH; but, Prop. XI. ADB + DK^2 = GKH; therefore the rect. LKD = the rect. ADB.

Q. E. D.

The ſame by Mr. Nicholſon.

Draw CF perpendicular to AB, and join LO, FO, GO, DG, HO, and DF; O being the centre. Then, by Prop. XV. DF is a tan. to the circle at F, conſequently DO : FO = GO :: FO = GO : CO; theref. Eu. VI. 6, the ∠ GDO = CGO = OHG; therefore the points H, O, G, D, are in a circle, the diameter of which is perpendicular to GH: but, ſince OGL is a right angle, OL is the diameter of the circle paſſing through H, O, G, D; wheref. the ∠COG = DLG, and ∠CGO = ∠OLG, conſe-

conſeq. the ∠ DLO = COG + CGO (= DCK);
therefore, by ſim. △s LD : DO :: DC : DK,
or, LDK = ODC = DF^2 = ADB.
Q. E. D.

PROP. XXIII.

Fig. 219, *Pl.* 15, *and Fig.* 224, *Pl.* 16.

Demonſtrated by Meſſrs. Campbell, Harris, Lowry, Nicholſon, *and* Swale.

ANALYSIS, by Meſſrs. Harris *and* Swale.

By hypotheſis the rect. ACB = rect. GCF,
that is, AC : FC :: GC : BC,
and the angles GCB, ACF are right ones;
therefore the △s GBC, FCA, are equiangular,
and the ∠ CGB = ∠ CAF:
but, the ∠ GFE = ∠ AFC;
therefore ∠ GEF = ACF = a right angle;
wherefore AB is the diameter of the circle.
Q. Q. V.

SYNTHESIS, by Meſſrs. Campbell, Lowry, *and* Nicholſon.

Since AB is the diameter of the circle,
the ∠ GEF is a right angle;
but, the ∠ ACF is a right angle:
therefore the ∠ GCB = ∠ CAF;
Hence, the △s GCB, ACF, are equiangular;
wherefore AC : CF :: CG : CB;
therefore the rect. ACB = rect. GCF.
Q. E. D.

PROP. XXIV.

Fig. 220, *Pl.* 15, and *Fig.* 225, *Pl.* 16.

Demonſtrated by Meſſrs. Campbell, Harris, Lowry, Nicholſon *and* Swale.

ANALYSIS, by Meſſrs. Harris *and* Swale.

Draw GD, and let it meet the circle in I, and join HI. By hypotheſis $LDM = DE^2 = DF^2$, that is, $DM^2 + LMD = DM^2 + FME$, or, $LMD = FME = GMN$; wherefore LM : MN :: MG : MD, therefore the △'s LNM, MGD, are equiangular; therefore ∠ NLM = MGD (NGI) = NHI; and theref. HI is parallel to DE, *ie.* perp. to AB; wheref. by Prop. VII. AC : CB :: AD : DB.

Q. Q. V.

SYNTHESIS.

By Meſſrs. Campbell, Lowry, *and* Nicholſon.

Since AC : CB :: AD : DB, by conv. Prop. VII. HI is perpendicular to AB, and therefore parallel to DE; theref. ∠ MGD = NGI = NHI = NLM; wherefore the △'s LNM, MGD, are equiangular; therefore ML : MN :: MG : MD; hence, $LMD = GMN = FME$, that is, $DM^2 + LMD = DM^2 + FME$, that is, $LDM = DF^2 = DE^2$.

Q. E. D.

PROP XXV. Fig. 226, 227, *Pl.* 16.

Demonſtrated by Meſſrs. Campbell *and* Lowry.

Draw the diameter DM, and join FM, FE; and let CD meet GB in L.

The

Then ∠ (DMF) GCL + (FDM) HFE = a right ∠,
and ∠ GCL + CGE = a right ∠,
therefore the ∠ CGE = ∠ HFE,
and therefore the △'s GEF, HFE, are equiangular.
Conſequently GE : EF (AE) :: (EF) AE : EH,
and therefore the reſt. GEH = AE².

Q. E. D.

The ſame by Mr. Harris.

ANALYSIS.

Draw the diameter FI, and join IC.
By hypotheſis GEH = AE² = EF²,
that is, GE : EF :: EF : EH;
therefore the △'s EHF, GFE, are equiangular.
Hence the △'s HDL, FIC, are alſo equiangular;
therefore ∠ HLD = GCI = a right angle,
and therefore CD is perpendicular to AB.

Q. Q. V.

The ſame by Mr. Swale.

ANALYSIS.

By hyp. (AH + HE) × AE = AE² = GEH = (GA + AE) × HE,
hence, HA × AE = GA × HE;
therefore HA : HE :: GA : AE,
that is, HA : 2HE :: GA : 2AE,
that is, HA : HB :: GA : GB;
wheref. Conv. Prop. VII. CD is perpendicular to AB.

Q. Q. V.

PROP. XXVI. Fig. 228, 229, *Pl.* 16.

Demonſtrat.

Join FD, DG; let O be the centre, and draw FO, GO.
Then (dem. Prop. IV.) ∠ FDC = CDG; conſe. ∠ KDF = GDL.
Alſo, by Prop. I. DC · CO = AC · CB = FC · CG;
therefore a circle will paſs thro' the points F, D, G, O;

 wherefore

wherefore $\angle FDG + FOG = 2$ right $\angle$'s $= \angle FDG + 2LDG$,
and $\angle FOG = 2KHG$;
therefore $\angle LDG = KHG$,
and therefore the $\triangle$'s LDG, LKH, are equiangular
consequently $LG : LD :: LK : LH$;
therefore $LG \cdot LH = DL \cdot KL = KD \cdot DL + DL^2$:
but, Prop. XI. $LG \cdot LH = DA \cdot DB + DL^2$,
consequently the rect. KDL = rect. ADB.

Q. E. D.

The same by Messrs. Harris *and* Swale.

ANALYSIS.

Through O, the centre of the circle, draw the diameter GOI, and join HI, DG.

By hypothesis the rect. LDK = rect. ADB,
and, by Prop. XI. $LD^2 + ADB = HLG$;
therefore $(LD^2 + LDK)$ or, $KLD = HLG$;
wherefore the points K, D, H, G, are in a circle;
consequently $\angle GHK = \text{sup. } GDK = GDL$;
therefore $\angle ODG = FHI = FGI = CGO$;
wherefore the $\triangle$'s OGC, ODG, are equiangular;
therefore DO : OG or OA :: OG or OA : OC,
and therefore DO − OA : DO + OA :: OA − OC : OA + OC,
that is, AD : DB :: AC : CB.

Q. Q. V.

The same by Mr. Lowry.

Draw DF to meet the circle in Q, and join GQ.
Then, since AC : CB :: AD : DB,
by Prop. VII. QG is perpendicular to AB, and parallel to DE, therefore $\angle KLG = QGH$:
But the points FHQG are in a circle,
therefore $\angle KFD = QGH = KLG$;
and therefore the points D, L, H, F, are in a circle,
Hence, DKL, or, $DK^2 + KDL = FKH$:
but Prop. XI. $DK^2 + ADB = FKH$,
and therefore the rect. KDL = rect. ADB.

Q. E. D.

PROP.

PROP. XXVII.

Fig. 230, 231, *Pl.* 16.

Demonſtrated by Meſſrs. Campbell *and* Lowry.

Draw KC to meet the circle in M, and join HM.
By hyp. $EC \cdot CF$, or, $EF \cdot FC + FC^2 = CD^2$;
theref. $EFC = CD^2 - FC^2 = (CD+FC) \cdot (CD-FC) = NFD = KFG$;
wherefore $EF : FK :: FG : FC$;
therefore the $\triangle$'s EFG, KFC, are equiangular;
hence, $\angle KCE = FGE = HMK$;
theref. HM is parallel to DN, and conſeq. perp. to AB;
conſequently, by Prop. VII. AL : LB :: AC : CB.

Q. E. D.

The ſame by Meſſrs. Harris *and* Swale.

Let O be the centre of the circle; draw the diameter HOI, and join IK, HC.

By hypotheſis AL : LB :: AC : CB,
that is, LO—AO : LO+AO :: AO—OC : AO+OC;
therefore LO : OA or OH :: OH : OC;
wherefore the $\triangle$'s OLH, OCH, are equiangular;
theref. the angles OHL, OCH are equal, and ſo are their complements, $\angle HCF = HIK = EGF$;
wherefore the points H, C, G, F, are in a circle;
conſequently rect. FEC = rect. HEG = rect. DEN,
or, $FEC + EC^2 = DEN + EC^2$,
that is, $ECF = DC^2$.

Q. Q. V.

ARTICLE XIV.

Three Propoſitions from Lawſon.

(To be anſwered in Number IX.)

PROP. XXXI.

LET AB touch a circle in B, and any line AE be drawn and therein be taken two points E and F on the ſame ſide of A, ſuch that the rectangle EAF may be equal to the ſquare of AB, and from A any line be drawn to meet the circle in C and D, and EC, FD be drawn meeting the circle again in G and H; GH being drawn will be parallel to AE.

PROP. XXXII.

Through any point A within a circle let a line be drawn meeting it in B and E, and therein two points F and G be taken ſuch on different ſides of A that the rectangle FAG may be equal to the rectangle BAE, and through A any line be drawn meeting the circle in C and D, and FC, GD be drawn to meet the circle again in H and K; then HK being drawn will be parallel to AB.

PROP. XXXIII.

Let AB be a line without a circle, and from A and B two lines be drawn to touch the circle in C and D, and let the ſquare of AB be equal to the ſum of the ſquares of AC and BD, and from A any line be drawn to meet the circle in E and F, and BE, BF be drawn meeting the circle again in G and H; the points A, G, H, will be in a right line.

ART.

ARTICLE XV.

Two Propoſitions from Stewart's *Theorems.*

(To be anſwered in Number IX.)

PROP. XXVII. THEO. XXIV.

ET there be any regular figure inſcribed in a
circle, and from all the angles of the figure
the centre of the circle let there be drawn right
s to any point; the ſum of the fourth powers of
lines drawn from the angles of the figure, will
qual to the multiple by the number of the ſides
he figure of the fourth power of the ſemi-
ieter of the circle, together with four times
multiple by the ſame number of the fourth power
he line whoſe ſquare is equal to the rectangle
ained by the ſemi-diameter and the line drawn
1 the centre, together with the multiple by the
: number of the fourth power of the line drawn
1 the centre.

PROP. XXVIII. THEO. XXV.

et there be any regular figure of a greater num-
of ſides than four circumſcribed about a circle,
from any point in the circumference of the
le let there be drawn perpendiculars to the ſides
he figure; eight times the ſum of the fourth
ers of the perpendiculars, will be equal to 35
es the multiple by the number of the ſides of the
ire of the fourth power of the ſemi-diameter of
circle.

ART-

ARTICLE XVI.

On the Deſcription of Parabolic Trajectories.

By Mr. WILLIAM WALLACE, *Aſſiſtant Teacher of the Mathematics in the Academy at Perth.*

PROP. I. THEOREM. Fig. 217, *Pl.* 15.

IF three ſtraight lines touch a parabola, a circle deſcribed through their interſections ſhall paſs through the focus of the parabola.

Let DC, DE, CE be three tangents to a parabola, a circle deſcribed about the triangle DCE ſhall paſs through F the focus of the parabola.

This Theorem was propoſed as a prize Queſtion, at page 309, vol. I. and the demonſtration thereof together with the demonſtrations of the following corollaries are given at pages 56 and 57, Vol. II.

Cor. 1. If CD, CE touch the curve at A and B, and if FA, FB be joined, the triangles AFC, CFB are ſimilar to the triangle DFE.

Cor. 2. Let AC, BC be two ſtraight lines given by poſition touching a given parabola, if any other tangent be drawn meeting them in D and E, and FD, FE be joined, the triangle DFE is given ſpecies.

Cor. 3. If AC, BC are two tangents to a parabola, and if a circle be deſcribed through the focus to touch BC, one of them, at C, it ſhall paſs through A the point where the other tangent meets the curve.

Cor. 4. If AC, BC, DE are tangents to a parabola at the points A, B, N; theſe lines are ſimilarly divided at their interſections and the points of contact, that is AD : DC = DN : NE = CE : EB.

The

The following problem is very ſimple, and only given for the ſake of thoſe that follow.

PROP. II. PROBLEM. Fig. 252, Pl. 16.

To deſcribe a parabola, having given its focus and two tangents to the curve.

Let H be the given focus, and PB, BQ the two given tangents. Draw HG, HK perpendicular to the tangents, and in the perpendiculars produced take GM equal to GH and KR equal to KH; join MR which will evidently be the directrix, and the focus being given, the curve may be deſcribed in the manner taught by the writers on Conic Sections.

PROP. III. PROBLEM. Fig. 252, Pl. 16.

To deſcribe a parabola that may touch four ſtraight lines given by poſition.

Let AB, AC, BF, DF be the four ſtraight lines, and let ABC, DBF be any two of the four triangles formed by the lines; let circles be deſcribed about theſe triangles, they will meet each other at H, the focus of the parabola. Having now given the focus and more than one tangent to the curve, the manner of deſcribing the parabola is evident from the laſt problem.

PROP. IV. PROBLEM. Fig. 252, Pl. 16.

Three tangents to a parabola, and the point at which one of them touches the curve, are given, to deſcribe the parabola.

Let AB, BC, AC be the three ſtraight lines given by poſition, and let P be the given point of contact. Deſcribe a circle about the triangle ABC, alſo through P deſcribe a circle that may touch BC, any

any one of the lines, at B; this circle will cut the other at H the focus of the parabola; having now the focus and more than one tangent to the curve the method of deſcribing it is obvious.

PROP. V. PROBLEM. Fig. 252, Pl. 16.

Two tangents to a parabola, and the points at which they touch the curve are given to deſcribe the parabola.

Let BP, BQ be the tangents, and P, Q the points of contact. Through the point P deſcribe a circle that may touch the tangent BQ at B; alſo through the point Q deſcribe a circle to touch BP at B; theſe circles ſhall meet each other at H the focus of the parabola required. There are now given two tangents and the focus, hence the method of deſcribing the curve is evident.

PROP. VI. PROBLEM. Fig. 252, Pl. 16.

To deſcribe a parabola that ſhall touch three given ſtraight lines and have its axis parallel to a ſtraight line given by poſition.

Let AB, BC, AC, be the three ſtraight lines given by poſition, and let *de* be the ſtraight line to which the axis is to be parallel.

Deſcribe a circle about the triangle ABC, let *de* meet the tangent AB in *d*, make the angle CBH equal to the angle *ed*B and H will be the focus of the parabola required.

Let the curve be deſcribed, and let it touch BA at P, join HP. By Prop. I. the angle HBC is equal to the angle HPB, but by conſtruction HBC is equal to the angle *ed* B, and it is evident that HPB is equal to the angle contained by the tangent AB and the axis, hence the axis and *de* make equal angles with AB, therefore the axis is parallel to *de* as was required.

ART

ARTICLE XVII.

Anſwers to the Mathematical Queſtions propoſed in ARTICLE LVIII. No. V. Vol. I.

I. QUESTION, 89, *anſwered by Mr.* J. Lochwood.

AS the ſide of the equilateral triangle inſcribed in the generating circle is given = 16, the diameter of the circle is readily found $= \frac{32}{3}\sqrt{3}$ = the axis of the cycloid; therefore when the cycloid revolves about a tangent parallel to its axe, the diameter of its circumſcribing cylinder will be expreſſed by $\frac{64}{3}\sqrt{3} \times 3{\cdot}1416 = 64 \times 1{\cdot}0472\sqrt{3}$. But by what is ſaid under the article *cycloid* in Dr. Hutton's Dictionary, this content is to the content of the ſolid generated by a revolution of the cycloid about a tangent parallel to its axe, as 4 : 3; therefore, $3072 \times (1{\cdot}0472)^2 \times {\cdot}7854 \times 32\sqrt{3} = 146649{\cdot}90222$, the content required.

The ſame anſwered by Mr. Ralph Simpſon, Croxdale, *near* Durham.

It has been ſhewn that the ſide of an equilateral triangle is to the radius of its circumſcribing circle as 1 to 57735027. Therefore as 1 : ·57735027 :: 16 : 9·23760432 = radius of the generating circle, which $\times 2 \times 3{\cdot}1416 = 58{\cdot}041715$ = its circumference; which, according to the property of the cycloid, is = the length of its baſe; and its height or axis = 18·4752 = the diameter of the circle.

According to Dr. Hutton's Mathematical and Philoſophical Dictionary, Vol. I. pa. 355, Torricelli has ſhewn that the content of a ſolid, formed by the rotation of a cycloid about a tangent parallel to its axis, is to the content of its circumſcribing cylinder as 3 to 4. Now the content of a cylinder whoſe altitude is 18·4752 = the axis of the cycloid, and diameter of the end 116·08343 = twice the cycloid's baſe, is eaſily found = 195533·15315. And therefore, by the above proportion, $195533{\cdot}15315 \times 3 \div 4 = 146649{\cdot}86486$ the content required.

Ingenious ſolutions were alſo given by Meſſrs. Boſworth, Gregory, Harris, Johnſton, Lowry, *and* Swale.

II. QUES-

II. QUESTION 90, *anſwered by Mr.* Johnſton, Birmingham.

Conſ. Let ABHI (fig. 232, pl. 16,) be the rectangular garden, and on AB produced take BD = AH; draw DK perp. to AD, and = $\frac{1}{20}$th of AB; join HK, upon which as a diameter let a circle be deſcribed interſecting AD in O, and AO will be the breadth of the walk.

Demon. Let S be the centre of the circle and draw SV parallel to AH meeting AD in V; alſo let L be the point where the circle cuts AH, and join OS, LS, and KL; on BI take BM = AO; then it is evident that the area of the walk is = BM × (AB + MI) = BM × OD; but by the property of the circle AO = DQ, and AL = DK = AB ÷ 20; therefore AO × OD (= BM × OD) = OA × AQ = LA × AH = $\frac{1}{20}$ AB × AH; that is, the area of the walk is = to $\frac{1}{20}$th of the area of the garden.

Calculation, DK = 4, LK = AD = 180, LH = 96, SG = 48, SV = 52, $OS^2 = LS^2 = 90^2 + 48^2$, and OV = $\sqrt{(OS^2 - SV^2)} = \sqrt{(90^2 + 48^2 - 52^2)} = \sqrt{7700} = 87.749643874$; hence, AO = 2.250356126 yards the breadth of the walk required.

The ſame by Mr. Richard Nicholſon, *Liverpool.*

Conſ. Fig. 233, pl. 16. Take AB, BC equal to the length and breadth of the garden reſpectively; biſect AC in O, and take AF to FB as the area of the walk to the area of the garden. Upon FC deſcribe a ſemi-circle to meet BD, a perp. to FC, in D; then with the centre O and diſtance OD let the ſemi-circle GDH be deſcribed, then AG or HC will be the breadth of the walk required.

Demon. AB·BC − GB·BH (FB·BC) = AF·BC : AB·BC :: AF : AB, the given ratio, as per *Conſ.*

Cal. BF is = 95, and BC = 80; hence $BD^2 = 95 \times 80 = 7600$: but OB is $\frac{180}{2} - 80 = 10$; therefore OD = $\sqrt{(BD^2 - OB^2)} = \sqrt{7700} = 87\frac{3}{4}$, and HC = OC − OD = $2\frac{1}{4}$ yards nearly.

The

The ſame algebraically by Mr. W. Marrat, *Lincoln.*

Put x for the breadth of the walk; then $100 - x =$ length of the middle walk, and $x(100-x) =$ its area; alſo $80x$ is the area of the end walk; and their ſum $x(100-x) + 80x = 8000 \div 20$, that is, $180x - x^2 = 400$; hence $x = 2\frac{1}{4}$ yards nearly.

The ſame otherwiſe by Mr. Gregory.

This Queſtion may be readily ſolved by a quadratic equation, and it will admit of a neat geometrical conſtruction; but I chooſe rather to anſwer it by the common rule of trial and errour, thus: Let it be ſuppoſed that 2 is the breadth of the walk, then is $100 - 2 = 98$, the length of the walk along the middle; hence, as 80 is the breadth of the Garden, $2 \times \overline{80 + 98} = 356$, for the content of the whole walk, inſtead of $\frac{100 \times 80}{20}$ or 400; an errour of 44 in defect. Again, ſuppoſe $2\frac{1}{2}$ the breadth; then $97\frac{1}{2}$ the length of the walk along the Middle; therefore $\overline{97\frac{1}{2} + 80} \times 2\frac{1}{2} = 443\frac{3}{4}$, inſtead of 400, an errour of $43\frac{3}{4}$ in exceſs. The ſum of the products of the firſt aſſumption into the ſecond errour, and the ſecond aſſumption into the firſt errour, is $197\frac{1}{2}$; which divided by $87\frac{1}{2}$, the ſum of the errours, gives $2\frac{88}{351}$ or nearly $2\frac{1}{4}$ for the width of the walk. If this be aſſumed for the width, the operation purſued as before, and the reſult corrected by either of the former aſſumptions and its reſult, we ſhall get 2·2503562 for the anſwer; agreeing with $90 - 10\sqrt{77}$, the anſwer obtained by a quadratic equation, to the 7th decimal place.

Other ingenious anſwers were alſo received from Meſſrs. Boſworth, Harris, Johnſton, Lockwood, Lowry, Reed, Swale, *and* Thornoby.

III. QUESTION 91, *anſwered by Mr.* Lowry.

Let QST (Fig. 234, pl. 16.) be a ſection of the conical glaſs, BPAR a ſection of the ſpheroidal egg paſſing through the centre C. Here CD the ſemi-conjugate $= \frac{1}{2}$, BA the tranverſe $= 2$, $TE = 4$, $QS = 2$, and $ES = 1$. Then by Emerſon's Conics I. 26, $AK \times BH = CD^2$: but $AT = 4\,AK$ and $TB = 4\,BH$; therefore $AT \times TB = 16AK \times BH = 16CD^2$.

On AB describe a semi-circle and draw the tangent FG; join CG, then $TG^2 = AT \times TB = 16CD^2$; therefore $TC^2 = CG^2 + 16CD^2$, or $TC = \sqrt{5}$; hence EC and consequently AE, BE are given; therefore, by the property of the ellipsis we have $BA^2 : FD^2 :: AE \times EB : NE^2$; therefore NE is given. Hence the content of the immersed part of the egg may be found, by Mensuration, from which deducting the content of the vacuity of the glass, (which may easily be found,) the remainder will be the quantity of water overflowing. But the numbers given in the question are such as shew the content of the vacuity of the glass to be greater than the content of the whole egg consequently no water will run over in the present instance.

The same by Mr. J. Lockwood, *Leeds.*

Let STQ (fig. 234, pl. 16.) represent the glass, APBR the spheroid, and put $BA = 2 = t$; conjugate axe $= c$; and $AM = x$; then from known principles the subtangent TM will be expressed by $(2tx - 2x^2) \div (t - 2x)$, and the ordinate PM by $(c \div t) \times \sqrt{(tx - x^2)}$: but by sim. $\triangle$s TM : MP :: 4 : 1, per question; therefore $(2tx - 2x^2) \div (t - 2x) = (4c \div t) \times \sqrt{(tx - x^2)}$, and by reduction $(t^2 + 16)\, x^2 - (16t + t^3)\, x + 4t^2 = 0$, or in numbers $x^2 - 2x + \frac{4}{5} = 0$; hence $x = 1 - \sqrt{(1 \div 5)}$. Now it will be very easy to determine TM, PM, TB, BE, AE and consequently NE. And then every thing else, if proper numbers had been given by the proposer.

The same by Mr. T. Reed, *Student in Mr.* Bulmer's *School at Sunderland, Durham.*

The solidity of the egg is $\overline{1}^2 \times 2 \times .5236 = 1.0472$; content of the whole cone $= \overline{2}^2 \times .7854 \times (4 \div 3) = 4.1888$; content of the water contained in it is $= \overline{1.75}^2 \times .7854 \times 3.5 \div 3 = 2.806168\,75$; therefore $4.1888 - 28061 = 1.3826 =$ the content of the vacuity of the glass, and as this exceeds the content of of the egg, no water will run over.

And thus the question is answered by Messrs. Bosworth, Gregory, Harris, Lowry, Marrat, Simpson, Swale, *and* Thornoby.

VI. QUESTION 92, *answered by Mr.* Lockwood.

Let ABCD (fig. 235, pl. 16.) represent the semi-circular field; P the given point in the diameter, and draw the lines as in the figure. Put

AB $= 2a$, OP $= b$ and AE $= x$; then by the property of circle AD $= \sqrt{2ax}$, ED $= \sqrt{(2ax - x^2)}$: alſo the area of ſector AOD, or BOC will be expreſſed by $(4\sqrt{2ax} - \sqrt{x - x^2})) \times (a \div 6)$; and the area of the $\triangle$ OPD, or OPC $\frac{1}{2}\sqrt{(2ax - x^2)}$; therefore the area of the ſpace PAD, or PBC be expreſſed by $(4\sqrt{2ax} - \sqrt{(2ax - x^2)}) \times (a \div 6) + \frac{1}{2}\sqrt{(2ax - x^2)} = 200 \div 3$ by the queſtion; which equation gives $x = 2{\cdot}68$, or $11{\cdot}6046$. Hence PD $= 16{\cdot}327$, and PC $= {\cdot}628$.

Alſo by Trigonometry, the $\angle$ APD is found $= 26^\circ\ 33'\ 39''$, the $\angle$ APC $= 63^\circ\ 16'\ 36''$.
Whence the expence of fencing will be 15l. 18s. 6d.

the ſame way it was anſwered by Meſſrs. Boſworth, Gregory, Johnſton, Lowry, Swale, *and* Thornoby.

V. QUESTION 93, *anſwered by Mr.* Samuel Thornoby.

Since this queſtion muſt be ſolved by ſome approximation or ther, perhaps the following is as ready as any. Since the areas of nilar ſegments of circles are as the ſquares of their verſed ſines, ıe eaſieſt method of finding the diameter, will be by firſt finding a nilar ſegment in a table of areas and verſed ſines.

This may eaſily be done by means of the large Table at the end Dr. Hutton's Menſuration, and by a few trials, the verſed ſine a ſegment ſimilar to that in the queſtion is ·031, its area being ·00720942; for $\cdot00720942 \div (\cdot031)^2$ is $= 7{\cdot}508$, and $\div 2^2 = 7{\cdot}5$, which quotients are ſufficiently near an equality. ence, then as ·031 : 2 :: 1 (tab. diam.) : $64\frac{1}{2}$ very near the true ameter, and therefore the chord of the ſegment is $= 22{\cdot}36$ ains.

Now, from the nature of the queſtion the ſpace GPCBG (fig. 36, pl. 16.) muſt be to the ſpace AIHFA as 40 to 30, or as to 3. Aſſume CL $= {\cdot}4$ chains; then

ıe area of the ſegment PCL $= 2{\cdot}701628$,
ıd the area of the rectangle GPLD $= 8{\cdot}1016$;
ence, 4 times the ſpace GPCBG $= 103{\cdot}212912$,
ıd 3 times the ſpace AIHFA $= 99{\cdot}6444$;
erefore the errour is 3.567512 in exceſs.

gain, ſuppoſe CL $= {\cdot}3$ chains, then proceeding after the ſame anner, the errour will be 2·504 too little. Hence we find ·38753

a near value of CL.—For a second approximation, ſuppoſe CL = ·38753, and proceeding as before, CL will be found = ·3893489 very nearly. Hence GA = 6·21 the ſide of the ſquare required.

The ſame anſwered by Mr. J. Lockwood, *Leeds.*

Put d D $= x$, CD being $= 2$; then CB $= \sqrt{(2x+4)}$, and AB $= 2\sqrt{2x}$; hence by menſuration and the nature of the queſtion $(2\sqrt{x} + (4 \div 3)\sqrt{(2x+4)} \times 8 = 300$. From this equation x is found $= 62\frac{1}{2}$ nearly; therefore dC $= 64\frac{1}{2}$ nearly, and AD $= 11{\cdot}18$. Now put CL $= x$, Cd $= a$; AD $= c$, 15 chains $= m$; then by the nature of the circle PC $= \sqrt{ax}$, 2PL $= 2\sqrt{(ax - x^2)}$; therefore the area of the ſegment LPC $= (2\sqrt{(ax -)} + (4 \div 3)\sqrt{ax}) \times (2x \div 10)$, and the area of the parallelogram LG $= (2-x)\sqrt{(ax-x^2)}$; therefore the area of the ſpace APHI $= (c - \sqrt{(ax-x^2)})^2 - m + (4x \div 10)\sqrt{(ax - x^2} + (8x \div 30)\sqrt{ax} + \sqrt{(ax-x^2)} \times (2-x)$, which at 3l. per ſquare chain comes to $3 \times (c - \sqrt{(ax^2 - x^2)}) - 3m + (12x \div 10)\sqrt{(ax-x^2)} + (8x \div 10)\sqrt{ax} + 3(2-x)\sqrt{(ax-x^2)}$, and the value of the ſpace AGP $= 4m - (16x \div 10)\sqrt{(ax-x^2)} - (32x \div 30)\sqrt{ax} - 4(2-x)\sqrt{(ax-x^2)}$; their ſum $= 3 \times (c - \sqrt{(ax-x^2)})^2 + (\cdot 6x - 2)\sqrt{(ax-x^2)} - (8x \div 30)\sqrt{ax} = 105$. Hence x is found $= {\cdot}3904$, and GA the ſide of the ſquare $= 6{\cdot}17$ and its content $= 3$A. 3R. 9P.

The ſame by Mr. J. H. Swale, *Cheſter.*

Since the area and verſed ſine of the ſegment are given, the chord and diameter are eaſily found $= 22{\cdot}36$ and $64{\cdot}5$ chains reſpectively. Now let ACB repreſent the field in its original form, and AGHI the ſame when in a ſquare. Draw the verſed ſine CD, and from the point P, where GH cuts the ſegment, draw PL parallel to CD meeting CD in L. Put the diameter $= 64\frac{1}{2} = 2m$, DC $= 2 = a$, DA $= 11{\cdot}18 = c$, the given area $= 30$ ſquare chains $= 2n$, $40 = 10v$, $30 = 10u$, and CL $= x$. Then LD $= PG = a - x$, DG $= \sqrt{(2mx - x^2)}$, and AG $=$ the ſide of the ſquare $= c - \sqrt{(2mx - x^2)}$. Again, the area of the ſemi-ſegment CPL $= (3\sqrt{(2mx - x^2)} + 2\sqrt{2mx}) \times (2 \div 15)x$, and the area of the rectangle DGPL $= (a-x)\sqrt{(2mx - x^2)}$, the ſum of which taken from the ſemiſegment ADC leaves the ſpace APG $= n - (a - (3 \div 5)x\sqrt{(2mx - x^2)} + (4 \div 15)x\sqrt{2mx})$, which deducted from the area of the ſquare leaves $(c - \sqrt{(2mx - x^2)})^2 - (n - (a - (3 \div 5)x\sqrt{(2mx - x^2)}$ +

$+(4 \div 15) \times \sqrt{2mx}))$ = area of the curvilinear ſpace AFPHI. Hence by the nature of the queſtion $(v - u)((3 \div 5)(x - a)\sqrt{(2ax - x^2)} - (4 \div 15) \times \sqrt{2mx} + n) + u(c - \sqrt{(2mx - x^2)})^3 = 2uv$. From which equation we obtain $x =$ 1904285 chains; conſequently DG = 5·002834, GA = ·177166 chains; alſo the area of the ſquare = 38·1565644, the area of the part AFPG = 6·061009, and the area of the exterior part AFPHIA = 32·095555 ſquare chains, all nearly as required.

According to one or other of theſe methods is the anſwers given by Meſſrs. Boſworth, Gregory, Johnſton, Lowry, Simpſon, *and* Thornoby.

VI. QUESTION 94, *anſwered by Mr.* Johnſton.

Let HR (fig. 183, pl. 13.) be the horizon, P the pole, Z the zenith, and A the ſun's place at the the time of obſervation. Then we have given the colat. PZ = 38° 28′, the ſum of the coalt. and odec. AP + AZ = 110° 12′, the ſum of the azimuth and hour angle AZP + APZ = 156° 35′, to find the angle APZ.

Now by Emerſon's Trigonometry, B. III. Prop. 37.

As tangent $\frac{1}{2}$PZ = 19° 14′
: tangent $\frac{1}{2}$(AP + AZ) = 55° 6′
:: coſine $\frac{1}{2}$(AZP + APZ) = 78° 26′ 30″
: coſine $\frac{1}{2}$(AZP − APZ) = 34° 36′ 30.

Hence the azimuth is = 113° 3′, and the hour angle = 43° 50′; conſequently the time from noon is 2 hours. 55 min. 20 ſec.

And thus the queſtion is anſwered by Meſſrs. Boſworth, Gregory, Harris, Lowry, Simpſon, Swale, *and* Thornoby.

VII. QUESTION 95, *anſwered by Mr.* Louis Hill.

Let ACBD (fig. 238, pl. 16.) be the given ellipſe, AB, CD its tranſverſe and conjugate axes reſpectively; F, G its foci. Suppoſe the point A to have arrived at any other point Q in the quadrant AQC; join QF, QG, and in QF produced take FP = QG, and P will be a point in the curve required. Let PE, QI be made perp. to AB and put AB = t, FG = a, FB = d, the abſciſſa BE = x and the correſponding ordinate PE = y. Then EF is $= d - x$, FP = $\sqrt{y^2 + (d-x)^2}$ and FQ = $t - \sqrt{y^2 + (d-x)^2}$; alſo, by ſim. $\triangle$s FP : FQ :: FE : FI = $t - \sqrt{y^2 + (d-x)^2} \times (d-x) \div \sqrt{y^2 + (a-x)^2}$. By the queſtion and the property of the figure QG + QF = PF + QF = AB,

$= AB$, and by Simpſon's Geometry B. II. Theo. IX. $FP^2 - QF^2 = GQ^2 - QF^2 = GI^2 - FI^2$, that is, $FP - QF : FG - GI + IF = FG + 2IF : AB$: but $FP - QF$ is $= 2FP - A = 2\sqrt{y^2 + (d-x)^2} - t$, therefore $2\sqrt{y^2 + (d-x)^2} - t : a :: a : + (2t - 2\sqrt{y^2 + (d-x)^2}) \times (d-x) \div \sqrt{y^2 + (d-x)^2} : t$, or $2t\sqrt{y^2 + (d-x^2)} - t^2 = a^2 + (2at - 2a\sqrt{y^2 + (d-x)^2}) \times (d-x) \div \sqrt{y^2 + (d-x)^2}$; this by reduction gives $2t(y^2 + \overline{d-x}^2) - 2at(d-x) = (t^2 + a^2 - 2a \times \overline{d-x})\sqrt{y^2 + (d-x)^2}$ for the equation of the curve required, which is a line of the 4th order.

Obs. When x is $= o$, y is $= o$, and when $x = d$, y is $= (t^2 + a^2) \div 2t = FH = FR$ the greateſt ordinate.

Similar anſwers were alſo received from Meſſrs. Boſworth, Gregory, Evans, Lowry, Surtees, *and* Swale.

VIII. QUESTION 96, *anſwered by Mr.* Olinthus Gregory, *Bookſeller and Teacher of the Mathematics, Cambridge.*

Let AB (fig. 239, pl. 16.) repreſent the vertical ſide of the veſſel, a, c, e, g, &c. the ſeveral apertures therein, at a foot diſtance from each other: on AB deſcribe a ſemicircle, and let the ſines ab, cd, ef, hg, &c. be drawn, then (as it is ſhewn by moſt writers on Hydroſtatics) twice ab, is the diſtance which water ſpouting out at a falls from B, on the horizontal BH, twice cd, is the diſtance which water ſpouting out at c falls from B, and ſo on. But it is a property of the circle that $Aa \times aB = ab^2$, $Ac \times cB = cd^2$, &c. hence the ſeveral diſtances to which the fluid will ſpout are readily found to be as follows:

$$2\sqrt{1 \times 9} = 2\sqrt{9} = 6.$$
$$2\sqrt{2 \times 8} = 2\sqrt{16} = 8.$$
$$2\sqrt{3 \times 7} = 2\sqrt{21} = 9.16515$$
$$2\sqrt{4 \times 6} = 2\sqrt{24} = 9.79795$$
$$2\sqrt{5 \times 5} = 2\sqrt{25} = 10.$$
$$2\sqrt{6 \times 4} = 2\sqrt{24} = 9.79795$$
$$2\sqrt{7 \times 3} = 2\sqrt{21} = 9.16515$$
$$2\sqrt{8 \times 2} = 2\sqrt{16} = 8.$$
$$2\sqrt{9 \times 1} = 2\sqrt{9} = 6.$$

nd the quantity of water discharged at all the holes,

t 1 foot deep; put $s = 16\frac{1}{12}$ feet, D = 1, depth

face of the water to the centre of the hole, F = feet, area of the hole, $t = 600''$ time of discharging:). *Prop.* 97. *Emerson's larger Mechanics*) $6{\cdot}128tF$ ·54946 ale gallons, discharged by the upper hole in

And as it is known that the quantities discharged from in equal times are as the square roots of their depths urface; 4·54946 the quantity discharged by the upper multiplied into the square roots of 2, 3, 4, 5, 6, 7, 8, pectively, will give 6·43391, 7·87990, 9·09892, 11·14386, 12·03674, 12·86782, and 13·64838 for quantities issuing at each of the holes; the sum of which 89 ale gallons, discharged by all the 9 holes in 10

the quantity of water discharged determined on the sup- at the velocity with which it spouts is equal to that which ody would acquire in falling through half the height of of the water above the aperture. But neither this hypo- r any other which has yet been advanced, agrees with ex- , when applied to ascertaining the measure of effluent deed scarcely any of the attempts which have been made the truth of the two most generally received hypotheses which is laid down by NEWTON in the first edition of his , and the other in the second and third editions] are satis- and, if the modes of reasoning which have been used are s, but little dependance can be placed on the results.

is the best information on this subject may be met with in s of *M. D'Alembert* and *D. Bernoulli*, and in an ingenious r the *Rev. M. Young*, *D. D.* inserted in the *Transactions yal Irish Academy* for 1788.

Since the above was written I have perused the second of *Dr. Hutton's* useful "Course of Mathematics", and l an answer to this question, in which the *distances* agree e in this solution, and the *quantity* discharged is determined 3·8849 gallons, on the supposition that the velocity equal to that acquired by a heavy body in falling through height of the water above the orifice. It may be worth observe that if the quantity discharged, as determined in ion be multiplied into $\sqrt{2}$, the product will very nearly h the quantity given in Dr. Hutton's answer.

us solutions to this question were also received from Messrs. sworth, Lowry, Marrat, *and* Simpson, *the proposer.*

To IX.

To IX. QUESTION 97, *no anſwer has been received.*

X. QUESTION 98, *anſwered by Mr.* Newton Boſworth, *Peterborough.*

The firſt perſon who appears to have treated the ſubject of this queſtion with any degree of accuracy, was the celebrated *Leibnitz*, in the *Acta Eruditorum* of Germany, anno 1685. His method of determining the abſolute and relative preſſure of the Globe on each of the planes, though ſcientific and ingenious, is yet, I preſume, diffuſive and inelegant. His paper, however, is a valuable one, and worth conſulting by thoſe who deſire information on the ſubject. An Engliſh tranſlation of it may be found in "*Acta Germanica*," or Literary Memoirs of Germany and the North, VOL. II. pa. 73.

This Problem has alſo been inveſtigated by the late Mr. DITTON, in a publication entitled "*The General Laws of Nature and Motion.*" As his ſolution is more conciſe than that of Leibnitz, though evidently founded upon ſimilar principles, I ſhall take the liberty of extracting it.

"Let the globe H reſt upon the two inclined planes (fig. 240, pl. 16.) AC and CD; and let the angle ACD be a right one. Let G expreſs the abſolute gravity, or total *momentum* of the globe, which is ſhared betwixt the two planes (each of which bears its part) but in what proportion we are now to determine. Let AC = L, AB = A, DC = *l*, and DE = *a*. The angle ACD made by the two planes, being a right one by the hypotheſis, 'tis evident, that the quadrilateral figure GHFC is a perfect ſquare; or, which is all one, that the lines IHF, KGH, which are the lines of direction of the globe's perpendicular preſſure upon each plane, are parallel to thoſe planes reſpectively; viz. IHF parallel to AC, and KHG parallel to DC. And that this could not be were the angle ACD an acute or an obtuſe one, is obvious to any one that conſiders the matter. From hence then it follows, that each plane performs to the other, the office of a power acting with a direction parallel to the plane. Thus the plane AC ſuſtains all that force of the globe, by which it endeavours to deſcend upon the inclined plane DC, and which force, if the plane AC were taken away, or powers (acting with the direction KG parallel to the plane DC) would ſuſtain. So likewiſe the plane DC, ſuſtains the *momentum* of the globe upon the plane AC, which *momentum* would be balanced by a power acting with the direction IF parallel to AC, if the plane DC were removed. But beſides theſe preſſures, each plane is burthened with one of another

r kind, and that is the difference between the total abſolute *tum* of the globe and that relative or partial *momentum* with i the globe endeavcurs to deſcend upon that plane. For, if : of the planes were taken away, and a power was ſubſtituted : room of it, then though that power ſuſtained the burden h preſſed upon the plane before, yet the body ſtill reſting i the other plane, that plane is preſſed by it; and the quan- of that *momentum* by which the plane is preſſed is evidently t was juſt now aſſerted. To ſtate theſe proportions therefore, certain that the *momentum* by which the globe endeavours to end upon the plane DC, is $= \frac{G \times a}{l}$; this the plane AC ains. And the *momentum* by which the globe endeavours to cend upon the plane AC, is $= \frac{G \times A}{L}$; this the plane DC lains. But the plane DC ſuſtains alſo the *momentum* $G - \frac{\times a}{l}$; and the plane AC ſuſtains alſo the *momentum* $G - \frac{\times A}{L}$. Therefore the whole *momentum*, with which the plane C is burdened, is $= \frac{G \times A}{L} + \frac{G \times l - G \times a}{l}$; and the whole *mentum* with which the plane AC is burdened, is $= \frac{G \times a}{l}$ $\frac{G \times L - G \times A}{L}$. And the ſum of theſe two aggregates f *momenta* (with which both planes are burdened) caſting out con- adictories, will be $= \frac{G \times l \times L + G \times l \times L}{L \times l} = 2G =$ to rice the abſolute *momentum* of the globe; which is abſurd and mpoſſible. For the abſolute *momentum* of the globe is parted :twixt the two planes, and what they ſuſtain cannot poſſibly xceed that. We are therefore to take the halfs of theſe ſums of *omenta*, or (which amounts to the ſame) an arithmetical mean etwixt them. And then they will be $\frac{G \times A}{2L} + \frac{G \times l - G \times a}{2l}$, nd $\frac{G \times a}{2l} + \frac{G \times L - G \times A}{2L}$; the ſum of which is $= G$,

the

the abſolute *momentum* of the globe. The *momentum* of the glo then, with reſpect to each plane, is determined. Q. E. F."

But I apprehend the queſtion will admit of a ſolution, mu more elegant and ſimple, and yet equally deciſive: this I ha attempted to make out as follows.—

Since two ſides of each of the right angled triangles ABC, CE (fig. 240, pl. 16.) are given, the other ſide, being eaſily foun may be ſaid to be given alſo.

In order to attain its preſent ſituation, ſuppoſe the globe to h rolled along the plane CD from D towards C, and that w arrived at F, it is ſtopped in its progreſs by the other plane A at G. Now it is evident that the preſſure upon the plane AC, equal in effect to a power capable of ſuſtaining the globe upon other plane, acting in the direction GK, parallel to CD. Mechanics, as the power at K is to preſſure on the plane CD, is DE to CE. Therefore the preſſure of the globe on the pl CD, is to that on the plane AC, as CE is to DE.

Again: Imagine the globe to have deſcended down the p AC, and that it is ſtopped and ſuſtained by the other plane C which bears a weight equal to a power acting at I, in the direc IF, and capable of holding up the globe if that plane were ta away. Then the preſſure on AC is to that on DC, as BC i AB, or, the preſſure on DC is to that on AC, as AB is to BC which is the ſame as before, for it may eaſily be proved that C DE :: AB : BC, and the ſum of the preſſures on both planes, determined by each of the above methods, is equal to the who weight, or total *momentum*, of the globe.

From hence, and the nature of the queſtion, it follows, th if the planes be of equal length, the quantity of preſſure ea ſuſtains will be inverſely as the altitudes; if the altitudes be ali the preſſure will be directly as their lengths; and if both planes of equal length and altitude, the preſſure will be equal alſo.

Thus the particulars of this intereſting queſtion are truly det mined, without the "alternatives" of LEIBNITZ, or the "co tradictories" of DITTON; and I preſume the ſuperior ſimplici of the method I have uſed, will juſtify my deviation from au rities ſo reſpectable.

Ingenious ſolutions to this queſtion were alſo received from Me Lowry, Swale, *and* Thornoby.

XI. QUESTION 99, *anſwered by Mr.* W. Marrat, *Lincol*

Let ABC (fig. 241, pl. 16.) be the horizontal plane, A*a*, Cc the poſts, and *b*D, *c*D, *a*D the rafters, alſo let *bf*, *ae*,

…to DH. Draw *cs*, ea parallel to BC, CA. Then, …a right angle, in the △ *bcs* we have given the sides *bs*, *cs* and …angle *s* to find the side *bc* and the ∠ *scb*; alſo in the △ *bc*D …the three ſides given to find the angles D*bc*, and D*cb*. …the ∠ D*bf* = ∠ D*bc* − ∠ *scb*; the ∠ D*cg* = ∠ D*cb* − …Hence, in the △ D*bf* we have given D*b* and all the ∠s …*bf*, the diſtance of the poſt *Bb* from the point H, which is …under D. And in the △ D*cg* we have D*c* and all the ∠s to …the diſtance of the poſt *Cc* from the point H. In the ſame …may the angles be found in the △ D*ae*, and then the …, which is the diſtance of the poſt *Aa*, from the point H.

The ſame anſwered by Mr. John Harris.

H, (fig. 241, pl. 16.) be the point ſought, and join AH, …CH; alſo join DH and on it let fall the ⊥s *ae*, *be*, *ce* from …of the poſts *Aa*, *Bb*, *Cc*. Now, in the △ ACB, all the …being given, the ſine and coſine of the ∠ CAB is eaſily …which put = *s* and *c* reſpectively; alſo put the coſine of …CAH = *y*; its ſine will be = $\sqrt{1-y^2}$ (the radius being 1.)

…the coſine of the ∠ BAH will be = $cy + s\sqrt{1-y^2}$.

…by a well-known theorem, we ſhall have

$$AC^2 + AH^2 - 2AC \times AH \times y = CH^2,$$

$$AB^2 + AH^2 - 2AB \times AH \times (cy + s\sqrt{1-y^2}) = BH^2.$$

…Eu. I. 47, $CH = cg^2 = cD^2 - Dg^2 = cD^2 - \overline{D^2 - AH^2\ (ae^2)} + eg^2) = cD^2 - aD^2 - eg^2 + AH^2$ …$\times \sqrt{aD^2 - AH^2}$ = (becauſe *eg* = poſt *Aa* − poſt …1) $cD^2 - aD^2 - eg^2 + AH^2 - 2\sqrt{aD^2 - AH^2}$.

…$AC + AH^2 - 2AC \times AH \times y = cD^2 - aD^2 - eg^2 +$ …$- 2\sqrt{aD^2 - AH^2}$, or $2AC \times AH \times y = 2\sqrt{aD^2 - AH^2}$ …$aD^2 + eg^2 + AC^2 - cD^2$, that is, $2bxy = 2\sqrt{a^2 - x^2}$ …(putting $aD = a$, $AC = b$, $aD^2 + eg^2 + AC^2 - cD^2$ …and $AH = x$);

…ently $y = (\sqrt{a^2 - x^2} + m) \div bx$.

Again,

Again, $BH^2 = bf^2 = bD^2 - (\sqrt{aD^2 - AH^2} + ef^2)^2 =$ (because $ef =$ poſt $Aa -$ poſt $B\dot{b} = \frac{1}{2}$) $bD^2 - aD^2 - ef^2 + AH^2 - \sqrt{aD^2 - AH^2}$; therefore $AB^2 + AH^2 - 2AB \times AH \times (cy + s\sqrt{1 - y^2}) = bD^2 - aD^2 - ef^2 + AH^2 - \sqrt{aD^2 - AH^2}$, or $2AB \times AH \times (cy + s\sqrt{1 - y^2}) = \sqrt{aD^2 - AH^2} + aD^2 + ef^2 + AB^2 - bD^2$, that is, $ex \times (cy + s\sqrt{1 + y^2}) = \sqrt{a^2 - x^2} + n$ (putting $2AB = e$, and $aD^2 + ef^2 + AB^2 - bD^2 = n$). Whence, by ſubſtituting $(\sqrt{a^2 - x^2} + m) \div bx$ for y in this equation and reducing it, we get

$$x = \sqrt{\left(\frac{2qr - p^2}{2q^2} + \sqrt{\left(\frac{2qr - p^2}{2q^2}\right)^2 - \frac{r^2 - p^2a^2}{q^2}}\right)}.$$

Where $p = (2nb - 2mec)(b - ec) + 2mc^2s^2$, $q = (b^2 + 1) \times c^2s^2 + (b - ec)^2$ and $r = (a^2 + m^2)c^2s^2 + (nb - mec)^2 + (b - ec)^2 \times a^2$.

XII. QUESTION 100, *anſwered by Mr.* Lowry.

Let AB (fig. 242, pl. 16.) repreſent the horizon, Z the zenith, P the pole; OP, PS, the hour circles making given angles ZPS, ZPO, with the meridian AZB; SZ, OZ, the azimuth circles. *mn* the parallel of declination, O and S the ſtar's poſition, and OP = SP the co-declination required.

Now while the ſtar paſſes over the portion OS of the parallel of declination, the increaſe of azimuth (not the increaſe of declination) muſt be the greateſt poſſible; therefore the angle OZS muſt be a *maximum*, while the ſide ZP and angles ZPS, ZPO are conſtant, and the other ſides variable. When this takes place it is evident, from *Cotes' De Eſtima Error in mixt Mathem. Prop. XVI.* that

ſin. ZO : ſin. POZ :: ſin. ZS : ſin. PSZ, and by trigo.
ſin. ZO : ſin. ZPO :: ſin. ZP : ſin. ZOP; therefore
ſin.^{2} ZO : ſin. ZPO :: ſin. ZS $\times$ ſin. ZP : ſin. PSZ. Again,
ſin. ZS : ſin. ZPS :: ſin. ZP : ſin. PSZ, therefore,
ſin.^{2} SZ : ſin. ZPS :: ſin. ZP $\times$ ſin. ZS : ſin. PSZ;
therefore by equality ſin.^{2} ZO : ſin. ZPO :: ſin.^{2} ZS : ſin. ZPS.
From Z, on the great circles PO, PS, demit the perpendiculars ZE, ZI, and lay off the arch FP = EP; make FH perpendicular

and = to EZ, and join SH with a great circle. Then the
s OZE, SFH are evidently equal in every reſpect, there-
e problem is reduced to this, *viz.*—From the given points
ſo to draw the arches ZS, HS to interſect in the circle FIS,
eir ſines may obtain a given ratio.
effect this, put a and b for the ſine and coſine of IF, m and n
: coſines of FH and ZI, p and q for the ſines of ZPO and
reſpectively, and x for the ſine SI, the coſine of which will
$(1 - x^2)$ to radius unity.
d by trigonometry the coſine of ZS $= n\sqrt{(1 - x^2)}$, the coſ.
$: b\sqrt{(1 - x^2)} - ax$, and coſ. SH $= bm\sqrt{(1 - x^2)} - max$;
fore, ſin.^{2} SH $= 1 - (bm\sqrt{(1 - x^2)} - max)^2$;
i. by queſt. $1 - x^2 (1 - x^2) : 1 - (bm\sqrt{(1 - x^2)} - max^2)^2 :: q : p$.
e by reduction, putting $v = (qma)^2 - q(bm)^2 - pn^2$, $w =$
a^2, and $r = q(bm)^2 - pn^2 + p - q$, we get $vx^2 + wx\sqrt{}$
$- x^2) = r$, whence, by completing the ſquare, x^2 is found $=$
$+ rv) \div (v^2 + w^2) \pm \sqrt{[(\frac{1}{2}w^2 + rv)^2 \div (v^2 + w^2)^2 -}$
$- (v^2 + w^2)]$. From this SI becomes known, and SP is
ly determined.

XIII. QUESTION 101, *anſwered.*

ANALYSIS, by Meſſrs. I. T. M'Doneld *and* Swale.

ig. 175, pl. 12, let ACB be the △ required to be conſtructed,
baſe AB and perp. CD of which are given; ABCE its cir-
ſcribing circle whoſe diameter is Cm; alſo let CKE be the
biſecting the vertical angle.
m by ſim. △s AC : Cm :: CD : CB,
and EC : Cm :: CD : CK;
ef. rect. ECK = rect. ACB = 2 rect. AKB, per queſtion.
rect. AKB = rect. EKC; theref. rect. ECK = 2 rect. EKC;
hence EC = 2EK, or EL = LN;
ef. EL is given, and a circle deſcribed through the three
nts A, E, B, will cut NC, drawn parallel to AB, in C the
ex of the △ required.

SYNTHESIS, by Meſſrs. Harris, Lowry, *and* Nicholſon.

Conſ. Take AB = the given baſe and biſect it at right angles
h the indefinite perp. FLN, then take LE, LN, each = to the
en perpendicular, and through the three points A, E, B, de-
be a circle meeting NC, drawn ‖ to AB, in C; join AC,
, and ACB is the △ required.
Demon. Draw CD ⊥ to AB and join CE. Then AB is the
en baſe, and CD = LN = the given perp.

And,

And, since EL = LN, EC is = 2EK;
therefore 2EK × KC = EC × CK; but EK × KC = AK × KB
therefore 2AK × KB = EC × CK = AC · CB.

Q. E. D.

XIV. QUESTION 102, *answered by Messrs.* Harris, Lowry, Nicholson, *and* Swale.

Conf. Take NL = the given perp. (fig. 175, pl. 12.) and LE such, that the rect. NLE may be = to the given rectangle. Then make NC perp. to EN and = to half the given diff. of the segments of the base; draw CE, and perp. thereto CF to meet EN produced in F, and through the points E, C, F, describe a circle to meet ALB, drawn through L parallel to NC, in A and B; join AC, CB, and ACB will be the △ required.

Demon. Draw CD perp. to AB, and conceive W to be the point of contact of the inscribed circle. Then CD = LN = the given perp. and AD — DB = 2DL = 2NC = the given diff of the seg. of the base; also, by Prop. VIII. Art. LII. Vol. I. the rect. AWB = rect. NLE = the given rectangle by construction.

Q. E. D.

XV. QUESTION 103, *answered by Mr.* Lowry.

This prob. is unlimited, any one of the three *data* being determinable from the other two. For, in the right ∠d △ LTQ (fig. 175, pl. 12.) LT is = to half the diff. of the sides, QT = to half the distance intercepted by the perpendiculars from the extremities of the base upon the line bisecting the vertical ∠: but any two of these being given the third is also given; therefore some other *datum* is necessary to limit the problem: such as the base, perp. line bisecting the base, line bisecting the vertical ∠, ratio of that line and base, &c. &c.

If the line bisecting the base be given, the △ may be determined thus. From L, to QT produced, apply LC = the given bisecting line, and draw CA, CB to make the ∠s ACQ, BCQ each = to half the vertical ∠; produce QL to meet AC in P, and take AP = LT = half the given diff. of the sides, and draw ALB, and ABC is evidently the △ required.

Mr. Swale, the proposer, says, he intended the question to have been as follows: "Given the segment of the line bisecting the vertical ∠ included between the perpendiculars from the ∠s at the base, and the difference of the sides, to construct the △, when the said bisecting line divides the distance between the middle of the base and the perp. from the vertical ∠ in a given ratio."

To which he gives the following

ANALYSIS.

ANALYSIS. Suppoſe it done, and ABC (fig. 243, pl. 16.) the Δ required; AU, BT the ⊥s upon the biſecting line CU, and L the middle of the baſe. Join LT, LU, and draw the ⊥ s CD, LQ; produce LQ to meet DV, drawn ‖ to TU, in V. Then becauſe the ratio of LK to KD is given, and LQ a given line, QV is given. And becauſe the circle LTDU is given in magnitude and poſition, and VD parallel to QT given by poſition, the point D is given, and from hence the Δ may be eaſily conſtructed.

Obſervations ſimilar to thoſe of Mr. Lowry's *were received from Meſſrs.* Harris, M'Doneld, *and* Nicholſon.

VI. QUESTION 104, *anſwered by Meſſrs.* Harris, Lowry, Nicholſon, *and* Swale.

Conſ. Take AF (fig. 244, pl. 16.) = the diff. between half the ſum of the ſquares of the ſides, and the ſquare of half the diff. of the ſegments of the baſe; alſo take AS = ½ AF, and draw SD perp. to AF and let it meet a ſemi-circle deſcribed thereon in D; draw AD, and on it continued take DC = AD, and DP = half the diff. of the ſegments of the baſe. At P erect the perp. PB to meet FB, drawn ‖ to AC, in B; join AB, BC, and ABC is the Δ required.

Demon. By *conſ.* AD = DC, therefore AP — PC = 2DP = the given diff. of the ſegments of the baſe. Again, $AB^2 + BC^2 = 2AD^2 + 2DB^2 = 2AD^2 + 2DP^2 + 2PB^2 = 2AF^2 + 2DP^2$; but, by *conſ.* AF^2 = the diff. between half the given ſum of the ſquares of the ſides, and the ſquare of half the diff. of the ſegments of the baſe (DP); therefore $AB^2 + BC^2$ = the given ſum of the ſquares. Moreover, ſince SF = 2AS; SD^2 is = $2AS^2$, and $DF^2 = 2AD^2$: therefore $DF^2 \times AD$ is a maximum by *Theo. XVIII. Simpſon on the max. et min.* conſequently $BP^2 \times AC$ is alſo a maximum. *Q. E. D.*

The ſame otherwiſe, by Mr. M'Doneld, *near Durham.*

Put s^2 = the ſum of the ſquares of the ſides, x = the leſs ſegment of the baſe, and d = the diff. of the ſegments; then the baſe = $d + 2x$, and the ſquare of the perp. = $\frac{1}{4}s^2 - \frac{1}{8}d^2 - dx - x^2$, the product of theſe, by the queſt. muſt be a maximum; therefore, its fluxion being made = o, the reſulting equation is $s^2 - \frac{1}{2}d^2 = 6(x + \frac{1}{2}d)^2$, that is, the ſum of the ſquares of the ſides *minus* half the ſquare of the diff. of the ſegments of the baſe is = to ſix times the ſquare of half the baſe.

Hence, if AF be taken ſuch that twice its ſquare may be = the given ſum of the ſquares — half the ſquare of the diff. of the ſegments of the baſe, and AS be made = ½AF, and SD be drawn ⊥ to AF, meeting a ſemi-circle deſcribed thereon in D, AD will be = half the baſe of the ∠, from hence the △ is readily determined.

For, $AS \cdot SF = SD^2 = 2AS^2$;
therefore $AD^2 = 3AS^2$ and $DF^2 = 6AS^2$;
hence $2AD = DF$, or $AF^2 = 3AD^2$;
conſequently $2AF^2 = s^2 - \frac{1}{2}d^2 = 6AD^2$.

Q. E. D.

In the ſame manner was this queſtion anſwered by Mr. Gregory.

XVII. QUESTION 105, *anſwered by Mr.* Nicholſon.

Conſ. Find DE (fig. 245. pl. 16.) by Eu. 29, VI, and Simpſon's Geometry, Prob. III. B. VI. ſuch, that the rectangle contained under 2DE and the ſum of twice the given diff. of the ſides, twice the given ſegment and 2DE, may be = to twice the diff. of the ſquare of the baſe, and the ſquare of the diff. of the ſides. In ED produced take EA = to the given diff. and DC = to the given ſegment; make AB = to the given baſe and BC = to CE; ſo ſhall ABC be the △ required.

Demon. Join BD; ſince AB = the given baſe, DC = the given ſegment, and AE = the given diff. of the ſides; we have only to prove that BD is perp. to AC, in order to which, we have,

by *conſ.* $2AB^2 - 2AE^2 = (2AE + 2CD + 2DE) \times 2DE$;
therefr. $AB^2 - AE^2 = 2AE \cdot ED + 2ED^2 + 2CD \cdot DE$,
and $AB^2 + DC^2 = (AE + ED)^2 + (ED + DC)^2$,
conſequently $AB^2 - BC^2 = AD^2 - DC^2$;
therefore BD is perpendicular to AC. *Q. E. D.*

The ſame by Mr. Lowry.

Conſ. Take CD = the given diſtance intercepted between the perp. and vertical ∠, DQ = the given diff. of the ſides; draw DB perp. to CD, and take DL = CD; alſo draw CLP and apply thereto QP ſuch, that QP^2 may be = to the ſum of the ſquares of the baſe and ſegment DC; demit the perp. PE, and make EA = DQ. To DB apply AB = the given baſe, and join CB; then ſhall ACB be the △ required.

Demon. By conſ. AB = the baſe, and BC = the given ſegment. And, ſince CD = DL, CE is = EP, and AD = EQ;
therefr. $EC^2 + EQ^2 (AD^2) = QP^2 = AB^2 + DC^2$, by conſ.

$BC^2 + AD^2 = AB^2 + DC^2$;
$EC^2 + AD^2 = BC^2 + AD^2$, and $EC = BC$;
$AC - BC = AE = DQ$ = the given diff. of the ſides.
Q. E. D.

The ſame by Mr. Swale.

Analyſis. Let us ſuppoſe the thing done, and ACB the △ to be determined. Demit upon the oppoſite ſide AC the perp. BD; make Dd = DC, CE = CB, and join Bd, BE. Then, by Eu. II. 12,

$AB^2 = AE^2 + EB^2 + 2AE \cdot ED$
$= AE^2 + Ed^2 + 2AE \cdot ED + 2Ed \cdot dD + EC^2$
$= AE^2 + (Ed + 2dD) \cdot Ed + EC^2 + 2AE \cdot ED$
$= AE^2 + (Ed + EC) \cdot EC + 2AE \cdot ED$
$= AE^2 + (EC + EA) \cdot 2ED$.

Whence $(EC + EA) \cdot 2ED = AB^2 - AE^2$.
Now make $AF = dC$, and $EG = \frac{1}{2}dC$,
then $EC + EA = dF$, and $2ED = 2dG$.
Hence, $2GD \cdot dF = AB^2 - AE^2$ = a given ſpace, and FG is a given line.

Conſequently the Prob. is reduced to the 29th Prop. of the 6th Book of Profeſſor Playfair's Edition of Euclid's Elements.

Mr. M'Doneld *alſo favoured us with an anſwer to this queſtion.*

XVIII. QUESTION 106, *anſwered by Mr.* Lowry.

Geometrical Analyſis. Suppoſe it effected, and that ACB (fig. 246, pl. 16.) is really the △ which was to be conſtructed. Let AHBG be the circumſcribing circle, having its diameter perp. to the baſe AB. Join AH, CG, and demit the ⊥s GD, HF, CK, CP and QB, the latter being perp. to CG; alſo join DK, IF, and IQ.

Then $RQ = QB$, and $AI = IB$,
theref. IQ is ∥ to AC, and = AD = FC = $\frac{1}{2}$ the given diff. of the ſides; conſequently IF is equal and parallel to QC.

In the ſame way it may be ſhewn that DK is ∥ to AH:
theref. ∠ DKI = ∠ DFI = $\frac{1}{2}$ the given vertical ∠ ACB;
wherefore the points D, I, F, K are in a circle,
and conſequently the rect. DEF = to the rect. IEK.

Again, becauſe the ratio of AP to PB is given, the ratio of AI to IP, that is, the ratio of AE to EC, that is, the ratio of IE to EK, is given, and IK = PC is given in magnitude;
therefore the rect. DEF = the rect. IEK, is given.

Hence, there is given AD = FC, the ratio of AE to EC, and

the rectangle DEF to determine the lines AE, EC, which is done at queſtion 61, Repoſitory.

The ſame by Mr. Nicholſon.

Conſ. In fig. 247, pl. 16, take M to the given perp. in the given ratio of the ſegments of the baſe, the antecedent being greater than the conſequent, and make the ſquare upon S = to the diff. of the ſquares upon M and the perp. Upon AE = the given diff. of the ſides erect the perp. AH = S, and join HE, and in AE produced take EI = the given perp. to HE, produced if neceſſary, apply IK = M, and draw HC ∥ to KI to meet AE in C. Now upon AC deſcribe a ſemi-circle, in which apply CD = the given perp. join AD, and produce it indefinitely; then to AD apply CB = CE, and ACB will be the △ required.

Demon. By *Conſ.* AE = AC — BC the given diff. and CD is perp. to AB and equal to the given one. Therefore we have only to prove that AD and DB have the given ratio.

By the conſtruction M = IK, and CD = EI;
theref. by ſim. △s, &c. CH : M :: CE : CD
or $AC^2 + M^2 = CD^2 : M^2 :: CB^2 : AC^2 - AD^2$,
and by diviſion $AC^2 - CD^2 : M^2 :: DB^2 : CD^2$,
that is, AD : M :: DB : CD,
or, AD : DB :: M : CD. *Q. E. D.*

Anſwers to this queſtion were alſo received from Meſſrs. Mc. Doneld, Swale, *and* Thornoby.

XIX. QUESTION 107, *anſwered by Mr.* Harris.

Conſ. From any point C (fig. 248, pl. 16.) draw C*a*, C*b* making the angle *a*C*b* with each other = the given vertical ∠; take C*a* of any length, and upon it conſtitute the parallelogram C*adb* = to double the given area, then biſect the ∠ *a*C*b* by the line COF, and take CO, CF ſuch, that the rect. OCF may be = to the rect. *a*C*b*, and OF = the given diſtance between the centres. Biſect OF in E, and about the centre E, with the diſtance EO or EF deſcribe a circle cutting the lines C*a*, C*b*, in A and B, and join AB; then ACB is the △ required.

Demon. Since EA, EO, EB are equal, it is evident that O is the centre of the circle inſcribed in the △ ACB, and F the centre of the circle touching the baſe and the continuation of the ſides CA, CB, and OF = the given diſtance.

Again, by *cor.* to Qu. 63, Rep. the rect. ACB = rect. OCF: but by *conſ.* rect. OCF = rect. *a*C*b*; ∴ rect. ACB = rect. *a*C*b*, and theref. the area of the △ ACB = area △ *a*C*b* = ½ the parall. C*adb* = the given area by *conſ.* *Q. E. D.*

The

The ſame by Mr. Lowry.

Conſ. Take OF (fig. 249, pl. 16.) = the given diſtance, and make ∠ FOD = ½ the vertical ∠. On OD drop the perp. FD, and continue DO, till the rect. ORD be to the given area as OD to FD, and complete the parallelogram RCDE, and from the centres O and F, deſcribe two circles to touch CE as at P and E; draw the tangents AB, AC, and ABC is the △ required.

Demon. OF is the given diſt. and the ∠ ACB = ACO + BCO = 2FOD = the given angle.

Again, by ſim. △s, OD : FD :: OR : OP :: ORD : OP · RD = OP · CE = the △ ABC:
but, by *conſ.* OD : FD :: rect. ORD : the given area,
therefore the △ ABC = the given area. *Q. E. D.*

This problem was conſtructed in a ſimilar manner by Mr. Gregory.

The ſame anſwered by Mr. Nicholſon.

Conſ. Upon PL = the given diſtance as a hypothenuſe, conſtruct the right ∠d △ LGP (fig. 250, pl. 16.) having the ∠ P = half the given vertical one. Take GM² = the given area, and GT to TP as GL to GP; join MT and produce it to meet PI drawn ‖ to GM. Biſect MI in H, and GP produced apply HC = to HI, and draw CD ‖ to PL to meet IP, GO produced in O and D. Then with centres O and D and radii OP, GD deſcribe the circles O and D, to touch which draw AB to meet CG in A, and CB (the tangent to both circles) in B; then ABC is the △ required.

Demon. Upon the diameter MI deſcribe a circle, which by the conſtruction will paſs through C; produce MG, CG to meet the circumference in K and E;
then by the circle EG = PC, and GK = PI;
and therefore GC × CP = MG × PI:
but by ſim. △s and *conſ.* GM:PI (:: GT : TP :: GL:GP) ::PO:PC,
and by diviſion, &c. GM² = GC × PO = the given area,
as it is well known that 2GC = AC + CB + AB;
and by parallels DO = LP = the given diſt. *Q. E. D.*

The ſame by Mr. Swale.

Analyſis. Let us ſuppoſe the △ ACB (fig. 192, pl. 13.) to be that required, and the ſeveral lines drawn as in the figure. Then ſince OS (=2OI=2AI=2BI) and the ∠ACB (=2∠IAB=2∠IBA) are given, AI(=IB) and the ∠IBA(=IAB) will be given; and conſequently

ſequently the △ AIB will be readily determined. Now AB be known, and the rect. AB × CR = a given one, CR will known, and conſequently the △ ACB.

The Geometrical Analyſis by Mr. M'Doneld, *was very li different from the above.*

XX. QUESTION 108, *anſwered by Mr.* Lowry.

Let (fig. 251, pl. 17.) be the place of the ring when the ro in a vertical poſition, then 2AS will be equal to the difference tween the length of the ſtring and rod. Now let the rod be ſ poſed to revolve with an uniform angular velocity about the pc A till it comes to the poſition AF, and let C be the place of ring at that time. Draw AC, FC, the latter meeting the verti line BAR in R; drop the perpendiculars FG, CQ, and contir the latter to meet FM, drawn parallel to AB in M. Then, Mechanics, the angles QAC, CFH are equal, therefore, th ſupplements, the angles ACQ, TCH, or QCR are equal; c ſequently the right angled △s RQC, AQC, will be equal in reſpects, for the ſide QC is common to them both. This bei premiſed, put $a = RF = AC + CF$ = the given length of ſtring, $b - AF = AB$ = the given length of the rod, $c = A$ $x = SQ$ the abſciſſa, and $y = QC$ the correſponding ordinate the required curve, then will $AQ = QR = x + c$, $RG + A = 2(x + c + AG)$.

And by Trigonom. $RG + AG : RF + AF :: RF - AF : A$ or, $2(x + c + AG) : a + b :: a - b : 2(x + c)$; theref $AG = (a^2 - b^2) \div 4(x + c) - x + c) = m^2 - 4(x \times$ m^2 being put $= a^2 - b^2$, hence, $RG = AR + AG = 2AQ$ $AG = 2(x + c) + (m^2 - 4(x + c)^2 \div 4(x + c) = (4(x$ $c)^2 + m^2) \div 4(x + c)$.

Then by ſimilar triangles $RG : RF :: x + c : AC$, or, $(4(x +$ $+ m^2) \div 4(x+c) : a :: x+c : 4a(x+c)^2 \div (4(x+c)^2+m^2) = A$ but by Eu. I. 47. $AC^2 - AQ^2 = QC^2$; hence $(4a(x+c)^2 \div (4(x+c)^2+m^2))^2 - (x+c)^2 = y^2$, or by reductic

$$\left(\frac{4a}{m^2 \div \overline{x+c}^2 + 4}\right)^2 - \overline{x+c}^2 = y^2,$$ the equation of the requi

curve.

Remarks. If $x = 0$, y is $= 0$; and when $x = b$, or $x +$ $= BS = SV$, y is alſo $= 0$, ſo that the curve paſſes through and V.

When x is $= \frac{1}{2}m$ the rod will be parallel to the horizon, a y *a maximum* and $=$ to $\frac{1}{2}b$.

XX

XXI. Or PRIZE QUESTION, 109, *anſwered by Mr.* Thornoby.

A triangle anſwering the conditions of the problem may be found thus. Let ASB (fig. 255, pl. 16.) be the circle given in magnitude and poſition, whoſe centre is O, and P the given point. Join PO and perp. thereto draw PS to meet the circle S; join OS, and through P draw APEB perp. thereto meeting the circle in A and B. Divide AB in Q ſo that AP : PB :: AQ^2 : QB^2, and on ES, (produced if neceſſary,) take ED = the given perp. and draw CD parallel to AB; then by the lemma on page 336, Simpſon's Algebra, deſcribe a circle to interſect CD in C ſo that if AC, BC, be drawn, AC may be to BC as AQ to QB, and ABC will be the △ required.

For the baſe AB paſſes through the given point P and its extremities falls in the circumference of the given circle, and ED = CI = the given perp. and by conſtruction AC^2 : BC^2 : AQ^2 : QB^2 :: AP : PB.

The ſame by Mr. John Surtees, *Sunderland.*

The baſe AB (fig. 256, pl. 16.) being drawn any how through P the given point, then AP, PB and AB are given. Put b = AE = EB, p = EH = CD the given perp. n = AP, m = PB and x = ED. Then $(b + x)^2 + p^2 = AC^2$, $(b - x)^2 + p^2 = BC^2$, and per queſt. $BC^2 \times m = AC^2 \times n$, or $\frac{m+n}{m-n} \times 2bx - x^2 = p^2 + b^2$, or $\frac{AB^2}{2PE} \times x - x^2 = AH^2$. Hence this

Conſtruction. Take EL = 2EP = the difference between the ſegments of the baſe made by the given point, and EM a third proportional to LE and AB. Upon LM, EM let the ſemicircles LGM, EFM be deſcribed, and erect the perp. EG and take therein EH = the given perp. and join AH; alſo in EG take EI = AH and through I draw IF parallel to AB. Drop the perp. FD which will meet HC, drawn parallel to AB, in C the vertex of the △ required; therefore join AC, CB, and ACB will be a △ anſwering the conditions of the queſtion.

Ingenious ſolutions were alſo received from Meſſrs. Harris, Hill, and Swale.

The Medal for ſolving the Mathematical Prize Queſtion is decided in favour of Mr. *S. Thornoby*, who will pleaſe to ſend for it to Mr. Glendinning's, by whom it will be delivered, free from expence, to any part in London.

ARTICLE

ARTICLE XVIII.

MATHEMATICAL QUESTIONS,

To be anſwered in Number IX.

I. QUESTION 131, *by Mr.* Thomas Milner, *Teacher of the Mathematics at Lartington, near Barnard-Caſtle.*

IF my age in days be equal to the leaſt whole number that can be divided by 15, 21, 27, and 29, the remainders will be 8, 11, 5, and 18 reſpectively. Query my age?

II. QUESTION 132, *by Mr.* J. Collins, *School-maſter, Kenſington.*

Given the diameter of a ſemi-circle = 24, 'tis required to inſcribe therein a parallelogram whoſe longeſt ſide ſhall be parallel to the diameter, ſo that the area of the ſame may be equal to the area of the greateſt circle that can be inſcribed in the remaining ſegment. Quere the dimenſions of each?

III. QUESTION 133, *by Mr.* Ralph Simpſon, *Croxdale.*

To find three numbers ſuch, that not only the product of every two, but alſo the product of all three added to a given ſquare number, ſhall make a ſquare number.

IV. QUESTION 134, *by* Matheſis, *near Bath.*

It is required to draw a chord through a given point within a given circle, ſuch, that the parts thereof intercepted by that point and the periphery may obtain a given ratio?

V. QUESTION 135, *by Mr.* James Lee, London.

Given the two ſides, and the ſum of the baſe and perp. of any plane △ to deſcribe it?

VI. QUESTION 136, *by Mr.* Olinthus Gregory.

The perimeter of an iſoſceles triangle is equal to $16 + 8\sqrt{5}$ and

and its vertical angle is 36°; it is required to find the length of each side of this triangle without having recourse to trigonometrical tables?

VII. QUESTION 137, *by Mr.* William Paſſman, *Teacher of the Mathematics, at Hull.*

Of all cones having the ſame external ſurface, excluſive of the baſe, required that, which will vibrate quickeſt when ſuſpended by the vertex?

VIII. QUESTION 138, *by Mr.* B. Haynes, *Land-Surveyor, at Saliſbury.*

Being employed to plan an Eſtate, part of which conſiſted of a mountain of conſiderable height, bordered on one ſide by a plain, now I would requeſt to be informed how I muſt proceed to lay down the ſaid mountain properly in my plan, and alſo how I may find the angle of its elevation with the horizon, making uſe of a Gunter's Chain only in the field?

IX. QUESTION 139, *by Mr.* John Harris.

An horizontal dial being made for one place; to ſet it up in another place that has a difference of longitude from the firſt, ſo, that it may ſhew the true hour of the day, in the place it was made for.

X. QUESTION 140, *by Mr.* W. Francis, Jun.

There is a light-houſe built upon a rock, one ſide of which ſtands perpendicular, facing the ſea ſhore, on the top whereof a pendulum, that meaſured 4·971 inches from the point of ſuſpenſion to the centre of its circular bob of ·6 inch radius, was obſerved to vibrate 50399 times in five hours; now admitting the earth to be a perfect globe 7958 miles in diameter, I would know the height of the light-houſe, being $\frac{2}{3}$ that of the rock; and alſo the diſtance of a privateer from the foot of the rock, at which I ſaw the fire from a gun, 20 vibrations of the pendulum, before the ſound reached my ear?

XI. QUESTION 141, *by the Rev. Mr.* L. Evans.

If the given trapeziod ABCD (fig. 253, pl. 16.) right angled at B and C, be divided into two equal parts, by the line CF, drawn from the point E biſecting BC, it will be as AB + CD : AD :: AB

AB : DF. But, if the line EF, be drawn from the point F biſecting AD, I ſay it will be as AB + CD : BC :: AB : CE. Query Demonſtration?

XII. QUESTION 142, *by Mr.* John Howard.

Given the ſegments of the baſe, and the ſum of the ſides to determine the great circle ſpherical triangle.

XIII. QUESTION 143, *by Mr.* Wm. Marrat.

If a tin can, in the form of a conical fruſtum, whoſe top diameter is $6\frac{1}{2}$, bottom diameter 8·7, and perp. altitude $11\frac{1}{2}$ inches reſpectively, be immerſed in water with its greater end downwards, and its axis perp. to the ſurface, and it ſink to the depth of 2 inches meaſured on the ſlant ſide: required the thickneſs of the tin; the ſpecific gravity of tin and water being 7300, and 1000?

XIV. QUESTION 144, *by Mr.* Samuel Thornoby.

It is required to cut a common playing card (the length being leſs than twice its breadth) into the three pieces ſo that when put together they may make a ſquare.

XV. QUESTION 145, *by Mr.* Gregory.

The circumference of the circle AFKG (fig. 254, pl. 16.) is divided into an even number of equal parts, as AB, BD, DF.... ....GF, &c. and the dividing points are connected by parallel chords, as CB, ED, GF, &c. Now the ſum of all the chords, is known to be $168 + 84\sqrt{3}$, and the length of the greateſt chord is 84: from which it is required to determine the number of chords, and the length of each.

XVI. QUESTION 146, *by* Philo, *Newcaſtle.*

Moſt authors who have written on the theory of Water Mills make the ratio of the ſtream (in underſhot wheels) to that of the water-wheel as 3 to 1 when the effect produced is the greateſt poſſible: But Mr. Waring in the 3rd vol. of the American Philoſophical Tranſactions, page 144, makes the ratio as 2 to 1. It is here required for the information of thoſe concerned in the conſtruction of ſuch works, to inveſtigate the truth or falſity of Mr. Waring's Theory.

XVII

XVII. QUESTION 147, *by Mr.* Haynes, *Salisbury.*

In a gentleman's park there is a piece of water on which I obſerved a ſmall ſkiff at anchor, having two maſts, the diſtance between which was known. By ſtanding on the brink of the water in a line with the maſts, I obſerved that a ray drawn from the top of the higheſt to the edge of the water cut the top of the loweſt. Now being only poſſeſſed of an inſtrument for meaſuring vertical angles, in what manner muſt I proceed to obtain the breadth of the water in the direction of the maſts, admitting that it is acceſſible on all ſides?

XVIII. QUESTION 148, *by Mr.* I. H. Swale.

It is required to find a point P, ſuch, that lines being drawn to it from the three angular points A, C, B, of a given triangle ACB, the ſum of the ſolids $PA^3 \times AC$, $PB^3 \times BC$, and $PC^3 \times AB$ may be equal to a given ſolid.

XIX. QUESTION 149, *by Mr.* I. T. M'Doneld.

Given the baſe, the vertical angle, and the ratio which the line biſecting the vertical angle has to the difference of the ſegments of the baſe made thereby to conſtruct the triangle.

XX. QUESTION 150, *by Mr.* William Peacock, *Land-Surveyor, at Birmingham.*

Given the baſe, the vertical angle, and the rectangle under the ſides and the line biſecting the baſe to conſtruct the plane triangle.

XXI. QUESTION 151, *by Mr.* John Lowry, *Exciſe-Officer, Birmingham.*

Given the line biſecting the baſe, and the line biſecting the vertical angle, to determine the triangle, when the ſolid, whoſe baſe is the ſquare of the perpendicular and altitude the difference of the ſegments of the baſe made by the biſecting lines, is a maximum.

XXII. QUESTION 152, *by Mr.* Geo. Brown, *Teacher of the Mathematics, at Howdon-pans, near North Shields.*

Let AB, AC be two ſtraight lines given by poſition, let B and C be given points in theſe lines, and let P be a given point without

them. It is required to draw two ſtraight lines PD, PE meeting the given lines in D and E ſo that the angle DPE may be of a given magnitude, and ſo that BD may have to CE a given ratio.

XXIII. QUESTION 153, *by Mr.* James Wolfenden, *Hollinwood, near Mancheſter.*

In a given circle to inſcribe a triangle ſuch, that the triangle formed by joining the three points, where perpendiculars from the extremities of the baſe and vertical angle, meet the line biſecting that angle and the baſe reſpectively, ſhall be a maximum.

XXIV. QUESTION 154, *by Mr.* Wm. Wallace, *Perth.*

It is required to deſcribe a triangle which may have its angles upon the circumferences of three given circles, and which may be ſimilar to the triangle formed by ſtraight lines joining their centres, and alſo, which may have one of its ſides paſſing through a given point.

XXV. QUESTION 155, *by Mr.* James North.

To aſſign the correct fluents of the equation $\dot{x}e^{\frac{2x}{a}} = -\frac{\dot{z}}{v^2}$;

where v is the coſine of the arc or angle z, to the radius 1, and $e = 2{\cdot}718281828$ the number whoſe hyp. log. is 1; x being $=0$ when $z =$ the given arc b. The fluent to be in finite terms of the ſine, coſine, and tangent of the arc z, and given quantities.

XXVI. PRIZE-QUESTION 156, *by Mr.* John Howard, *Mathematician, Newcaſtle-upon-Tyne.*

Suppoſe a ſphere to deſcend by its own gravity along any given curve, placed in a plane that is perpendicular to the horizon; it is required to determine that point where it will quit the curve?

ART.

ARTICLE XIX.

MATHEMATICAL LUCUBRATIONS.

Mr. WILLIAM WALLACE, *Aſſiſtant-Teacher of the Mathematics in Perth Academy.*

)POSITION I. THEOREM. Fig. 258, *Pl.* 17.

ET ABC be a triangle inſcribed in a circle; from any point V in the circumference, let e be drawn perpendiculars VD, VE, VF to the i of the triangle: the points D, F, E lie in a ght line.

oin AV, BV, CV; join alſo DF and EF. Be- e the angles VFC, VEC are right angles, a le may be deſcribed about the quadrilateral CE: in like manner it appears that a circle may deſcribed about the quadrilateral VFDA; efore the angles VAD and VFD are together al to two right angles; but, becauſe VABC is a drilateral in a circle, VAD is equal to VCE. ch, (ſince the points V, F, C, E are in a circle) qual to VFE; therefore the angles VFE, VFD together equal to two right angles; therefore and FE lie in a ſtraight line.

or. The triangle DVE is ſimilar to the triangle C, alſo DVF is ſimilar to BVC, and EVF is lar to BVA.

PROP. II. LOCUS. Fig. 259, *Pl.* 17.

et ABC be a given triangle, let a point V be n ſo, that if VD, VE be drawn perpendicular AB, BC the ſides of the triangle, and VF per- licular to AC the baſe, and FD, FE be joined, ratio of FD to FE may be given; the point V be in the circumference of a given circle.

Join AV, CV and draw DG and BH perpendicular to AC. Becauſe the angles ADV, AFV are right angles, the points A, D. V, F are in the circumference of a circle, therefore the angle VAD is equal to VFD, that is to the alternate angle FDG, now ADV, FGD are right angles, therefore the triangles ADV, DGF are ſimilar, hence
FD:AV=DG:DA=BH:BA, and by alternation
FD:BH=AV:BA: in like manner it appears that
BH:FE=BC:VC. Therefore
FD:FE=BC·AV:BA·CV, and becauſe the ratio of FD to FE is given by hypotheſis, the ratio of BC·AV to BA·CV is given, and ſince BC, BA are given lines, the ratio of AV to CV is alſo given, now the points A and C are given, therefore, by a well known propoſition, the point V is in the circumference of a given circle.

From the preceding analyſis it appears that the given ratio of DF to FE muſt not be the ſame with the ratio of BC to BA, for in that caſe AV would be equal to VC and the point V would be in a given ſtraight line.

The *locus* of V may be found by the following conſtruction: Let the given ratio be that of P, a given line to AB a ſide of the triangle, but P muſt not be equal to BC: divide AC at K and *k* ſo that P:BC=AK:KC=A*k*:*k*C; a circle deſcribed on K*k* as a diameter ſhall be the locus required: that is, if from any point V in the circumference there be drawn perpendiculars VD, VE to the ſides of the triangle and VF perpendicular to the diameter of the circle, and if FD, FE be joined, FD ſhall be to FE in the given ratio of P to AB, For the points K, *k* being found as in the preceding conſtruction, it is well known that if ſtraight lines be drawn from any point V in the circumference to the points A and C, AV:VC=AK:KC=Ak:kC, i. e. by conſt. =P:BC, and theref. BC·AV:BA·CV=BC·P:BA·BC=P:BA;

and

and it has been ſhewn in the preceding analyſis that BC · AV : BA · CV = DF : EF ; therefore DF : EF = P : AB.

Q. E. D.

From this propoſition the truth of the following *Local Theorem* is obvious.

PROP. III. LOCAL THEO. Fig. 259, 260, *Pl.* 17.

Let AB, CB be two ſtraight lines given by poſition, meeting K*k* the diameter of a given circle in A and C, ſo that AK : KC = A*k* : *k*C ; from any point V in the circumference let there be drawn VD, VE perpendicular to the given lines and VF perpendicular to the diameter of the circle and let DF and EF be joined. DF ſhall be to EF in the given ratio of AK · BC to CK · BA.

Cor. If the given ſtraight lines AB, CB meet at a point B in the circumference (fig. 260.) then AK : KC = AB : BC and AK · BC = AB · CK, therefore alſo FD = FE.

In the ſame way that Prop. II. is derived from a well known circular locus, may the following propoſitions concerning Loci be derived from other known propoſitions ; their demonſtrations therefore are not added, but left as exercies to the Student in Geometry.

PROP. IV. LOCUS. Fig. 261, *Pl.* 17.

Let ABC be a given triangle ; let a point V be taken, ſuch, that, if VF be drawn perpendicular to BC, the baſe of the triangle ; and VD, VE perpendicular to AB, AC the ſides ; and FD, FE be joined, the difference between the ſquares of DF and EF may be equal to a given ſpace ; the point V ſhall be in the circumference of a given circle.

PROP. V. LOCUS, Fig. 261, Pl. 17.

Let ABC be a given triangle; let V be ſo taken, that perpendiculars being drawn as in the laſt propoſition, and alſo FD, FE being joined, the ſum of the ſquares of FD, and FE may be equal to a given ſpace; the point V ſhall be in the circumference of a given circle.

PROP. VI. LOCUS. Fig. 262, Pl. 17.

Let ABC be a given triangle, let a point V be ſo taken, that, if VD, VE, VF be drawn perpendicular to the ſides of the triangle, and the points D, E, F be joined, the ſum of the ſquares of the ſides of the triangle DEF may be equal to a given ſpace; the point V is in the circumference of a given circle.

A ſimilar propoſition may be extended to the figure formed by drawing perpendiculars to the ſides of any rectilineal figure whatever.

From the converſion of theſe LOCI elegant LOCAL THEOREMS as well as a number of curious PORISMS may evidently be formed: but theſe, as well as the ſolutions of the two following problems, we ſhall omit at preſent, recommending them however to the conſideration of the Student in Geometry.

Prob. 1. To find a point ſuch that if perpendiculars be drawn from it to the ſides of a given triangle, the ſtraight lines which join the points where theſe perpendiculars meet the ſides ſhall form a triangle ſimilar to a given triangle.

Prob. 2. Fig. 259, Pl. 17. To find a point V in the circumference of a given circle, ſuch, that if VD, VE be drawn perpendicular to two ſtraight lines given by poſition, and VF perpendicular to the diameter of the circle, and FD, FE be joined, the

the rectangle FD · FE may be equal to a given ſpace; alſo to determine the point V ſo that the rectangle may be the greateſt poſſible.

PROP. VII. PROBLEM.

Having given the ſum of the ſides of a triangle, as alſo the ſum of their ſquares, and either the ſum of their cubes, or the ſolid contained by the three ſides, to find the area of the triangle.

Let the ſides of the triangle be a, b, c.

Let $A = a + b + c$,
$B = a^2 + b^2 + c^2$,
$C = a^3 + b^3 + c^3$.

The relation between the area of a triangle and its ſides is expreſſed by this equation,

$$\frac{A}{2} \cdot \left(\frac{A}{2} - a\right) \cdot \left(\frac{A}{2} - b\right) \cdot \left(\frac{A}{2} - c\right) = (\text{area})^2, \text{ that is,}$$

$$\frac{A^4}{16} - (a+b+c)\frac{A^3}{8} + (ab+ac+bc)\frac{A^2}{4} - abc\frac{A}{2} = (\text{area})^2.$$

Now $a + b + c = A$,

and $ab+ac+bc = \frac{(a+b+c)^2 - (a^2+b^2+c^2)}{2} = \frac{A^2 - B}{2}$,

alſo $abc = \frac{a^3+b^3+c^3+(a+b+c)(ab+ac+bc-a^2-b^2-c^2)}{3}$

$$= \frac{2C - 3AB + A^3}{6}$$

ence $(\text{area})^2 = \frac{A^4 - 2A^2B - 8Aabc}{16}$,

$(\text{area})^2 = \frac{-A^4 + 6A^2B - 8AC}{48}$,

and

and the area $\begin{cases} = \dfrac{\sqrt{A^4 - 2A^2B - 8Aabc}}{4}. \\ = \sqrt{\dfrac{-A^4 + 6A^2B - 8AC}{48}}. \end{cases}$

COROLLARY. Hence in any triangle,

The radius of the inscribed circle $\begin{cases} = \dfrac{\sqrt{A^3 - 2AB - 8abc}}{2A} \\ = \sqrt{\dfrac{-A^3 + 6AB - 8C}{12A}} \end{cases}$

PROP. VIII. THEO. Fig. 263, 264, *Pl.* 17.

Let ABCD be a quadrilateral described in a circle, let two of its oppoſite ſides AC, BD, when produced, meet at E, let the diagonals AD, BC be drawn, interſecting each other at F, and let EF be joined, meeting AB at H. AH is to HB as the rectangle AC · AE to the rectangle BD · BE.

Draw LEK parallel to AB, meeting BC, AD produced in L and K.

Then AB : EL = AC : CE,
and EK : AB = ED : DB,
therefore EK : EL = AC · ED : CE · DB.
But ED : EC = AE : EB,
therefore AC · ED : EC · DB = AC · AE : EB · DB,
therefore EK : EL, or, AH : HB = AC · AE : EB · DB.

Q. E. D.

PROP. IX. PROBLEM. Fig. 265, *Pl.* 17.

Let ABC be a given triangle, a ſtraight line FG, given by poſition, may be found, ſuch, that if from any

ny point V in AC one of the ſides of the triangle,
ere be drawn VE, VD parallel to AB, BC, the
ier two ſides, ſo as to form the parallelogram
EBD, and the diagonal DE be drawn, meeting
, the line which may be found, in H; the dia-
al DE ſhall be divided at H into ſegments DH,
, having to each other a given ratio.

ppoſe the poriſm true, and that FG, the line to
ound, meets AB, BC in K and L; let DV meet
n N. Becauſe the ratio of DH to HE is given,
atio of DN to LE is alſo given, and ſince this
be univerſally true, it is evident that if the
V were taken in AC, ſo that the point D might
ide with K, then alſo E would coincide with
ience it appears that KL is the diagonal of a
lelogram KPLB inſcribed in the triangle
.

ow the given ratio of DH to HE, or of DN to
is compounded of the ratios of BL to LE and of
o BL, that is of the ratios of AP to PV and
K to BK, or of the ratios of AP to PV and
V to PC; therefore DH: HE = AP : PC,
ne ratio of DH to HE is by hypotheſis given,
fore the ratio of AP to PC is given, and, the
AC being given, the point P is alſo given.

ence the following conſtruction.

t AC be the ſide of the triangle in which the
V is always to be taken, and let the given
of DH to HE be that of α to β; Find P in
o that AP : PC = α : β; draw PK parallel to
meeting AB in K, and PL parallel to AB,
ing BC in L; draw the diagonal KL, which
be the line that may be found, that is, if from
y point in AC there be drawn VD, VE, parallel
e other ſides of the triangle, ſo as to form the
lelogram DVEB, and the diagonal DE be
n, meeting KL in H; DH : HE = AP : PC =

The

The Synthetic Demonſtration is evident from the preceding Analyſis.

This Poriſm may be rendered more general as follows.

PROP. X. PORISM. Fig. 266, *Pl.* 17.

Let ABC be a given triangle, a ſtraight line FG, given by poſition, may be found, ſuch, that if from V, any point in AC a ſide of the triangle, there be drawn VD, VE, meeting the other ſides AB, BC and making with them given angles at D and E, and if DE be joined, meeting FG, the line which may be found, in H, the ſtraight line DE ſhall be divided at H into ſegments DH, HE having to each other a given ratio.

Let FG, the line to be found, meet AB and BC in K and L: By taking a particular caſe of the poriſm, as in the laſt propoſition, it appears that when the point D coincides with K, then alſo E muſt coincide with L; hence it follows that ſtraight lines KP, LP, drawn parallel to DV, EV, muſt meet at P, a point in AC. Draw BQ parallel to DV or KP, meeting AC in Q, and draw QR parallel VE or PL meeting BC in R; becauſe the angle ABQ is given, the point Q is given; and becauſe the angle QRC is given, the point R is alſo given. Draw DN parallel to BC meeting KL in N, then, DH : HE = DN : LE; now, the ratio of DN to LE is compounded of the ratios of BL to LE, and DN to BL; and DN : BL = DK : BK = VP : PQ = LE : ER; therefore the ratio of DN to LE is compounded of the ratios BL to LE and of LE to ER, wherefore DN : EL = BL : LR. But the ratio DN to LE, or of DH to HE is by hypotheſis given, therefore the ratio of BL to LR is given, and, the points B, R being given, the point L is alſo given; and becauſe LP, PK make given angles with BC,

, BA, the point K is given, and the ſtraight line is given by poſition, as was to be ſhewn.

Conſtruction. Let the given ratio be that of α to draw BQ, meeting AC in Q, ſo that the angle Q may be equal to the angle which VD is to ke with AB, and draw QR, meeting BC in R, hat the angle QRC may be equal to the angle ich VE is to make with BC, take L in BR, ſo t BL : LR $= \alpha : \beta$, draw LP parallel to RQ, eting AC in P, and draw PK parallel to BQ, eting AB in K, through the points K, L draw ſtraight line FG, which is the line to be found ; t is, if from V any point in AC, ſtraight lines), VE be drawn parallel to PK, PL meeting AB, in D and E, and DE be joined, meeting KL in DH ſhall be to HE in the given ratio of α to β. The Demonſtration evidently follows from the alyſis, and for the ſake of brevity is here omitted.

(To be continued.)

ARTICLE XX.

tracts from a Paper on the Trigonometrical Tables of the Brahmins.

By JOHN PLAYFAIR, F. R. S. Edin.

d Profeſſor of Mathematics in the Univerſity of Edinburgh.

m the Tranſactions of the Royal Society of Edinburgh, Vol. IV.)

f the ſecond volume of the Aſiatic reſearches an extract is given from the *Surya Siddhanta*, the ancient book which has long, though obſcurely, pointed out as the ſource of the nomical knowledge of the Brahmins. The Surya Siddhanta the Sanſcrit language: It is one of the Saſtras or inſpired wri- of the Hindoos, and is called the *Jyotiſh* or Aſtronomical ra. It profeſſes, as we learn from Mr. DAVIS, the ingenious tranſlator,

tranſlator, to be a revelation from heaven, communicated to MEY. a man of great ſanctity, about four millions of years ago, towa the cloſe of the *Satya Jug*, or of the golden age of the Indi mythologiſts, a period at which man is ſaid to have been incor parably better than he is at preſent; when his ſtature exceed twenty-one cubits, and his life extended to ten thouſand years.

Interwoven, however, with all theſe extravagant fictions, t ſingular book contains a very ſober and rational ſyſtem of Aſtron mical calculations; and even the principles and rules of Trig nometry, a ſcience of all others the moſt remote from fable, a the leaſt ſuſceptible of poetical decoration.

The circumference of the circle is here divided into 360 eq parts, each of which is again divided into 60, and ſo on; the ſa diviſion was followed by the Greek mathematicians, and this co cidence is the more to be remarked, as it relates to a matter mere arbitrary arrangement, and one by no means neceſſarily co nected with the properties of the circle. Since this agreeme cannot well be attributed to chance, it might be ſuppoſed to reſ from ſome communication having taken place between the t nations, were it not that another very probable cauſe may aſſigned for it. In Greece, and no doubt in every other count the diviſion of the circle, into equal parts, is of a much older d than the origin of trigonometry, and muſt be as ancient as the fi circular inſtruments uſed for meaſuring angles in the heavens. T inventors of theſe inſtruments naturally ſought to make the di ſions on them correſpond to the ſpace which the ſun deſcribed dai in the ecliptic; and they could eaſily diſcover, without a very preciſe knowledge of the length of the year, th this might be nearly effected by making each of them the 360 part of the whole circumference. This principle may therefo have directed the aſtronomers, both of the Eaſt and Weſt, to d ſame diviſion of the circle, without any intercourſe having tak place between them.

The next thing to be mentioned is alſo a matter of arbitrary a rangement, but one in which the Brahmins follow a meth peculiar to themſelves. They expreſs the radius of the circle parts of the circumference, and ſuppoſe it equal to 3438 minut or 60ths of a degree. In this they are quite ſingular. Ptolem and the Greek mathematicians, after dividing the circumferen as we have already deſcribed, ſuppoſed the radius to be divided in 60 equal parts, without ſeeking to aſcertain in this diviſion any thi of the relation of the diameter to the circumference; and th throughout the whole of their tables, the chords are expreſſed ſexageſimals of the radius, and the arches in ſexageſimals of t circumference. They had, therefore, two meaſures, and t units; one for the circumference, and another for the diamet

T

loo mathematicians, again, have but one meaſure and one both, *viz.* a minute of a degree, or one of thoſe parts the circumference contains 21600. From this identity es they derive no inconſiderable advantage in many cal- though it muſt be confeſſed, that the meaſuring a ſtraight radius or diameter of a circle, by parts of a curve line, he circumference, is a refinement not at all obvious, and bly been ſuggeſted to them by ſome very particular view y have taken, of the nature and properties of the circle. ure here aſſigned to the radius, *viz.* 3438 of the parts of : circumference contains 21600, is true to the neareſt nd this is all the exactneſs aimed at in theſe trigonome- es; the author of them, however, muſt have known the he diameter to the circumference to a greater degree of for it appears from the Inſtitutes of AKBAR * that the : knew it to be that of 1 to 3·1416, which is much nearer than the preceeding. Calculating, as we may ſuppoſe, or ſome other proportion, not leſs exact, the authors of the nd, that the radius contained in truth 3437′· 44″· 48‴, as the fraction of a minute is here more than a half, they their conſtant cuſtom is, the integer next above, and called 3438 minutes.

Trigonometrical tables are two, the one of ſines and the verſed ſines. The ſine of an arch they call *cramajya* of , and the verſed ſine *utcramajya*, they alſo make uſe of e or *bhujajya*. Theſe terms ſeem all to be derived from *jya*, which ſignifies the chord of an arch, from which the the radius, or ſine of 90°, viz. *trijya*, is alſo taken. This y in their trigonometrical language is a circumſtance not y of remark. But what is of more conſequence to be , is, that the uſe of ſines, as it was unknown to the Greeks, culated by the help of the chords, forms a ſtriking difference the Indian trigonometry and theirs. The uſe of the ſine, f the chord, is an improvement which our modern trigono- es, as we have hitherto been taught to believe, to the and it is certainly one of the acquiſitions which the mathe- ſciences made, when, on their expulſion from Europe, they age in the Eaſt. But whether the Arabs are the authors of ntion, or whether they themſelves received it, as they did erical characters, from India, is a queſtion, which a more nowledge of Hindoo literature will probably enable us to

ention is made in this trigonometry of tangents, or ſecants; ſtance not wonderful, when we conſider that the uſe of . II. L theſe

* Tranſactions R. S. Edin. Vol. II. page 135, Phyſical

theſe was introduced in Europe no longer ago than the middle of the ſixteenth century. It is on the other hand not a little ſingular, that we ſhould find a table of verſed ſines in the Surya Siddhanta; for neither the Greek nor the Arabian mathematicians had any ſuch, nor had we in modern Europe, till after the time of PITISCUS, who wrote about the end of the century juſt mentioned.

Next, as to the extent and accuracy of theſe tables. The firſt of them exhibits the ſines to every twenty-fourth part of the quadrant, that is, the ſine of 3° 45′, and of all the multiples of that arch, viz. 7° 30′, 11° 15′, &c. up to 90°. The table of verſed ſines does the ſame. In each, the ſine, or verſed ſine, is expreſſed in minutes of the circumference, but without any fractions of a minute; and when the fraction that ought to have been ſet down is greater than ½, the integer next greater is placed in the table. Thus the ſine of 3° 45′ being, when accurately expreſſed in their way, 224′ 49″, is put down 225′; and ſo of the reſt. The numbers, therefore, are only ſo far exact as never to differ more than half a minute from the truth, and this very limited degree of accuracy gives, no doubt, to their trigonometry the appearance of an infant ſcience. But when, on the other hand, we conſider the principles and rules of their calculations, rather than the numbers actually calculated, we find the marks of a ſcience in full vigour and maturity: and we will acknowledge, that the Hindoo mathematicians did not ſatisfy themſelves with the degree of accuracy above mentioned, from any incapacity of attaining greater exactneſs.

Their rules for conſtructing their tables of ſines may be reduced to two, *viz.* the one for finding the ſine of the leaſt arch in the table, that of 3° 45′, and the other for finding the ſines of the multiples of that arch, its triple, quadruple, &c.

With reſpect to the firſt, the method proceeds by the continual biſection of the arch of 30°, and correſpondent extractions of the ſquare root, to find the ſine and coſine of its half, its fourth part and ſo on. The rule, when the ſine of an arch is given, to find that of half the arch, is preciſely the ſame with our own: "The ſine of an arch being given, find the coſine, and thence the verſed ſine of the ſame arch; then multiply half the radius into the verſed ſine and the ſquare root of the product is the ſine of half the given arch." Now as the ſine of 30° was well known to theſe mathematicians to be half the radius, from thence was found the ſine of 15°, then the ſine of 7° 30′, and laſtly of 3° 45′. which is the ſine required. Thus the ſine of 3° 45′ would be found equal to 224° 44″, or 225 nearly, and the ſine of 7° 30′ equal to 448′ 39″, or 449 nearly.

When

en the fine of 3° 45′, or of 225′, was found equal to 225,
of the table was conſtructed by a rule, that for its ſimplicity
gance, as well as for ſome other reaſons, is entitled to par-
attention. It is as follows " Divide the firſt jyapinda.
225; the quotient 1, deducted from the dividend, leaves
which added to the firſt jyapinda, or fine, gives the ſecond,
ine of 7° 30′, equal to 449′. Divide the ſecond jyapinda.
is thus found, by 225, and deduct 2, the neareſt integer to
tient, from the former remainder 224′, and the new remain-
′, added to the ſecond jyapinda, will give the third jyapinda
671′. Divide this laſt by 225, and ſubtract 3, the neareſt
to the quotient, from the former remainder 222′, and there
left 219, which added to the third jyapinda, gives the
and ſo on to the twenty-fourth or laſt.

not immediately obvious on what geometrical principle this
founded, but a ſlight change in the enunciation will remove
iculty. The rule may be expreſſed more generally thus:
any fine by 225, and ſubtract the quotient, or the integer
the quotient, from the difference between that fine and the
t leſs; the remainder is the difference between the ſame fine
fine next greater: and, therefore if it be added to the
will give the latter. If then (fig. 267, pl. 17.) GA, GC,
three contiguous arches in the table, of which the differences
F. of conſequence are equal, and of which the fines are
D, and EF, the rule as laſt ſtated gives us CD — AB —

$$\text{: FF} - \text{CD, hence EF} + \text{AB} = \text{CD}\left(2 - \frac{1}{225}\right) = \text{CD}$$

But 225 is the fine of the arch 3° 45′, and 449 of twice,

ch, as already ſhewn, and therefore, according to this rule
e be three arches, of which the common difference is
, the fine of the mean arch will always have to the ſum of
es of the extreme arches a given ratio, that namely, which
e of 3° 45′ has to the fine of twice 3° 45′, or of 7° 30′;
is is a true propoſition; and, therefore, we are in poſſeſſion
principle on which the Hindoo canon is conſtructed. The
trical theorem which is thus ſhewn to be the foundation of
gonometry of Hindoſtan, may alſo be more generally enun-

" If there be three arches in arithmetical progreſſion, the
the middle arch is to the ſum of the fines of the two ex-
arches, as the fine of the difference of the arches to the fine

of twice that difference." This theorem is well known m and is juſtly reckoned a very remarkable property of the ci

Now it is worth remarking that this property of ſines, has been ſo long known in the Eaſt, was not obſerved by thematicians of Europe till about two hundred years ago. Theorem indeed concerning the circle, from which it is under one ſhape or another, has been known to them f early period, and may be traced up to the writings of Eu where a propoſition nearly related to it forms the 97th of the " If a ſtraight line be drawn within a circle given in mag cutting off a ſegment containing a given angle, and if the the ſegment be biſected by a ſtraight line produced till it m circumference: the ſtraight lines, which contain the given ſhall both of them together have a given ratio to the ſtrai which biſects the angle." This theorem differs from the rule, only by affirming a relation to hold of their ſines given by EUCLID as uſeful for the conſtruction of geo problems; and trigonometry being then unknown, he prob not think of any other application of it. But what may ſ traordinary is, that when about 400 years afterwards, PTO the aſtronomer, conſtructed a ſet of trigonometrical tal never conſidered Euclid's theorem, though he was proba ignorant of it, as having any connection with the matter i He, therefore, founded his calculations on another prop containing a property of quadrilateral figures inſcribed in which he ſeems to have inveſtigated on purpoſe, and which diſtinguiſhed by his name. This propoſition contains, EUCLID's, and of courſe the Hindoo theorem, as a pa caſe; and though this caſe would have been the moſt u PTOLEMY, of all others, it appears to have eſcaped hi vation; on which account he did not perceive that every in his tables might be calculated from the two preceding m by an operation extremely ſimple, and every where the ſan therefore his method of conſtructing them is infinitely more and complicated than it needed to have been.

Not only did this eſcape PTOLEMY, but it remained un by the mathematicians, both Europeans and Arabians, wh after him, though they applied the force of their minds to more than to Trigonometry, and actually enriched that ſci many valuable diſcoveries. They continued to conſtruct the by the methods PTOLEMY had employed, till about the the ſixteenth century, when the theorem in queſtion, on wh Hindoo rule is founded, was diſcovered by VIETA; it is pu in his *Teatiſe on Angular Sections*, not with his own demonſ

ist with one given by an ingenious mathematician of our own country, ALEXANDER ANDERSON, of Aberdeen. It was then regarded as a theorem entirely new, and I know not that any of the geometers of that age remarked its affinity to the propoſitions of EUCLID and PTOLEMY. It was ſoon after applied in Europe, as it had been for ſo many ages before in Hindoſtan, and quickly gave to the conſtruction of the trigonemetrical canon, all the ſimplicity it ſeems capable of attaining. From all this I think it might fairly be concluded, even if we had no knowledge of the antiquity of the Surya Siddhanta, that the trigonometry contained in it is not borrowed from Greece or Arabia, as its fundamental rule was unknown to the geometers of both thoſe countries, and is greatly preferable to that which they employed.

ARTICLE XXI.

ATWOOD'S INVESTIGATIONS ON WATCH BALANCES.

(Continued from Page 363, Vol. I.)

THE vibration of a balance impelled by a ſingle ſpiral ſpring only, has been the ſubject of the preceding inveſtigations; but caſes occur in which two or more ſprings are employed in giving vibratory motion to the balances of watches. Not to mention preceding inſtances, Mr. *Mudge*, an eminent watch-maker of the preſent times, has invented a method of combining the action of ſpiral ſprings, to impel the balance in each ſemiarc of vibration, on a principle not more remarkable for the novelty than it is for the ingenuity of the contrivance. The conſideration of this additional caſe will therefore not be thought foreign to the preſent ſubject, eſpecially as it may contribute to elucidate ſome circumſtances reſpecting the effect of ſprings on the vibrations of balances, which at the firſt view are not at all obvious.

Let

Let two ſpiral ſprings be applied to act on a balance in the ſame direction; if the two ſpri unwinding themſelves, by turning the balance to the ſame point of quieſcence, or, in other if the accelerative forces of both ſprings ceaſe middle point of the vibration, whatever be the r ſtrength of the two ſprings, they will act balance preciſely in the ſame manner as if one only had been applied, of equal ſtrength wit (the ſprings being here ſuppoſed ſimilar, in of the law of the elaſtic forces and tenſions.) when the points of quieſcence of the two do not coincide, that is, when one ſpring cor to accelerate the balance in its vibration, aft acceleration of the other ſpring has ceaſed, th of a ſemivibration muſt be obtained from a ſe inveſtigation.

Let the circumference of a balance (fig. 82, be impelled by the action of a ſpiral ſpring tl the ſemiarc of vibration BO, the forces of this being always in proportion to the angular di from the point of quieſcence O; let a ſecond auxiliary ſpring alſo act on the balance from tl tremity of the ſemiarc B as far as the point which point all acceleration of the auxiliary ceaſes, the forces of the auxiliary ſpring vary the angular diſtances from the point of quie Q. Suppoſe that the accelerative force of the cipal or balance ſpring on the circumference balance at the diſtance from quieſcence OD is take $Qd = OD$, and let the accelerative fo the balance ſpring on the circumference o balance be to that of the auxiliary ſpring, whe ſprings are wound to the ſame angle $OCD =$ in the proportion of 1 to n; then the accele force of the auxiliary ſpring at the angular di

from quieſcence Qd will $= nf$; let $BO = b$, $BQ = c$, $QO = d = b - c$; alſo let OD or $Qd = a$.

Suppoſe the balance to have deſcribed the arc BH by the joint action of both the ſprings, and let the arc BH be repreſented by x. Then becauſe the accelerative force of the principal or balance ſpring at the angular diſtance from quieſcence $OD = a$ is f; the accelerative force of the ſame ſpring at the diſtance OH is $= f(b - x) \div a$; and ſince the force of the auxiliary ſpring at the angular diſtance from quieſcence $Qd = a$ is nf, the accelerative force of this ſpring at the angular diſtance from quieſcence QH will be $= nf(c - x) \div a$; wherefore the joint force of both ſprings to accelerate the circumference when at the diſtance OH from the point of quieſcence O, that is, when the balance has deſcribed the arc BH, will be $= (f \div a) \times (b - x + cn - n\dot{x})$. Let u be the ſpace through which a body falls freely from reſt by the acceleration of gravity, to acquire the velocity of the circumference when it has deſcribed the arc BH; this will give the following equation $\dot{u} = (f \div a) \times (b\dot{x} - x\dot{x} + cn\dot{x} - n\dot{x}\dot{x})$; and $u = (f \div 2a) \times (2bx - x^2 + 2cnx - nx^2)$; and if $l = 193$ inches, the velocity of the circumference when the arc BH has been deſcribed, is $= \sqrt{(2lf \div a)} \times \sqrt{(2bx - x^2 + 2cnx - nx^2)}$. Let t repreſent the time in which the balance deſcribes the arc BH; then $\dot{t} = \sqrt{(a \div 2lf)} \times \dot{x} \div \sqrt{(2bx - x^2 + 2cnx - nx^2)}$ and the fluent or $t = \sqrt{(2a \div lf(n + 1))} \times$ into a circular arc of which the ſine is $\sqrt{(x(n + 1) \div (2b + 2nc))}$ to radius $= 1$; which is the time of deſcribing the arc BH. When $x = c$, the time of deſcribing the arc BQ, is $\sqrt{(2a \div lf(n + 1))} \times$ into a circular arc of which the ſine $= \sqrt{(c(n + 1) \div (2b + 2nc))}$.

The time of deſcribing the remaining part of the ſemiarc QO, (fig. 82, pl. 5.) is next to be deter-

 mined,

mined. In the caſes to which this inveſtigation applied, the auxiliary ſpring ceaſes entirely to a on the balance after it has deſcribed the arc B(This being ſtated, while the balance deſcribes the r maining arc of the ſemivibration QO, it will be im pelled by the balance ſpring only. To aſcertain th time of deſcribing the arc QO, it is firſt to b obſerved, that when the circumference has de ſcribed the arc BQ, it will have acquired a velocity equal to that of a body which has fallen from reſt by the acceleration of gravity through a ſpace * = $(f \div 2a) \times (2bc - c^2 + nc^2)$.

Suppoſe the balance to have proceeded through the arc QR, (fig. 82, pl. 5.) and let OR = x; the force by which the circumference is accelerated at R = $fx \div a$: and if u is the ſpace through which a body falls freely from reſt by the acceleration of gravity to acquire the velocity of the circumference in the point R, $\dot{u} = -f\dot{x}x \div a$; taking the fluents ſo that u may become = $(f \div 2a) \times (2bc - c^2 + nc^2)$ when $x = d$, $u = (f \div 2a) \times (2bc - c^2 + nc^2 + d^2 - x^2)$; or becauſe $d^2 = b^2 - 2bc + c^2$; $u = (f \div 2a) \times (b^2 + nc^2 - x^2)$; and the velocity of the circumference at R (fig. 82, pl. 5.) = $\sqrt{(2f \div a) \times (b^2 + nc^2 - x^2)}$: let t be the time of deſcribing QR; then $\dot{t} = \sqrt{(a \div 2f)} \times -\dot{x} \div \sqrt{(b^2 + nc^2 - x^2)}$, and $t =$ $\sqrt{(a \div 2f)} \times$ into a circular arc of which the coſine is $x \div \sqrt{(b^2 + nc^2)}$ to radius = 1, which ſhould = 0 when $x = d$; wherefore t or the time of deſcribing QR = $\sqrt{(a \div 2f)}$ ×

* When the circumference has deſcribed the arc BH = z, it will have acquired a velocity equal to that of a body which has fallen freely from reſt by the acceleration of gravity through a ſpace = $(f \div 2a) \times (2bx - x^2 + 2cnx - nx^2)$, as appears from the inveſtigation in page 127; and when $x = c$, this ſpace becomes $(f \div 2a) \times (2bc - c^2 + 2c^2n - nc^2) = (f \div 2a) \times (2bc - c^2 + nc^2)$.

into a circular arc of which the cosine is $x \div (b^2 + nc^2) - \sqrt{(a \div 2lf)} \times$ into a circular arc of hich the cosine is $d \div \sqrt{(b^2 + nc^2)}$; and when $= o$, that is, when the entire arc QO has been de-ribed, the time $t = \sqrt{(a \div 2lf)} \times$ into a circular c of which the sine is $d \div \sqrt{(b^2 + nc^2)}$.

The result of this investigation is, that the time which the balance describes the semiarc BO, by e joint action of both springs through the arc BQ, id by the action of the balance spring only through

e arc QO, is $= \sqrt{\frac{2a}{lf(n+1)}} \times$ an arc, of

hich the sine is $\sqrt{\frac{c(n+1)}{2b+2nc}} + \sqrt{\frac{a}{2lf}} \times$ an

c, of which the sine is $\frac{d}{\sqrt{b^2 + nc^2}}$ (to radius

= 1) expressed in parts of a second *.

* When $n = o$, this expression becomes $\sqrt{(a \div 2lf)} \times$ wice the arc, of which the sine is $\sqrt{(c \div 2b)} +$ an arc, of hich the sine is $d \div b$); but since $c = b - d$, the two arcs here entioned will be exactly $= 90^\circ = p \div 2$; and the time of a nivibration $= \sqrt{(a \div 2lf)} \times p \div 2 = \sqrt{(ap^2 \div 8lf)}$, reeing with the solution in page 162, Vol. I. Suppose $d = o$; ice in this case $b = c$, the time of a semivibration becomes $\sqrt{(2a \div lf(n+1))} \times$ an arc, of which the sine is $\sqrt{(b \div 2b)}$ hich arc is $= 45^\circ$, or $p \div 4$; wherefore the time of a semi-bration in this case $= \sqrt{(ap^2 \div 8lf(n+1))}$, which is the true due, according to the solution in page 162, Vol. I. See also age 136, of the present volume.

If the points of quiescence are in the first semiarc of vibration, id $c = o$, the point B will coincide with Q (fig. 82, pl. 5.), from hich point the vibration will commence; in this case the expres-on for the time of a semivibration will become $t = \sqrt{(a \div 2lf)}$, : an arc of which the sine is $d \div d$, or an arc of $90^\circ = p \div$, or $t = \sqrt{(ap^2 \div 8lf)}$; which agrees entirely with the solution page 162, Vol. I. for in this case the auxiliary spring not acting n the balance while it describes the arc QON, the balance will

vibrate

ſituated in the latter ſemiarc of vibra
pl. 5.), between O and E, inſtead of be
B, as in the former ſolution. Accor
condition, making BQ = c, BO = b,
notation remaining as before, it appear
veſtigation no ways differing from th
that the time in which the balance deſcri
arc BO will be $t = \sqrt{(2a \div f)}$ (
cular arc, of which the ſine is $\sqrt{}$
$(2b + 2nc)$) expreſſed in parts of a ſec

This reſult expreſſes the time in whic
deſcribes the ſemiarc BO, (fig. 83, pl.
celerative force of two ſprings, namely
ſpring, of which the point of quieſcen
an auxiliary ſpring, of which the point
Q. On conſidering this caſe more fu
plied to the actual vibrations of a ba
appear evident, that the action of a th
the balance while it is deſcribing the
muſt be taken into the calculation, i
the two ſprings already mentioned, in
tain a ſolution entirely correſpondent
cumſtances of the caſe, when the po
cence of the auxiliary ſprings are ſi

that when the points of quieſcence of the balance and auxiliary ſprings are coincident; the auxiliary ſpring commencing its action from the extremity B of the arc BO, (fig. 84, pl. 5.) continues to accelerate the balance till it arrives at the quieſcent poſition O, at which point the action of the auxiliary ſpring entirely ceaſes; on this account it is plain that another auxiliary ſpring, equal and ſimilar to the former, having alſo the point of quieſcence coincident with O, muſt be applied to act by retardation on the balance while it deſcribes the arc OE, in order that the times of deſcribing the arcs BO and OE, as well as theſe arcs themſelves, may be equal. According, therefore, to this diſpoſition of the auxiliary ſprings, the balance will deſcribe each ſemiarc of its vibrations preciſely in the ſame manner as if it was impelled by one ſpiral ſpring only, the ſtrength of which is equal to that of the balance and either auxiliary ſpring, when wound to the ſame tenſion. Suppoſe the balance to vibrate from B to E, and, for the ſake of diſtinction, let the auxiliary ſpring which accelerates the balance from the extremity of the arc B, in the direction BO, be called u; and let the other auxiliary * ſpring which retards the balance in the ſemiarc OE be denoted by v; conſequently, when the balance vibrates in the contrary direction from E to B, the auxiliary ſpring v will accelerate the balance from E to O, and the other auxiliary ſpring u will retard it from O to B. In reſpect, therefore, to the ſpring u, BO is the firſt ſemiarc, and OE is the latter ſemiarc of vibration; and on a ſimilar principal in reſpect to the ſpring v, EO is the firſt ſemiarc, and OB is the latter ſemiarc of vibration.

* The circular arcs which are drawn interior to the circumference of the balance in the figures 82, 83, 84, pl. 5, are intended to repreſent thoſe portions of the balance's vibration in which the auxiliary ſprings reſpectively act.

In the preceding pages, the time has been inv gated in which a balance vibrates in the ſen BO, (fig. 82, pl. 5.) when the point of quieſc Q, of the auxiliary ſpring *u*, is ſituated betwe and O, that is in the firſt ſemiarc of vibration. this caſe, in order that the time of deſcribing the ſemiarc OE, by retardation, may be equal to th deſcribing the firſt ſemiarc BO by accelerated mc the quieſcent point of the auxiliary ſpring *v* mu placed at N, between O and E, ſo that EN ma equal to BQ; according to this diſpoſition o auxiliary ſprings, the point of quieſcence Q o auxiliary ſpring *u* will be ſituated in the firſt arc of vibration, conſidering the balance as v ting from B to E; and the point of quieſcence the auxiliary ſpring *v*, will be in the firſt ſemia vibration, conſidering the balance as vibrating E to B.

The third caſe which remains to be more inveſtigated is, when the points of quieſcence c auxiliary ſprings are ſituated in the latter ſemia the reſpective vibrations; that is, when the quie point Q of the auxiliary ſpring *u* is ſituated i latter arc OE, (fig. 83, pl. 5.) while the balanc brates from B to E, and the point of quieſcence the auxiliary ſpring *v* is ſituated in the latter arc while the balance vibrates from E to B. In thi it is evident, that while the balance vibrates fr to O, it deſcribes the arc BN by the accelerat two ſprings; namely, the balance ſpring, of v the point of quieſcence is O, and the auxiliary ſ *u*, of which the point of quieſcence is Q. A balance deſcribes the arc NO by the combined of three ſprings; namely, by that of the balance ſ of which the point of quieſcence is O, that auxiliary ſpring *u*, of which the point of quieſce Q, both of which accelerate the balance, and t the auxiliary ſpring *v*, of which the point of cence is N, acting by retarding the balance.

The time in which the balance deſcribes the ſemi-arc BO, by the combined action of theſe three ſprings, will be obtained by the following inveſtigation.

Reſuming the former notation, let DO (fig. 83, pl. 5.) $= a = Qd = Ne$, BO $= b$, BQ $= c$, OQ $= ON = d = c - b$. The accelerative force of the balance ſpring on the circumference of the balance at the tenſion OD $= f$; the accelerative force of the auxiliary ſprings at the ſame tenſion Qd or $Ne = nf$. From page 127 and 128, it appears, that the time in which the balance deſcribes the arc BN by the joint action of the balance ſpring and auxiliary ſpring $u = \sqrt{(2a \div lf(n+1))} \times$ a circular arc of which the ſine $= \sqrt{((b-d)(n+1) \div (2b + 2nc))}$; and the velocity acquired by the circumference at N is equal to that of a body which has fallen freely from reſt by the acceleration of gravity through a ſpace* $= (f \div 2a) \times (b^2 - d^2 + nb^2 - 3nd^2 + 2nbd)$. To find the time of deſcribing the arc NO, ſuppoſe the balance to have proceeded in its vibration from N to R, (fig. 83, pl. 5.) and make NR $= x$.

The accelerative force of the balance ſpring on the circumference of the balance at the point R is - - $+ (f \div a)(d - x)$.

The

* In the inveſtigation, page 127, it is ſhewn, that when the balance has deſcribed the arc BH $= x$, the ſpace through which a body muſt fall freely from reſt to acquire the velocity of the circumference at the point H, or $u = (f \div 2a) \times (2bx - x^2 + 2cnx - nx^2)$, the expreſſion being the ſame, whether the point Q is on one ſide of O or on the other, provided BQ $= c$. In the preſent caſe, $c = b + d$, and when the circumference has deſcribed the arc BN, $x = b - d$; wherefore if $b + d$ is ſubſtituted for c, and $b - d$ for x in the equation, $u = (f \div 2a) \times (2bx - x^2 + 2cnx - nx^2)$, the reſult will be $u = (f \div 2a) \times (b^2 - d^2 + nb^2 - 3nd^2 + 2nbd)$.

The accelerative force of the auxiliary ſpring u, at the tenſion or diſtance from quieſcence $Qd = a$ being nf, the force of the ſame ſpring at the tenſion or diſtance $QR = 2d - \dot{x}$, is - - $+(nf \div a)(2d$

The force of the auxiliary ſpring v, at the tenſion $NE = a$ being nf, at the tenſion NR, the force of this ſpring acting by retardation will be $-(nf \div$

Sum of theſe forces acting on the circumference of the balance, when it has deſcribed the arc $NR = (f \div a) \times (d - x + 2dn -$

Let u be the ſpace through which a body falls from reſt by the acceleration of gravity, to ac the velocity of the circumference at R; the p ples of acceleration give this equation; $u = (f \times (d\dot{x} - x\dot{x} + 2dn\dot{x} - 2nx\dot{x})$; and taking the fl ſo that when $x = 0$, u may be $= (f \div 2$ $(b^2 - d^2 + nb^2 - 3nd^2 + 2nbd)$; $u = (f - \times (b^2 - d^2 + nb^2 - 3nd^2 + 2nbd + (2d + x - (1 + 2n)\, x^2)$; if t is put to repreſent the ti deſcribing the arc NR, $\dot{t} = \sqrt{(a \div 2lf)} \times \dot{x} - (b^2 - d^2 + nb^2 - 3nd^2 + 2nbd + (2d + 4$ $- (1 + 2n)\, x^2)$, and taking the fluents ſo that $x = 0$, $t = 0$, and making $x = d$, the time o ſcribing the arc $NO = \sqrt{(a \div 2lf(1 + 2n$ a circular arc of which the ſine $= \sqrt{(d^2 (1 +$ $\div (b^2 + n\,(b + d)^2 - 2nd^2)$, or becauſe $b +$ c, $t = \sqrt{(a \div 2lf(1 + 2n))} \times$ a circular arc of w the ſine $= \sqrt{(d^2 (1 + 2n) \div (b^2 + nc^2 - 2nd$

The reſult is, that when the points of quieſc of the auxiliary ſprings are ſituated in the

ſem

ſemiarcs of the reſpective vibrations, the time in which the balance deſcribes the ſemiarc BO will be $= \sqrt{2a \div lf\,(n+1)} \times$ a circular arc of which the ſine is $\sqrt{((b-d)\,(n+1) \div (2b+2nc))} + \sqrt{a \div 2lf\,(1+2n)} \times$ a circular arc of which the the ſine is $\sqrt{(d^2\,(1+2n) \div (b^2+nc^2-2nd^2)}$.

Corollary.—If the points of quieſcence of the auxiliary ſprings are in the latter ſemiarcs of vibration, and the vibration commences from the point N (fig. 83, pl. 5.), in this caſe $d = b$, or $c = 2d$, and by the ſolution above, the time of a ſemivibration becomes $\sqrt{(a \div 2lf\,(1+2n))} \times$ a circular arc of which the ſine is $\sqrt{(d^2(1+2n) \div (d^2\,(1+2n)}$, or an arc of $90^\circ = p \div 2$: wherefore the time of a ſemivibration $= \sqrt{(ap^2 \div 8lf\,(1+2n))}$. We obſerve, therefore, that whether the points of quieſcence of the auxiliary ſprings are ſituated in the firſt or latter ſemiarcs of vibration, if the ſemiarc of vibration ſhould be $=$ to the diſtance of the ſaid points from the point of quieſcence of the balance ſpring, the times of vibration will be the ſame whatever be the magnitude of that ſemiarc.

The balance of Mr. MUDGE's time-keeper deſcribes the ſemiarc BO, by the joint action of two ſprings, *i. e.* the balance * ſpring, and an auxiliary ſpring; each ſpring is wound through the ſame arc BO, and comes to the ſame point of quieſcence O (fig. 84, pl. 5.); conſequently the action of the two ſprings is the ſame with that of a ſingle ſpring of equal ſtrength with both. In this caſe, the time of a ſemivibration through the arc BO will be obtained from a former ſolution; for referring to page 162 Vol. I. and making OD $= a$, and the force of th balanc

* A double ſpiral ſpring is applied in the balance of M MUDGE's time-keeper, but as theſe two ſprings act as one ſpri they are here conſidered as ſuch.

balance ſpring and auxiliary ſpring at the diſtan from quieſcence OD, $= f + nf = F$; the time a ſemivibration is

$$= \sqrt{\frac{ap^2}{8f(n+1)}} = \sqrt{\frac{ap^2}{8lF}}$$

(To be continued.)

ARTICLE XXII.

Demonſtrations to Dr. STEWART'S *Propoſitions propoſed in* ARTICLE XLI. Vol. I.

PROP. XXI. THEO. XVIII. *Fig.* 164, 165, 166, *Pl.* 11.

Demonſtrated by Mr. JOHN LOWRY.

Caſe I. WHEN the lines given by poſition are parallel to each other, fig. 164.

Let there be any number of right lines AL, BM CN, DO, &c. given by poſition, and parallel to each other, and let *a*, *b*, *c*, *d*, &c. be given magnitudes, as many in number as there are right lines given by poſition; three right lines XQ, YR, ZS, may be found, that will be given by poſition, ſuch, that if from any point P there be drawn perpendiculars PA, PB, PC, PD, &c. to all the right lines given by poſition, and likewiſe perpendiculars PX, PY, PZ, to the three lines found; the ſquare of PA, together with the ſpace to which the ſquare of PB has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of PC has the ſame ratio that *a* has to *c*, and ſo on, will be

be equal to the ſpace to which the ſum of the ſquares of PX, PY, PZ, has the ſame ratio that thrice a has to the ſum of a, b, c, &c.

From the point P draw PABCD, &c. perpendicular to the right lines given by poſition, and by Prop. X. find the point W for the points A, B, C, D, &c. and for the given magnitudes a, b, c, d, &c. In the right line PD take any point X, and divide XW in a, ſo that the ſpace to which the rectangle XaW has the ſame ratio that thrice a has to twice the ſum of a, b, c, &c. may be equal to the difference between the ſpace to which the ſquare of XW has the ſame ratio that thrice a has to twice the ſum of a, b, c, &c. and the ſquare of WA, together with the ſpace to which the ſquare of WB has the ſame ratio that a has to b, together with the ſpace to which the ſquare of WC has the ſame ratio that a has to c, and ſo on. Make WY $=$ Wa, and WZ $=$ aX, and through the three points X, Y, Z, draw three right lines XQ, YR, ZS, parallel to the right lines given by poſition, and they will be three ſuch lines as are required.

It may be ſhewn as in Prop. XVII. Caſe I. that the ſpace to which the ſum of the ſquares of XW, YW, and ZW has the ſame ratio that thrice a has to the ſum of a, b, c, &c. is equal to the ſquare of AW, together with the ſpace to which the ſquare of BW has the ſame ratio that a has to b, together with the ſpace to which the ſquare of CW has the ſame ratio that a has to c, and ſo on,

But, by Prop. X. the ſquare of PA, together with the ſpace to which the ſquare of PB has the ſame ratio that a has to b, together with the ſpace to which the ſquare of PC has the ſame ratio that a has to c, and ſo on, is equal to the ſquare of AW, together with the ſpace to which the ſquare of BW has the ſame ratio that a has to b, together with the ſpace to which

which the ſquare of CW has the ſame ratio has to *c*, and ſo on, together with the ſpace tc the ſquare of PW has the ſame ratio that *a* the ſum of *a*, *b*, *c*, &c. Therefore the ſq PA, together with the ſpace to which the ſq PB has the ſame ratio that *a* has to *b*, togethe the ſpace to which the ſquare of PC has tl ratio that *a* has to *c*, and ſo on, is equal to th to which the ſum of the ſquares of XW, and ZW, together with thrice the ſquare o has the ſame ratio that thrice *a* has to the ſu *b*, *c*, &c. But, ſince XW is equal to the ſum and ZW, it follows from Prop. IX. that the the ſquares of PX, PY, and PZ, is equal to of the ſquares of XW, YW, and ZW, t with thrice the ſquare of PW; therefore the of PA, together with the ſpace to which th of PB has the ſame ratio that *a* has to *b*, togetl the ſpace to which the ſquare of PC has the ſa that *a* has to *c*, and ſo on, is equal to the which the ſum of the ſquares of PX, PY, a has the ſame ratio that thrice *a* has to the ſu *b*, *c*, &c.

Caſe II. When the lines given by poſitio ſect each other in a point, fig. 165.

Let there be any number of right lines A AD, AE, &c. given by poſition, and inte each other in the point A, and let *a*, *b*, *c*, be given magnitudes, as many in number a are right lines given by poſition; three rig AX, AY, AZ, may be found, that will be gi poſition, ſuch, that if from any point P t drawn perpendiculars PB, PC, PD, PE, & the right lines given by poſition, and likew pendiculars PX, PY, PZ, to the three lines the ſquare of PB, together with the ſpace t the ſquare of PC has the ſame ratio that *a* h

t

together with the ſpace to which the ſquare of PD has the ſame ratio that a has to c, and ſo on, is equal to the ſpace to which the ſum of the ſquares of PX, PY, PZ, has the ſame ratio that thrice a has to the ſum of a, b, c, &c.

Draw AP, and upon it, as a diameter, let a circle be deſcribed, interſecting the lines given by poſition in B, C, D, E, &c. By Prop. X. find a point R for the points B, C, D, E, &c. and the given magnitudes a, b, c, d, &c. and by Prop. XIX. find two right lines AI, AK, for the point P, the right lines AB, AC, AD, AE, &c. given by poſition, and the given magnitudes a, b, c, d, &c. From any point Z, in the circumference of the circle, draw ZR, and continue it, ſo that RZ may be double of RS. Let O be the centre of the circle, and join OS, and let XSY be drawn perpendicular to SO, meeting the circle in X and Y. Join AX, AY, AZ, and they will be three ſuch lines as are required.

By Prop. XIX. the ſquare of PB, together with the ſpace to which the ſquare of PC has the ſame ratio that a has to b, together with the ſpace to which the ſquare of PD has the ſame ratio that a has to c, and ſo on, is equal to the ſpace to which the ſum of the ſquares of PI, PK, has the ſame ratio that twice a has to the ſum of a, b, c, &c. And it may be ſhewn, in the ſame way as in Prop. XVII. Caſe 2, that twice the ſum of the ſquares of PI, PX, is equal to the ſpace to which the ſum of the ſquares of PX, PY, and PZ, has the ſame ratio that 3 has to 2; therefore the ſquare of PB, together with the ſpace to which the ſquare of PC has the ſame ratio that a has to b, together with the ſpace to which the ſquare of PD has the ſame ratio that a has to c, and ſo on, is equal to the ſpace to which the ſum of the ſquares of PX, PY, and PZ, has the ſame ratio that thrice a has to the ſum of a, b, c, &c.

Caſe III. When the lines given by poſition are neither all parallel, nor interſecting each other in one point, fig. 166.

Let there be any number of right lines AB, BC, CD, AD, &c. given by poſition, that are neither all parallel, nor interſecting each other in one point, and let *a*, *b*, *c*, *d*, &c. be given magnitudes as many in number as there are right lines given by poſition; three right lines Y*m*, Z*n*, K*r* may be found, that will be given by poſition, ſuch, that if from any point X there be drawn perpendiculars XF, XE, XG, XH, &c. to all the right lines given by poſition, and likewiſe perpendiculars X*m*, X*n*, X*r*, to the three lines found, the ſquare of XE, together with the ſpace to which the ſquare of XF has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of XG has the ſame ratio that *a* has to *c*, and ſo on, is equal to the ſpace to which the ſum of the ſquares of X*m*, X*n*, and X*r*, has the ſame ratio that thrice *a* has to the ſum of *a*, *b*, *c*, &c.

Find by Prop. XX. two right lines PV, PW, for the point X, the right lines AB, BC, CD, AD, &c. given by poſition, and the given magnitudes *a*, *b*, *c*, *d*, &c. From the point P draw PM, PO, PR, PQ, &c. perpendicular, and PL, PN, PS, PU, &c. parallel to AB, BC, CD, AD, &c. and let PL, PN, PS, PU, &c. meet the perpendiculars XE, XF, XG, XH, &c. drawn from the point X to the right lines AB, BC, CD, AD, &c. in L, N, S, U, &c. Find, by the preceding caſe, three right lines P*a*, P*b*, P*c*, for the point X, the right lines PL, PN, PS, PU, &c. given by poſition, and the given magnitudes *a*, *b*, *c*, *d*, &c. and from any point, as *c*, in P*c* draw C*q* parallel to P*a* meeting P*b* in *q*. Then, by Prop. XVI. find the interſection *x* of two right lines, for the point X and the right lines P*q*, P*c*, *cq*; and from *x* draw *xy*, *xz*, perpendicular to *cq*, P*c*, P*q*.

Let

Let a ſquare be found to which the ſquare of PM together with the ſpace to which the ſquare of PO has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of PR has the ſame ratio that *a* has to *c*, and ſo on, has the ſame ratio that the ſum of *a*, *b*, *c*, *d*, &c. has to thrice *a*. Divide this ſquare into three others, whoſe ſides ſhall have the ſame ratio to each other as the three lines *wf*, *wg*, *wh*. Draw PK, PY, PZ, parallel to *wf*, *wg*, *wh*, and thereon take PK, PY, PZ, equal to the ſides of the three ſquares juſt found, that is, on lines which are parallel to thoſe to which the ſides of the ſquares are reſpectively proportional. Then draw Y*m*, Z*n*, K*r*, perpendiculars to PY PZ, PK, and they will be three ſuch lines as are required.

Draw X*m*, X*n*, X*r*, X*a*, X*b*, X*c*, XV, XW, reſpectively parallel to Y*m*, Z*n*, K*r*, P*a*, P*b*, P*c*, PV, PW.

It appears, in the ſame way as in Prop. XVII. that the ſquare of XE, together with the ſpace to which the ſquare of XF has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of XG has the ſame ratio that *a* has to *c*, and ſo on, is equal to the ſquare of PM, together with the ſpace to which the ſquare of PO has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of PR has the ſame ratio that *a* has to *c*, and ſo on, together with the ſquare of XL, together with the ſpace to which the ſquare of XN has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of XS has the ſame ratio that *a* has to *c*, and ſo on. Alſo, that the ſum of the ſquares of X*m*, X*n*, X*r*, is equal to the ſum of the ſquares of X*a*, X*b*, X*c*, together with the ſum of the ſquares of PY, PZ, PK. Moreover, by the preceding caſe, the ſquare of XL, together with the ſpace to which the ſquare of XN has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of XS

has the ſame ratio that *a* has to *c*, and ſo on, is equal to the ſpace to which the ſum of the ſquares of X*a*, X*b*, X*c*, has the ſame ratio that thrice *a* has to the ſum of *a*, *b*, *c*, &c. and, by conſtruction, the ſquare of PM, together with the ſpace to which the ſquare of PO has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of PR has the ſame ratio that *a* has to *c*, and ſo on, is equal to which the ſum of the ſquares of PY, PZ, PK, has the ſame ratio that thrice *a* has to the ſum of *a*, *b*, *c*, &c. Therefore the ſquare of PM, together with the ſpace to which the ſquare of PO has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of PR has the ſame ratio that *a* has to *c*, and ſo on, together with the ſquare of XL, together with the ſpace to which the ſquare of XN has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of XS has the ſame ratio that *a* has to *c*, and ſo on, is equal to the ſpace to which the ſum of the ſquares of X*a*, X*b*, X*c*, has the ſame ratio that thrice *a* has to the ſum of *a*, *b*, *c*, &c. together with the ſpace to which the ſum of the ſquares of PY, PZ, PK, has the ſame ratio that thrice *a* has to the ſum of *a*, *b*, *c*, &c. Therefore the ſquare of XE, together with the ſpace to which the ſquare XF has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of XG has the ſame ratio that *a* has to *c*, and ſo on, is equal to the ſpace to which the ſum of the ſquares of X*m*, X*n*, X*r*, has the ſame ratio that thrice *a* has to the ſum of *a*, *b*, *c*, &c.

The ſame demonſtrated by Mr. Swale, *Fig.* 205, *Pl.* 15.

Let there be any number of right lines, AB, CD, EF, &c. given by poſition, and let *a*, *b*, *c*, &c. be given magnitudes, as many in number as there are right lines given by poſition; three right lines QR, ST,

ST, UV, may be found that will be given by poſition, ſuch, that if from any point P there be drawn perpendiculars PA, PC, PE, &c. to all the right lines given by poſition, and likewiſe there be drawn perpendiculars PS, PQ, PU, to the three lines found, the ſquare of PA, together with the ſpace to which the ſquare of PC has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of PE has the ſame ratio that *a* has to *c*, and ſo on, will be equal to the ſpace to which the ſum of the ſquares of PS, PU, PQ, has the ſame ratio that thrice *a* has to the ſum of *a*, *b*, *c*, &c.

Suppoſe three right lines AB, CD, EF, given by poſition. From any point P, demit the ⊥s PA, PC, PE; join AC, and take AG : GC as *b* : *a*; join EG, and take GH : HE as C : the ſum of *a*, *b*; join PG, PH; erect the ⊥s GI, HK, to meet ſemicircles deſcribed upon AC, EG, in I, K; take GL : GC as EK : EG, and let a ſemicircle deſcribed on AL meet GI in N; then in PG, PH, and any other line PU, taking PQ, PS, PU, equal to the ſides of the ſquares $3GN^2$, $3PH^2$, $3HK^2$, reſpectively, and drawing QR, ST, UV, ⊥ thereto, they will be three ſuch lines as are required.

By Prop. XX. the ſquare of PA, together with the ſpace to which the ſquare of PC has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of PE has the ſame ratio that *a* has to *c*, is equal to the ſpace to which the ſum of the ſquares PH, KH, has the ſame ratio that *a* has to the ſum of *a*, *b*, *c*, together with the ſpace to which the ſquare of GI has the ſame ratio that *a* has to the ſum of *a*, *b*. But the ſquare of GN is to the ſquare of GI, as GL to GC, or, as EH to EG, or, as the ſum of *a*, *b*, to the ſum of *a*, *b*, *c*; therefore the ſpace to which the ſquare of GI has the ſame ratio that *a* has to the ſum of *a*, *b*, is equal to the ſpace to which the

ſquare of GN has the ſame ratio that *a* has to the ſum of *a*, *b*, *c*. Wherefore, the ſquare of PA, together with the ſpace to which the ſquare of PC has the ſame ratio that *a* has to *b*, together with the ſpace to which the ſquare of PE has the ſame ratio that *a* has to *c*, is equal to the ſpace to which the ſum of the ſquares of PH, HK, GN, has the ſame ratio that *a* has to the ſum of *a*, *b*, *c*, that is, equal to the ſpace to which thrice the ſum of the ſquares of PH, HK, GN, has the ſame ratio that thrice *a* has to the ſum of *a*, *b*, *c*; that is, equal to the ſpace to which thrice the ſum of the ſquares of PH, HK, GN, has the ſame ratio that thrice *a* has to the ſum of *a*, *b*, *c*; that is, equal to the ſpace to which the ſum of the ſquares of PS, PU, PQ has the ſame ratio that thrice *a* has to the ſum of *a*, *b*, *c*.

Cor. Let there be any number of right lines given by poſition; three right lines may be found that will be given by poſition, ſuch, that if from any point there be drawn right lines in given angles to all the right lines given by poſition, and likewiſe there be drawn perperpendiculars to the three lines found, the ſum of the ſquares of the lines drawn in given angles to the right lines given by poſition, will be equal to the ſpace to which the ſum of the ſquares of the perpendiculars, drawn to the three lines found, has a given ratio.

PROP. E. THEO.

Added by Mr. Lowry.

In the right line AB (fig. 221, pl. 15.) let there be three points A, D, B; then I ſay, if D be between A and B, that the cube on AB will be equal to the ſum of the cubes on AD, DB, together with thrice

the

the ſolid whoſe baſe is the rectangle ADB, and altitude AB: But if B be between A and D, then I ſay, that the cube on AB, together with thrice the ſolid whoſe baſe is the rectangle ADB, and altitude AB, will be equal to the difference of the cubes on AD, DB.

Caſe I. When D is between A and B.

Let the cube AFKH, whoſe ſide is AB, be interſected by the plane D*q*LN parallel to the ſides AEIK, BFHG, of the cube and paſſing through the point D. Let the cubes AW*bc*, DPUS, whoſe ſides are AD, DB, be deſcribed, and the lines drawn as in the figure. Then the ſolid EDIN is equal to the ſum of the ſolids EWI*n*, XD*m*N. But the ſolid XD*m*N is equal to the cube on AD, together with the ſolid *adm*N; therefore the ſolid EDIN is equal to the ſum of the ſolids EWI*n*, *adm*N, together with the cube on AD. But AX and A*b* are each equal to AD, and AE is equal to AK, therefore EX and *b*K are each equal to DB; wherefore, the baſes EW, K*d*, of the ſolids EWI*n*, *adm*N, are each equal to the rectangle ADB, and their altitudes AK and *ab*, to AB and AD reſpectively. Therefore the ſum of the ſolids EWI*n*, *adm*N, is equal to the ſolid whoſe baſe is the rectangle ADB, and altitude the ſum of AB, AD, therefore the ſolid EDIN, is equal to the cube on AD, together with the ſolid whoſe baſe is the rectangle ADB, and altitude the ſum of AB, AD.

In the ſame way it may be ſhewn, that the ſolid *q*BLG is equal to the cube on DB, together with the ſolid whoſe baſe is the rectangle ADB and altitude AB, BD. Therefore the ſum of the ſolids EDIN, *q*BLG, that is, the cube on AB, is equal to the ſum of the cubes on AD, DB, together with the ſolid whoſe baſe is the rectangle ADB and altitude the ſum of AB, AD, together with the ſolid whoſe baſe is the

the rectangle ADB and altitude AB, BD. But the ſum of AB, AD, together with the ſum of AB, BD, is equal to thrice AB, therefore, the cube on AB is equal to the ſum of the cubes on AD, DB, together with thrice the ſolid whoſe baſe is the rectangle ADB, and altitude AB.

Caſe II. When B is between A and D. Fig. 222, pl. 15.

It has been ſhewn, that the cube on AD is equal to the ſum of the cubes on AB, BD, together with thrice the ſolid whoſe baſe is the rectangle ABD, and altitude AD. But the ſolid whoſe baſe is the rectangle ABD, and altitude AD is evidently equal to the ſolid whoſe baſe is the rectangle ADB, and altitude AB, therefore the cube on AD is equal to the ſum of the cubes on AB, DB, together with thrice the ſolid whoſe baſe is the rectangle ADB, and altitude AB. Wherefore, the cube on AB, together with thrice the ſolid whoſe baſe is the rectangle ADB, and altitude AB is equal to the difference of the cubes on AD and DB.

Cor. The cube on any right line is equal to eight times the cube on half that line.

PROP. F. THEO.

Added by Mr. LOWRY.

In the right line AB, let there be four points A, B, C, D, and let the point C, be equidiſtant from A and B: then, if D be between A and B, the ſum of the cubes on AD, DB, will be equal to twice the cube AC or CB, together with thrice the ſolid whoſe baſe is the ſquare of DC and altitude AB: but if B be between A and D, the difference of the cubes on AD, DB, will be equal to twice the cube on AC or CB, together with thrice the ſolid whoſe baſe is the ſquare of CD, and altitude AB.

Caſe

Caſe I. When D is between A and B. Fig. 221, pl. 15.

By Prop. E. the ſum of the cubes on AD, DB, together with thrice the ſolid whoſe baſe is the rectangle ADB, and altitude AB, is equal to the cube on AB, that is, *(by cor. ibid.)* equal to eight times the cube on AC. But the rectangle ADB, is equal to the exceſs of the ſquare on BC above the ſquare on DC, therefore the ſolid whoſe baſe is the rectangle ADB, and altitude AB, is equal to the ſolid whoſe baſe is the exceſs of the ſquare on BC above the ſquare on DC, and altitude AB, that is, equal to the exceſs of twice the cube on BC above the ſolid whoſe baſe is the ſquare of DC, and altitude AB, therefore, thrice the ſolid whoſe baſe is the rectangle ADB, and altitude AB is equal to the exceſs of ſix times the cube on BC above thrice the ſolid whoſe baſe is the ſquare of DC, and altitude AB. Wherefore the ſum of the cubes on AD, DB, is equal to the cube on AC, or BC, together with thrice the ſolid whoſe baſe is the ſquare of DC, and altitude AB.

Caſe II. When B is between A and D.

This follows in the ſame manner from *Caſe* 2, of Prop. E.

PROP. XXII. THEO. XIX. Fig. 268, 269. Pl. 17.

Demonſtrated by Mr. Lowry.

Caſe I. When the number of the ſides of the figure circumſcribed about the circle is even, fig. 268.

Let ABCDEF, &c. be any regular figure of an even number of ſides circumſcribed about a circle, and from any point G, in the circumference, let there be drawn GH, GK, GL, GM, GN, GO, &c. perpendicular to the ſides of the figure, and let *a* be the centre of the circle, and join G*a*; twice the ſum

of

of the cubes on GH, GK, GL, GM, GN, GO, &c, will be equal to five times the multiple of the cube on the ſemidiameter aG, of the circle, by the number of the ſides of the figure.

Let the circumſcribing figure touch the circle in P, Q, R, S, T, V, &c. and join aP, aQ, aR, &c. and draw GX, GY, GZ, &c. perpendicular to aP, aQ, aR, &c.

Becauſe the number of the ſides of the circumſcribing figure is even, it is plain, that aP, aQ, aR, &c. will paſs through the oppoſite points of contact, that is, through the points S, T, V, &c. and that PS, QT, RV, &c. are biſected in a; therefore, (Prop. F.) the ſum of the cubes on GH, GM, that is, the ſum of the cubes on PX, XS, is equal to twice the cube on aP, together with ſix times the ſolid whoſe baſe is the ſquare of aX and altitude aP. In the ſame way it is ſhewn, that the ſum of the cubes on GK, GN, is equal to twice the cube on aP, together with ſix times the ſolid whoſe baſe is the ſquare of aY, and altitude aP; and likewiſe, that the ſum of the cubes on GO, GL, is equal to twice the cube on aP. together with ſix times the ſolid whoſe baſe is the ſquare of aZ and altitude aP; and ſo on. Therefore, the ſum of the cubes on GH, GK, GL, GM, GN, GO, &c. is equal to the multiple of the cube on aP by the number of the ſides of the figure, together with ſix times the multiple, by the ſame number, of the ſolid whoſe baſe is the ſum of the ſquares of aX, aY, aZ, &c. and altitude aP.

Again, becauſe the angles GXa, GYa, GZa, &c, are right angles, the points X, Y, Z, &c. will be in the circumference of the circle whoſe diameter is Ga; and becauſe the circle paſſes through the point a, the circumference will be divided into equal parts in the points X, Y, Z, &c. as many in number as the right lines aP, aQ, aR, &c. (Lem. II.).

There-

Cor. Therefore (Prop. IV.) the ſum of the ſquares of aX, aY, aZ, &c. is equal to twice the multiple of the ſquare of half aG, by the number of the lines aP, aQ, aR. &c. that is, equal to the multiple of the ſquare of half aG, by the number of the ſides of the circumſcribing figure. Therefore, twelve times the multiple, by the number of the ſides of the circumſcribing figure, of the ſolid whoſe baſe is the ſum of the ſquares of aX, aY, aZ, &c. and altitude aP, is equal to thrice the multiple, by the ſame number, of the cube on aP: and therefore twice the ſum of the cubes on GH, GK, GL, GM, GN, GO, &c. is equal to five times the multiple of the cube on aP, by the number of the ſides of the figure.

Caſe II. When the number of the ſides of the figure circumſcribed about the circle is odd, fig. 269.

Let ABCDE, &c. be any regular figure of an odd number of ſides circumſcribed about a circle, and from any point in the circumference, let there be drawn PH, PK, PL, PM, PN, &c. perpendicular to the ſides of the figure, and let a be the centre of the circle, and join Pa; twice the ſum of the cubes on PH, PK, PL, PM, PN, &c. will be equal to five times the multiple of the cube on the ſemidiameter, aP, of the circle by the number of the ſides of the figure.

Let the circumſcribing figure touch the circle in the points Q, R, S, T, U, &c. and join aQ, aR, aS, aT, aU, &c.: join alſo PQ, PR, PS, PT, PU, &c. and draw PV, PW, PX, PY, PZ, &c. perpendicular to aQ, aR, aS, aT, aU, &c. The cube on PH or QV is equal to the ſolid whoſe baſe is the difference of the ſquares of PQ, PV, and altitude QV: but the ſquare of PV is equal to the difference of the ſquares of aP, aV, and the ſquare of PQ is equal to twice the rectangle aPH (Lem. I.); therefore the difference of the ſquares of PQ and PV is

equal

equal to the difference of twice the rectangle *a*PH, together with the ſquare of *a*V, and the ſquare of *a*P; therefore the cube on PH is equal to the ſolid whoſe baſe is the difference of twice the rectangle *a*PH, together with the ſquare of *a*V, and the ſquare of *a*P, and altitude PH. The ſame way it is ſhewn, that the cube on PK is equal to the ſolid whoſe baſe is the difference of twice the rectangle *a*PK, together with the ſquare of *a*W, and the ſquare of *a*P, and altitude PK; that the cube on PL is equal to the ſolid whoſe baſe is the difference of twice the rectangle *a*PL, together with the ſquare of *a*X, and the ſquare of *a*P, and altitude PL; that the cube on PM is equal to the ſolid whoſe baſe is the difference of twice the rectangle *a*PM, together with the ſquare of *a*Y, and the ſquare of *a*P, and altitude PM; that the cube on PN is equal to the ſolid whoſe baſe is the difference of twice the rectangle *a*PN, together with the ſquare of *a*Z, and the ſquare of *a*P, and altitude PN; and ſo on. Now, the ſolid whoſe baſe is twice the rectangle *a*PH, and altitude PH, together with the ſolid whoſe baſe is twice the rectangle *a*PK, and altitude PK, together with the ſolid whoſe baſe is twice the rectangle *a*PL, and altitude PL, together with the ſolid whoſe baſe is twice the rectangle *a*PM, and altitude PM, together with the ſolid whoſe baſe is twice the rectangle *a*PN, and altitude PN, and ſo on, is equal to the ſolid whoſe baſe is twice the ſum of the ſquares of PH, PK, PL, PM, PN, &c. and altitude *a*P. But the ſum of the ſquares of PH, PK, PL, PM, PN, &c. is equal to thrice the multiple of the ſquare of *a*P by the number of the ſides of the figure (Prop. V.) Therefore the ſolid whoſe baſe is twice the ſum of the ſquares of PH, PK, PL, PM, PN, &c. and altitude *a*P, is equal to thrice the multiple of the cube on *a*P, by the number of the ſides of the figure.

Again,

Again, the ſolid whoſe baſe is the ſquare of aP, and altitude the ſum of PH, PK, PL, PM, PN, &c. is equal to the multiple of the cube on aP, by the number of the ſides of the figure. Therefore twice the ſum of the cubes on PH, PK, PL, PM, PN, &c. is equal to four times the multiple of the cube on aP, by the number of the ſides of the figure, together with the ſolid whoſe baſe is twice the ſquare of aV, and altitude PH, together with the ſolid whoſe baſe is twice the ſquare of aW, and altitude PK, together with the ſolid whoſe baſe is twice the ſquare of aX, and altitude PL, together with the ſolid whoſe baſe is twice the ſquare of aY, and altitude PM, together with the ſolid whoſe baſe is twice the ſquare of aZ, and altitude PN, and ſo on. Now, it is eaſily ſhewn, that the ſquare of aV, together with the ſpace to which the ſquare aW has the ſame ratio that PH has to PK, together with the ſpace to which the ſquare of aX has the ſame ratio that PH has to PL, together with the ſpace to which the ſquare of aY has the ſame ratio that PH has to PM, together with the ſpace to which the ſquare of aZ has the ſame ratio that PH has to PN, and ſo on, is equal to the ſpace to which the ſquare of aP has the ſame ratio that twice PH has to the ſum of PH, PK, PL, PM, PN, &c. that is, the ſame ratio that twice PH has to the multiple of aP by the number of the ſides of the figure. Therefore the ſolid whoſe baſe is twice the ſquare of aV, and altitude PH, together with the ſolid whoſe baſe is twice the ſquare of aW, and altitude PK, and ſo on, is equal to the multiple of the cube on aP, by the number of the ſides of the figure. And, therefore, twice the ſum of the cubes on PH, PK, PL, PM, PN, &c. is equal to five times the multiple of the cube on aP, by the number of the ſides of the figure.

The ſame demonſtrated by Mr. **Swale**, *Fig.* 270, 271. *Pl.* 17.

Caſe I. When the number of the ſides is even, fig. 270.

Let ABCDEFA, be a regular figure of ſix ſides circumſcribed about the circle GHIKLM, whoſe centre is O. From any point P, in the circumferrence, let there be drawn PQ, PR, PS, PT, PV, PW, perpendicular to the ſides of the figure; twice the ſum of the cubes on the perpendiculars PQ, PR, PS, PT, PV, PW, will be equal to five times the multiple of the cube on the ſemidiameter, OP, of the circle, by the number AB, BC, CD, DE, EF, FA, of the ſides of the figure.

Draw OG, OH, OI, OK, OL, OM, to the points of contact; alſo join OP, and demit, upon OK, OI, OH. the perpendiculars P*d*, P*m*, P*n*. Becauſe the figure has an even number of ſides, it is evident, that AB, DE, BC, EF; and CD, FA, will be reſpectively parallel to each other, and conſequently, GOK, HOL, LOM, joining the points of contact, will be ſtraight lines, and each equal to the diameter.

Alſo, PQ $=$ *d*G; PR $=$ *n*H; PS $=$ *m*I, PT $=$ *d*K; PV $=$ *n*H; PW $=$ *m*M; and conſequently, twice the ſum of the cubes on the $\perp$s PQ, PR, PS, FT, PV, PW is equal to twice the ſum of the cubes on *d*G, *d*K, *n*H, *n*L, *m*I, *m*M.

The ſum of the cubes on *d*G, *d*K, is equal to the ſolid whoſe baſe is the difference between the ſum of the ſquares *d*G, *d*K, and the rectangle G*d*K, and altitude GK, that is, equal to the ſolid whoſe baſe is the difference between the ſquare of GK, and thrice the rectangle G*d*K and altitude GK, that is, equal to the ſolid whoſe baſe is the difference between the ſquare of GK, and thrice the ſquare of *d*P, and altitude

altitude GK; therefore twice the ſum of the cubes on *d*G, DK, is equal to the ſolid whoſe baſe is the difference between the ſquare of GK, and thrice the ſquare of *d*P, and altitude twice GK. In the ſame way it is ſhewn, that twice the ſum of the cubes on *nH*, *n*L, is equal to the ſolid whoſe baſe is the difference between the ſquare of HL, and thrice the ſquare of *n*P, and altitude twice HL; that is twice the ſum of the cubes on *m*I, M*m*, is equal to the ſolid whoſe baſe is the difference between the ſquare of IM and thrice the ſquare of *m*P, and altitude twice IM. Therefore twice the ſum of the cubes on *d*G, *d*K, *n*K, *n*L, *m*I, *m*M, is equal to thrice the ſolid whoſe baſe is the difference between the ſquare of GK, and the ſum of the ſquares of *d*P, *n*P, *m*P, and altitude twice GK, that is, equal to the ſolid whoſe baſe is the difference between twice the ſquare of GK, and twice the ſum of the ſquares of *d*P, *n*P, *m*P, and altitude thrice GK. But twice the ſquare of GK is equal to eight times the ſquare of OP; and ſince the angles about the point O are equal, twice the ſum of the ſquares of *d*P, *n*P, *m*P, is equal to thrice the ſquare of OP (Prop. XIV.); alſo thrice the diameter GK is equal to ſix times the radius OP. Therefore, twice the ſum of the cubes on *d*G, *d*K, *n*H, *n*L, *m*I, *m*M, is equal to the ſolid whoſe baſe is the difference between eight times the ſquare of OP, and altitude ſix times OP, that is, equal to the ſolid whoſe baſe is five times the ſquare of OP, and altitude ſix times OP, that is, equal to five times the multiple of the cube on OP, by the number ſix. And, therefore, twice the ſum of the cubes on PQ, PR, PS, PT, PV, PW, is equal to five times the multiple of the cube on the ſemidiameter, OP, of the circle, by the number of the ſides of the figure.

Caſe II. When the number of the ſides is odd, (fig. 271.)

Let ABCDEA, be a regular figure of five ſides circumſcribed about the circle LNQRS, whoſe centre is O. From any point P, in the circumference, let there be drawn PG, PH, PI, PK, PF, perpendicular to the ſides of the figure; twice the ſum of the cubes on the perpendiculars PG, PH, PI, PK, PF, will be equal to five times the multiple of the cube on the ſemidiameter, OP, of the circle, by the number AB, BC, CD, DE, EA, of the ſides of the figure.

Join the centre O, and the points of contact L, N, Q, R, S; join alſo P and the points L, N, Q, R, S, and upon OL, ON, OQ, OR, OS, demit the perpendiculars PT, PV, PW, PX, PM. Then will PG = TL, PH = VN, PI = WQ, FK = RX, PF = SM; and PO = LO = NO = QO = RO = SO, and twice the ſum of the cubes on PG, PH, PI, PK, PF, is equal to twice the ſum of the cubes on TL, VN, WQ, XR, MS.

Now, twice the ſum of the cubes on LT, LO, is equal to the ſolid whoſe baſe is the difference between twice the ſum of the ſquares of LT, LO, and twice the rectangle TLO, and altitude the ſum of LT, LO; twice the ſum of the cubes on NV, LO is equal to the ſolid whoſe baſe is the difference between twice the ſum of the ſquares of NV, LO, and twice the rectangle VNO, and altitude the ſum of NV, LO; twice the ſum of the cubes on QW, LO, is equal to the ſolid whoſe baſe is the difference between twice the ſum of the ſquares of QW, LO, and twice the rectangle WQO, and altitude the ſum of WQ, LO; twice the ſum of the cubes on RX, LO, is equal to the ſolid whoſe baſe is the difference between

between twice the ſum of the ſquares of RX, LO, and twice the rectangle XRO, and altitude the ſum of XR, LO; and twice the ſum of the cubes on SM, LO, is equal to the ſolid whoſe baſe is the difference between twice the ſum of the ſquares of SM, LO, and twice the rectangle MSO, and altitude the ſum of SM, LO. Therefore, twice the ſum of the cubes on LT, NV, QW, RX, SM, together with ten times the cube on LO, is equal to the ſolid whoſe baſe is the difference between twice the ſum of the ſquares of LT, LO, and twice the rectangle TLO, and altitude the ſum of LT, LO, together with the ſolid whoſe baſe is the difference between twice the ſum of the ſquares of NV, LO, and twice the rectangle VNO, and altitude the ſum of NV, LO, together with the ſolid whoſe baſe is the difference between twice the ſum of the ſquares of QW, LO, and twice the rectangle WQO, and altitude the ſum of WQ, LO, together with the ſolid whoſe baſe is the difference between twice the ſum of the ſquares of RX, LO, and twice the rectangle XRO, and altitude the ſum of XR, LO, together with the ſolid whoſe baſe is the difference between twice the ſum of the ſquares of SM, LO, and twice the rectangle MSO, and altitude the ſum of SM, LO, that is, equal to the ſolid whoſe baſe is the ſquare of PL, together with twice the ſquare of OT, and altitude the ſum of LT, LO, together with the ſolid whoſe baſe is the ſquare of NP, together with twice the ſquare of VO, and altitude the ſum of NV, LO, together with the ſolid whoſe baſe is the ſquare of QP, together with twice the ſquare of WO, and altitude the ſum of QW, LO; together with the ſolid whoſe baſe is the ſquare of RP, together with twice the ſquare of XO, and altitude the ſum of RX, LO; together with the ſolid whoſe baſe is the ſquare of SP, together with twice the ſquare of

[illegible] the [illegible] of SM, LO, that is, equal [illegible] whose base is the sum of the squares of PL, PN, PQ, PR, PS, together with twice the sum [illegible] LT, OV, OW, OX, OM, and altitude [illegible] together with the solid whose base is the [illegible] PL together with twice the square of OT, [illegible] LT together with the solid whose base [illegible] PN together with twice the square [illegible] NV, together with the solid [illegible] square of PQ, together with twice [illegible] QW, together with [illegible] the square of PR, together with [illegible] ON, and altitude RX, together [illegible] whose base is the square of PS, together [illegible] square of OM, and altitude SM, that is, equal to the solid whose base is the difference between [illegible] the squares of PL, PN, PQ, PR, PS, [illegible] times the square of LO, and twice the [illegible] squares of PT, PV, PW, PX, PM, [illegible] LO, together with the solid whose base [illegible] square of LO, and altitude the sum of [illegible] NV, QW, RX, SM, together with the difference [illegible] whose base is twice the sum of the squares of PL, PN, PQ, PR, PS, and altitude LO, and the [illegible] solids whose bases are twice [illegible] PT, PV, PW, PX, PM, and altitudes [illegible] NV, QW, RX, SM, respectively. But (Prop. IV. the [illegible] of the squares of PL, PN, PQ, PR, PS, is equal to ten times the square of LO; and (Prop. XIV. twice the sum of the squares of PT, PV, PW, PX, PM, is equal to five times the square of LO, and (Prop. III. the sum of LT, NV, QW, RX, SM, is equal to five times LO; and (Prop. V. Cor. I.) twice the sum of the squares of TL, NV, QW, RX, SM, is equal to fifteen times the square of LO. Therefore, the solid whose base is the sum of the squares of PL, PN, PQ, PR, PS, and altitude LO,

LO, is equal to ten times the cube on LO; the ſolid whoſe baſe is ten times the ſquare of LO, and altitude LO, is equal to ten times the cube on LO; the ſolid whoſe baſe is twice the ſum of the ſquares of PT, PV, PW, PX, PM, and altitude LO, is equal to five times the cube on LO; the ſolid whoſe baſe is twice the ſquare of LO, and altitude the ſum of LT, NV, QW, RX, SM, is equal to ten times the cube on LO; the ſolid whoſe baſe is twice the ſum of the ſquares of TL, NV, QW, RX, SM, and altitude LO, is equal to fifteen times the cube on LO; and the ſolid whoſe baſe is twice the ſquare of PT, and altitude LT, together with the ſolid whoſe baſe is twice the ſquare of PV, and altitude NV, together with the ſolid whoſe baſe is twice the ſquare of PW, and altitude QW, together with the ſolid whoſe baſe is twice the ſquare of PX, and altitude RX, together with the ſolid whoſe baſe is twice the ſquare of PM, and altitude SM, is equal to five times the cube on LO. And, therefore, twice the ſum of the cubes on LT, NV, QW, RX, SM, that is, twice the ſum of the cubes on PG, PH, PI, PK, PF, together with ten times the cube on LO, is equal to the difference between ten times the cube on LO, together with ten times the cube on LO, together with ten times the cube on LO, together with fifteen times the cube on LO, and five times the cube on LO, together with five times the cube on LO, that is, equal to the difference between forty-five times the cube on LO, and ten times the cube on LO, that is, equal to thirty-five times the cube on LO. Therefore, twice the ſum of the cubes on the perpendiculars PG, PH, PI, PK, PF, is equal to the difference between thirty-five times the cube on LO, and ten times the cube on LO, that is, equal to twenty-five times the cube on LO, that is, equal to five times the multiple of the cube on the ſemidiameter, OP, of the circle, by the number of the ſides of the figure.

ARTICLE

ARTICLE XXIII.

Demonſtrations to Lawſon's *Propoſitions propoſed in* ARTICLE III.

PROP. XXVIII. Fig. 272, 273, Pl. 17.

Demonſtrated by Mr. Colin Campbell.

JOIN CH, CK, and draw the radii OH, OK.
Since, the rect. ACB = the rect. ECF,
CFE, or ECF + CF^2 = ACB + CF^2 = KFG (Prop. XI.);
and FEC, or ECF + CE^2 = ACB + CE^2 = HEG (Prop. XI.):
wherefore FK : FE :: FC : FG, and EH : EF :: EC : EG;
conſequently ∠ FCK = EGF = ECH.
Hence ∠HOK + HCK = HCK + 2FCK = 2 right ∠s,
therefore the points H, O, K, C, are in a circle;
wherefore CLO = HLK = ALB.
Hence AL : LB :: AC : CB (Conv. Prop. I).
Q. E. D.

The ſame by Mr. Harris.

ANALYSIS.

Join CK and let it meet the circle again in I; join alſo CH, IH.—Since AL : LB :: AC : CB,
by Prop. VII. HI. is ⊥ to AB, and conſeq. ‖ to CD;
therefore the ∠FCK = HIK = HGK or EGF;
but, by Prop. IV. ∠FCK = ECH; hence, ∠ECH = EGF;
conſequently the points C, H, K, G, are in a circle;
therefore the rect. CEF = the rect. HEG:
but, by Prop. XI. ACB + CE^2 = HEG;
therefore CEF, or ECF + CE = ACB + CE^2;
and therefore the rect. ECF = the rect. ACB.
Q. Q. V.

The

The ſame by Mr. Lowry.

Draw CK to meet the circle in I and join HI.
By hypotheſis, ECF = ACB ; add CF^2 to each,
then $CFE = ECF^2 + CF^2 = ACB + CF^2 = KFG$ (Prop. XI.) ;
therefore the △s EFG, CFK, are equiangular ;
hence, ∠FCK = HGK = HIC ;
theref. HI is parallel to EF, and perpend. to AB,
conſequently, by Prop. VII. AL : LB :: AC : CB.
Q. E. D.

PROP. XXIX. Fig. 274, 275. *Pl.* 17.

Demonſtrated by Meſſrs. Campbell, Harris, *and* Lowry.

ANALYSIS, by Mr. Harris.

Since FG is parallel to AE by hypotheſis,
the ∠AEC = EFG = CDG or ADE ;
therefore the △s AEC, ADE are equiangular,
wherfore AD : AE :: AE : AC ;
hence $AE^2 = DAC = AB$; ∴ AE = AB.
Q. Q. V.

SYNTHESIS, by Meſſrs. Campbell *and* Lowry.

Becauſe AE = AB, $AE^2 = AB^2 = CAD$,
therefore DA : AE :: AE : AC ;
wherefore the △s ADE, ACE, are equiangular ;
hence, the ∠ AEC = ADE = EFG.
and FG is parallel to AE. *Q. E. D.*

PROP.

PROP. XXX. Fig. 276, 277, Pl. 17.

Demonſtrated by Meſſrs. Campbell, Harris, *and* Lowry.

ANALYSIS, by Mr. Harris.

Becauſe GH is parallel to AB, by hypotheſis,
the $\angle AEC = CGH = CDH$;
wherefore the points E, C, D, F, are in a circle;
hence, rect. $EAF =$ rect. $CAD = AB^2$. *Q. Q. V.*

SYNTHESIS, by Meſſrs. Campbell *and* Lowry.

By Analyſis, $EAF = AB^2 = CAD$;
therefore AC : AF :: AE : AD;
wherefore the $\triangle$s ACE, ADF are equiangular,
therefore, $\angle AEC = ADF = EGH$;
conſequently GH is parallel to AB. *Q. E. D.*

ARTICLE XXIV.

To the Editor of the Mathematical Repoſitory.

SIR,

IN almoſt every book of fluxions, methods hav
been pointed out for finding the fluent of a fluxio

of this form, *viz.* $\frac{x^m \dot{x}}{x^n - px^{n-1} + qx^{n-2} - \&c.}$ whe

all the roots of the equation $xn - px^{n-1} + qx^{n-2} - \&c. =$
are ſuppoſed to be poſſible. The following mann
of inveſtigating this fluent is different from any I ha

ſee

ſeen, and if you think that ſuch a trifle deſerves a place in your REPOSITORY, amongſt the many valuable papers to be found there, by inſerting it you will oblige,

Sir, your humble Servant,

A. B.

Required the fluent of $\frac{x^m \dot{x}}{x^n - px^{n-1} + qx^{n-2} - \&c.}$?

Let the roots of the equation $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$, be equal to $a, b, c, d.$ &c. and ſuppoſe

$$\frac{x^m \dot{x}}{x^n - px^{n-1} + qx^{n-2} \&c.} = K \times \frac{x^m \dot{x}}{x-a} + L \times \frac{x^m \dot{x}}{x-b} + M \times \frac{x^m \dot{x}}{x-c} + \&c.$$

then by diviſion we ſhall have

$$\frac{x^m \dot{x}}{x^n - px^{n-1} + qx^{n-2} - \&c.} =$$

$$\begin{cases} +K \times \left(x^{m-1}\dot{x} + ax^{m-2}\dot{x} + a^2 x^{m-3}\dot{x} \ldots\ldots + \frac{a^m \dot{x}}{x-a} \right) \\ +L \times \left(x^{m-1}\dot{x} + bx^{m-2}\dot{x} + b^2 x^{m-3}\dot{x} \ldots\ldots + \frac{b^m \dot{x}}{x-b} \right) \\ +M \times \left(x^{m-1}\dot{x} + cx^{m-2}\dot{x} + c^2 x^{m-3}\dot{x} \ldots\ldots + \frac{c^m \dot{x}}{x-c} \right) \\ + \quad \&c. \qquad\qquad\qquad\qquad \&c. \end{cases}$$

but $\frac{K}{x-a} + \frac{L}{x-b} + \frac{M}{x-c} + \frac{N}{x-d} + \&c. = \frac{1}{x^n - px^{n-1} + qx^{n-2} - \&c.}$,

therefore by reducing the fractions to a common denominator, we have

Kx

$$
\begin{array}{llll}
 & -b & +bc & \\
 & -e & +bd & -bcd \\
n-1 & -d \times Kx^{n-2} & +be & -bce \\
 & -e & +cd \times Kx^{n-3} & -bde \times Kx^{n-4} \ldots\ldots \pm bcde \,\&c \\
 & -\&c. & +ce & -cde \\
 & & +de & -\&c. \\
 & & +\&c. &
\end{array}
$$

$$
\begin{array}{llll}
 & -a & +ac & \\
 & -c & +ad & -acd \\
n-1 & -d \times Lx^{n-2} & +ae & -ade \\
 & -e & +cd \times Lx^{n-3} & -ace \times Lx^{n-4} \ldots\ldots \pm acde \,\&c \\
 & -\&c. & +ce & -cde \\
 & & +de & -\&c. \\
 & & +\&c. &
\end{array}
$$

$$
\begin{array}{llll}
 & -a & +ab & \\
 & -b & +ad & -abd \\
n-1 & -d \times Mx^{n-2} & +ae & -ade \\
 & -e & +bd \times Mx^{n-3} & -abe \times Mx^{n-4} \ldots\ldots \pm abde \,\&c \\
 & -\&c. & +be & -bde \\
 & & +de & -\&c. \\
 & & +\&c. &
\end{array}
$$

$$
\begin{array}{llll}
 & -a & +ab & \\
 & -b & +ac & -abc \\
n-1 & -c \times Nx^{n-2} & +ae & -abe \\
 & -e & +bc \times Nx^{n-3} & -ace \times Nx^{n-4} \ldots\ldots \pm abce \,\&c \\
 & -\&c. & +be & -bce \\
 & & +ce & -\&c. \\
 & & +\&c. &
\end{array}
$$

$$
\begin{array}{llll}
 & -a & +ab & \\
 & -b & +ac & -abc \\
n-1 & -c \times Px^{n-2} & +ad & -abd \\
 & -d & +bc \times Px^{n-3} & -acd \times Px^{n-4} \ldots\ldots \pm abcd \,\&c \\
 & -\&c. & +bd & -bcd \\
 & & +cd & -\&c. \qquad -1 \\
 & & +\&c. &
\end{array}
$$

&c.

t is evident, from the laſt equation that
+L+M+N+P+&c. = o, and likewiſe that
-a)+L(p-b)+M(p-c)+N(p-d)+&c. = o; but
+Lp+Mp+Np+&c. = o, therefore
+Lb+Mc+Nd+Pe+&c. = o.
Becauſe $q = ab+ac+ad+ae+$ &c. $+bc+bd$
$be+$, &c. $+cd+ce+$ &c.; if q' = the co-

cient of Kx^{n-3}, then $a(p-a)+q' = q$, or $q' = q-ap+a^2$.
In like manner if q'', q''', &c. be put for the co-
cients of Lx^{n-3}, Mx^{n-3}, &c. then will $q'' = q-$
$+b^2$, $q''' = q-pc+c^2$, &c.; hence
$q-pa+a^2)+L(q-pb+b^2)+M(q-pc+c^2)+$&c. = o.
But $Kq+Lq+Mq+$ &c. = o, therefore
$pa-a^2)+L(pb-b^2)+M(pc-c^2)+$ &c. = o;
d it has juſt been proved that
$\times pa+L\times pb+M\times pc+$, &c. = o, therefore
$\times a^2+L\times b^2+M\times c^2+$ &c. = o.

Put r', r'', r''', &c. for the reſpective coefficients
f Kx^{n-4}, Lx^{n-4}, Mx^{n-4}, &c. Then becauſe r
$= abc+abd+abe$, &c. $+bcd+bce$, &c. $+cde$,
&c. + &c. $aq'+r' = r$, or $r' = r-aq' = r-$
$q+pa^2-a^3$; in like manner $r'' = r-qb+$
b^2-b^3, $r''' = r-qc+pc^2-c^3$, &c. and Kr'
$+Lr''+Mr'''+$ &c. = o, therefore $K(r-qa$
$+pa^2-a^3)+L(r-qb+pb^2-b^3)+M(r-$
$c+pc^2-c^3)+$, &c. = o: but from the above
t will appear that $K(r-qa+pa^2)+L(r-qb+$
$b^2)+M(r-qc+pc^2)+$ &c. = o, therefore
$a^3+Lb^3+Mc^3+$ &c. = o. In the very ſame

manner we may proceed to $Ka^{n-2}+Lb^{n-2}+Mc^{n-2}$
+&c. = o; for there are $n-1$ columns which
vaniſh, but it is evident the laſt does not. Hence

we have $\frac{x^m \dot{x}}{x^n - px^{n-1} + qx^{n-2} - \&c.} =$

$$K \times \left(a^{n-1} x^{m-n} \dot{x} + a^{n-2} x^{m-n+1} \dot{x} \ldots\ldots + a^{m-1} \dot{x} + \frac{a^m \dot{x}}{x-a} \right)$$

$$+ L \times \left(b^{n-1} x^{m-n} \dot{x} + b^{n-2} x^{m-n+1} \dot{x} \ldots\ldots + b^{m-1} \dot{x} + \frac{b^m \dot{x}}{x-b} \right)$$

$$+ M \times \left(c^{n-1} x^{m-n} \dot{x} + c^{n-2} x^{m-n+1} \dot{x} \ldots\ldots + c^{m-1} \dot{x} + \frac{c^m \dot{x}}{x-c} \right)$$

$+ \&c.$ $+ \&c.$

If m be leſs than n, then $\frac{x^m \dot{x}}{x^n - px^{n-1} + qx^{n-2} - \&c.} =$

$K a^m \times \frac{\dot{x}}{x-a} + L b^m \times \frac{\dot{x}}{x-b} + M c^m \times \frac{\dot{x}}{x-c} + \&c.$ all the preceding terms vaniſhing; the fluent in each caſe is therefore, eaſily found.

Ex. I. Required the fluent of $\frac{x^5 \dot{x}}{x^3 - px^2 + qx - r}$?

Here $n = 3, m = 5$; therefore, by ſubſtitution,

$$+ K \times \left(a^2 x^2 \dot{x} + a^2 x \dot{x} + a^4 \dot{x} + a^5 \times \frac{\dot{x}}{x-a} \right)$$

$$\frac{x^5 \dot{x}}{x^3 - px^2 + qx - r} = + L \times \left(b^2 x^2 \dot{x} + b^2 x \dot{x} + b^4 \dot{x} + b^5 \times \frac{\dot{x}}{x-b} \right)$$

$$+ M \times \left(c^2 x^2 \dot{x} + c^2 x \dot{x} + c^4 \dot{x} + b^5 \times \frac{\dot{x}}{x-c} \right)$$

$+ K$

$$+K\times\left(\frac{a^{n}x^{3}}{3}+\frac{a^{2}x^{2}}{2}+a^{n}x+a^{5}\times \text{h.l.}\overline{x-a}\right)$$

$$\text{If } \frac{x^{5}\dot{x}}{x^{3}-px^{2}+qx-r}=+L\times\left(\frac{b^{n}x^{3}}{3}+\frac{b^{3}x^{2}}{2}+b^{n}x+b^{5}\times \text{h.l.}\overline{x-b}\right)$$

$$+M\times\left(\frac{c^{n}x^{3}}{3}+\frac{c^{3}x^{2}}{2}+c^{n}x+c^{5}\times \text{h.l.}\overline{x-c}\right).$$

Ex. II. Required the fluent of $\frac{x^{3}\dot{x}}{x^{5}-px^{4}+qx^{3}-rx^{2}+sx-t}$?

In this example $n=5$, $m=3$, therefore the fluent of

$$\frac{x^{3}\dot{x}}{x^{5}-px^{4}+qx^{3}-rx^{2}+sx-t}=Ka^{3}\times \text{h.l.}(x-a)+Lb^{3}\times \text{h.l.}(x-b)$$

$$+Mc^{3}\times \text{h.l.}(x-c)+Nd^{3}\times \text{h.l.}(x-d)+Pe^{3}\times \text{h.l.}(x-e).$$

The manner of determining the values of K, L, M, &c. is pointed out in every book of fluxions.

If n, be a large number, it is obvious that this method of finding the fluent can ſcarcely ever be made uſe of.

ARTICLE XXV.

Three Propoſitions from Lawſon.

(To be anſwered in Number X.)

PROP. XXXIV.

LET AB meet a circle in C and D, and A be without and B within the ſame, and let the rectangle CAD be equal to the ſquare of AB, together with the rectangle CBD, and through A any line be drawn meeting the circle in E and F, and BE, BF, be drawn meeting the circle again in G and H; then the points A, G, H, are in a right line.

PROP. XXXV.

From the extremes of AB, let two lines AC, BD be drawn to touch a circle in C and D, and in AB let a point E be taken on the ſame ſide of A with B, ſuch that the rectangle BAE may be equal to the ſquare of AC, and alſo in AB another point F on the ſame ſide of B with E ſuch that the rectangle EBF may be equal to the ſquare of BD, and through A any line be drawn meeting the circle in G and H, and BG, BH be drawn meeting the circle again in K and L; then the points L, K, F, are in a right line.

PROP. XXXVI.

If from A, the vertex of a triangle ABC, be drawn AD to any point D in the baſe, and DE be drawn parallel to AC, and DF to AB. I ſay the ſum of the rectangles BAF, CAF will be equal to the ſquare of AD, together with the rectangle BDC.

ARTICLE XXVI.

An old PROBLEM *with a* NEW SOLUTION.

By Mr. JOHN SURTEES, SUNDERLAND.

PROBLEM.

GIVEN *the day of the month, or ſun's declination to find the latitude of the place where the duration of twilight is the leaſt poſſible.*

SOLUTION. *Fig.* 303, *Plate* 18.

Let HH repreſent the horizon, Z the zenith, N the nadir, O'CO the parallel of declination, AOA the circle of twilight, ZBN an azimuth circle paſſing through

through O′ the place or point of the horizon where the ſun ſets, and O the place where twilight ends.

Then ſince the portion O′BO of the azimuth circle, is the ſhorteſt that can be comprehended between the horizon and circle of twilight, it is evident that O′CO will be the ſhorteſt portion of the parallel of declination.

Put t = tangent of BO′ = half the depreſſion, s = ſine of the declination = coſine of SO′, and radius = 1.

In the right angled ſpherical triangle SBO′, as coſine $BO' = \frac{1}{\sqrt{1+tt}}$: radius = 1 :: coſine of SO′ = s : coſine of BS = $s\sqrt{1+tt}$, and in the right angled ſpherical triangle SBN, as radius = 1 : coſine $BN = \frac{t}{\sqrt{1+tt}}$ :: coſine of BS = $s\sqrt{1+tt}$: coſine of SN = st = the ſine of the latitude, that is, as radius is to tangent of half the depreſſion, ſo is the ſine of the declination to the ſine of the latitude; which is different from what *Emerſon* and others have aſſerted.

N. B. If the latitude is north, the declination muſt be ſouth, and *vice verſa*.

Mr. *Emerſon* has erred in his ſolution of this problem.----The angle pvN (ſee his figure, page 492, Miſcellanies) is greater than 90°, when twilight is ſhorteſt, ſo of conſequence its coſine = $\frac{sx-l}{cy} = \frac{l}{y}$, and $sx = l \times \sqrt{1+c}$, which agree exactly with the above.

ARTICLE XXVII.

Anſwers to the Mathematical Queſtions propoſed in ARTICLE X. No. VI.

I. QUESTION 110, *anſwered by Mr.* **Olinthus Gregory,** *Bookſeller and Teacher of the Mathematics, Cambridge.*

IN order to give an accurate ſolution to this queſtion, it will be neceſſary to conſider the piece of fir, in each of the two inſtances, as immerſed in, and ſuſtained by two media, of different ſpecific gravities; in the firſt caſe *air* and *water*, and in the next, *air* and *oil of turpentine*. The following theorem from Mr. MARTIN (which perhaps is not ſo well known as it ought to be) will then enable us to determine the requiſite particulars: "The part of the ſolid in the heavier fluid, is to the whole ſolid, as the difference between the ſpecific gravity of the ſolid and lighter fluid, to the difference between the ſpecific gravity of the two fluids."——Now, the ſpecific gravity of water, oil of turpentine and fir, being as expreſſed, in the queſtion, and that of air being 1·25, we have this proportion, (conſidering the contents of priſms of equal baſes, as proportional to their altitudes): as 1000—1·25 (difference between the ſpecific gravity of water and air): 550—1·25, (difference between the ſpecific gravity of fir and air) :: 6 Inches (altitude of the part immerſed in water) : 10·92027335 inches, the altitude or ſide of the cubic piece of fir. Again by means of the ſame theorem, as 800—1·25 (difference between the ſpecific gravity of oil of turpentine and air) : 550 — 1·25 :: 10·92027335 inches : 7·50234742 inches, (altitude of the part immerſed in oil of turpentine). Had this queſtion been ſolved according to the common rule, which is only ſtrictly true *in vacuo*, it would have given $10\frac{10}{11}$ inches for the ſide of the cube, and $7\frac{7}{8}$ inches for the depth of the part immerſed in oil of turpentine.

N. B. In queſtions of this nature, another theorem of Mr. MARTIN's may often be had recourſe to with advantage: for the benefit of the young ſtudent who is unacquainted with the rule it is here added: "as the part of the ſolid within the heavier fluid, is to the part contained within the lighter: ſo is the difference of the ſpecific gravity of the ſolid and lighter fluid, to the difference between the ſpecific gravity of the ſolid and heavier." The inveſtigation of theſe two rules is given by Mr. MARTIN, and would have been repeated here, had it not been conſidered as a pleaſing exerciſe for the Tyro to trace out the rationale of them himſelf.

The

The ſame by Mr. John Byerly, *Ripon, Yorkſhire.*

By VINCE's Hydroſtatics, Prop. XVI. we have,
As the ſpecific gravity of fir 550 :
Is to the ſpecific gravity of water 1000 ::
So is the part immerſed in water :
To the whole cube ::
So is the depth of the immerſed part, 6 inches :
To the ſide of the cube, $10\frac{10}{11}$ inches.

Again, by the corollary to the ſame propoſition, we have this theorem, viz. "If the ſame body float upon two different fluids, the parts immerſed will be inverſely as the ſpecific gravities of the fluids." Hence, as 800 : 1000 :: 6 : $7\frac{1}{2}$, or 800 : 550 :: $10\frac{10}{11}$: $7\frac{1}{2}$ inches, the depth immerſed in oil of turpentine.

N. B. The part above the fluid is here ſuppoſed to be in vacuo. But when a body floats upon a fluid, the part above the fluid, being in the air, this rule will not be accurately true; near enough ſo, however, for all practical purpoſes. The effect of air, if neceſſary, may be computed by the following theorem, viz. "If a lighter fluid reſt upon a heavier, and their ſpecific gravities be as $a : b$, and a body, whoſe ſpecific gravity is c, reſt with one part P in the upper fluid, and the other part Q in the lower, then $P : Q :: b - c : c - a$, or $Q : P + Q :: c - a : b - a$. Vide VINCE's Hydroſtatics, Prop. XXI.

And thus nearly was the anſwer given by Mr. Johnſton.

The ſame anſwered by the Rev. L. Evans, *Royal Academy, Woolwich.*

Let $x =$ the ſide of the cube of fir ſought, then $550x^3 =$ its weight, and $6000x^2 =$ the weight of water diſplaced by the fir floating in it. Therefore $550x^3 = 6000x^2$. Hence $x = 10\frac{10}{11}$ inches, the ſide of the cube. Then as 800 : 550 :: $10\frac{10}{11}$: $7\frac{1}{2}$ inches, the depth immerſed in oil of turpentine.

Meſſrs. Boſworth (*the Propoſer*), Cantabrigus, Harris, Lowry, Marrat, Peacock, Simpſon, *and* Swale *anſwered it exactly like Mr.* Evans.

II. QUESTION 111, *anſwered by* Cantabrigus.

In this queſtion are given the diameters of a veſſel in form of a conic fruſtum, 18 and 14 inches reſpectively, and the altitude 28 inches, to find which baſe muſt be downwards that the liquor contained therein may be longeſt in running out of a circular orifice an inch in diameter; and how much longer time the liquor would be exhauſting with one end downwards than with the other. Here we may

end, and $\left(\frac{84+2x}{x}+9\right) \div 2 =$ the mean girt of the piece having the greater end. Consequently

$$\overline{\tfrac{1}{4}}^{2} \times \overline{\frac{42+2x}{x}}^{2} \times 12 + \overline{\tfrac{1}{4}}^{2} \times \overline{\frac{42+5\frac{1}{2}x}{x}}^{2} \times \overline{x-12} = \overline{\tfrac{1}{4}}^{2} \times 5\tfrac{1}{2}^{2} \times x + 3.$$

Whence $x = 17{\cdot}81$ feet, the answer.

The same by Mr. John Harris, *Land Surveyor, and Teacher of the Mathematics, Caermarthen.*

Put $a =$ lesser girt, $= 2$, $b =$ greater girt, $= 9$, $c = 12 =$ AG (fig. 279, pl. 18) the distance, of the section required, from the lesser end, $e = 3$, $x =$ AH, the length of the piece of timber, and $\pi = 3{\cdot}1416$. Then $\frac{a+b}{2}$ = the girt in the middle of the whole piece, and the square of $\frac{1}{4}$th of it $= \frac{\overline{a+b}^{2}}{64}$, which multiplied by x (the length) gives $\frac{\overline{a+b}^{2}}{64} \times x$, for the content of the whole piece. . . . Now, $\frac{a}{\pi}$ = the diameter AC, and $\frac{b}{\pi}$ = the diameter BD, and by similar triangles x (AH) : $\frac{b-a}{2\pi}$ (BH) :: c (AG) : $\frac{b-a}{2\pi x} \times c$ = EG; therefore the diameter EF $(= 2\text{EG} + \text{AC}) = \frac{b-a}{\pi x} \times c + \frac{a}{\pi}$ consequently the girt at the section EF $= \frac{b-a}{x} \times c + a$, and the girt at the middle of the lesser piece AEFC $= \frac{b-a}{2x} + a$; the square of $\frac{1}{4}$th of this is $= \frac{\overline{b-a}^{2}}{64x^{2}} \times c^{2} + \frac{b-a}{16x} \times ac + \frac{a^{2}}{16}$ which multiplied by c (AG), gives $\frac{\overline{b-a}^{2}}{64x^{2}} \times c^{3} + \frac{b-a}{16x} \times ac^{2} + \frac{a^{2}c}{16}$ = the content of the frustum AEFC.

Again,

Again, $\frac{b-a}{2x} \times c + \frac{a+b}{2}$ = the girt at the middle of the greater piece EBDF, and the ſquare of $\frac{1}{4}$th of this is $= \frac{\overline{b-a}^2}{64x^2} \times c^2 + \frac{b^2-a^2}{32x} \times c + \frac{\overline{b+a}^2}{64}$ which multiplied by $\overline{x-c}$ (GH), gives $\frac{\overline{b-a}^2}{64x} \times c^2 + \frac{b^2-a^2}{32} \times c + \frac{\overline{b+a}^2}{64} \times x - \frac{\overline{b-a}^2}{64x^2} \times c^2 - \frac{b^2-a^2}{32x} \times c^2 - \frac{\overline{b+a}^2}{64} \times c$, for the content of the fruſtum EBDF.

Now, the fruſtum AF $= \frac{\overline{b-a}^2}{64x^2} \times c^3 + \frac{b-a}{16x} \times ac^2 + \frac{a^2c}{16}$, and ED $= \frac{\overline{b-a}^2}{64x} \times c^2 + \frac{b^2-a^2}{32} \times c + \frac{\overline{a+b}^2}{64} \times x - \frac{\overline{b-a}^2}{64x^2} \times c^3 - \frac{b^2-a^2}{32x} \times c^2 - \frac{\overline{a+b}^2}{64} \times c$; their ſum $= \frac{\overline{b-a}^2}{64x^2} \times c^3 - \frac{\overline{b-a}^2}{64x^2} \times c^3 + \frac{b-a}{16x} \times ac^2 + \frac{\overline{b-a}^2}{64x} \times c^2 - \frac{b^2-a^2}{32x} \times c^2 + \frac{a^2c}{16} + \frac{b^2-a^2}{32} \times c - \frac{\overline{a+b}^2}{64} \times c + \frac{\overline{a+b}^2}{64} \times x$, which, per queſtion muſt be $= \frac{\overline{a+b}^2}{64} \times x$ (the content of the whole) $+ e$; therefore, by taking away what is common, &c. we have

$$\frac{b-a}{16x} \times ac^2 + \frac{\overline{b-a}^2}{64x} \times c^2 - \frac{b^2-a^2}{32x} \times c^2 + \frac{a^2c}{16} + \frac{b^2-a^2}{32} \times c - \frac{\overline{a+b}^2}{64} \times c = e.$$

Now, $\frac{b-a}{16x} \times ac^2 = (4ba - 4a^2) \times \frac{c^2}{64x^2}$,

$$\frac{\overline{b-a}^2}{64x} \times c^2 = (b^2 - 2ab + a^2) \times \frac{c^2}{64x^2},$$

and $- \frac{b^2-a^2}{32x} \times c^2 = -(2b^2 + 2a^2) \times \frac{c^2}{64x^2}$;

and

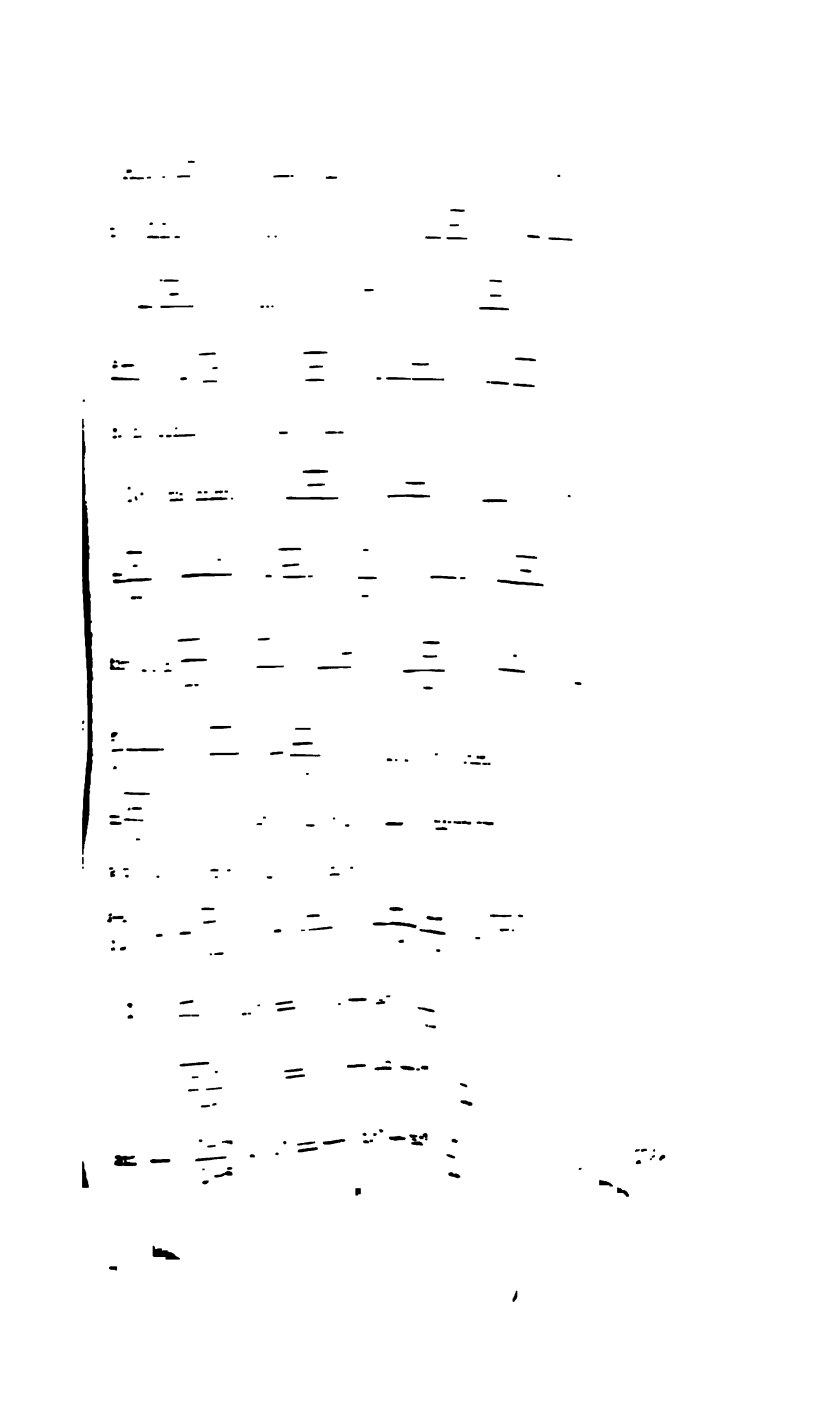
end, and $\left(\frac{84 + 2x}{x} + 9\right) \div 2 =$ the mean girt of the piece having the greater end. Consequently

$$\overline{\tfrac{1}{4}}^{2} \times \overline{\frac{42 + 2x}{x}}^{2} \times 12 + \overline{\tfrac{1}{4}}^{2} \times \overline{\frac{42 + 5\frac{1}{2}x}{x}}^{2} \times \overline{x - 12} = \overline{\tfrac{1}{4}}^{2} \times 5\tfrac{1}{2}^{2} \times x + 3$$

Whence $x = 17{\cdot}81$ feet, the answer.

The same by Mr. John Harris, *Land Surveyor, and Teacher of the Mathematics, Caermarthen.*

Put $a =$ lesser girt, $= 2$, $b =$ greater girt, $= 9$, $c = 12 =$ AG (fig. 279, pl. 18) the distance, of the section required, from the lesser end, $e = 3$, $x =$ AH, the length of the piece of timber, and $\pi = 3{\cdot}1416$. Then $\frac{a + b}{2} =$ the girt in the middle of the whole piece, and the square of $\frac{1}{4}$th of it $= \frac{\overline{a + b}^{2}}{64}$, which multiplied by x (the length) gives $\frac{\overline{a + b}^{2}}{64} \times x$, for the content of the whole piece. . . . Now, $\frac{a}{\pi} =$ the diameter AC, and $\frac{b}{\pi} =$ the diameter BD, and by similar triangles x (AH) : $\frac{b - a}{2\pi}$ (BH) :: c (AG) : $\frac{b - a}{2\pi x} \times c = $ EG; therefore the diameter EF $(= 2\text{EG} + \text{AC}) = \frac{b - a}{\pi x} \times c + \frac{a}{\pi}$, consequently the girt at the section EF $= \frac{b - a}{x} \times c + a$, and the girt at the middle of the lesser piece AEFC $= \frac{b - a}{2x} + a$; the square of $\frac{1}{4}$th of this is $= \frac{\overline{b - a}^{2}}{64x^{2}} \times c^{2} + \frac{b - a}{16x} \times ac + \frac{a^{2}}{16}$ which multiplied by c (AG), gives $\frac{\overline{b - a}^{2}}{64x^{2}} \times c^{3} + \frac{b - a}{16x} \times ac^{2} + \frac{a^{2}c}{16} =$ the content of the frustum AEFC.



and their ſum $= -(b^2 - 2ba + a^2) \times \frac{c^2}{64x} = -\frac{\overline{b-a}^2}{64x} \times c^2$.

Again, $\frac{b^2-a^2}{32} \times c = (2b^2 - 2a^2) \times \frac{c}{64}$,

$\frac{a^2}{16} \times c = 4a^2 \times \frac{c}{64}$,

and $-\frac{\overline{a+b}^2}{64} \times c = -(a^2 + 2ab + b^2) \times \frac{c}{64}$;

and their ſum $= (a^2 - 2ab + b^2) \times \frac{c}{64} = \frac{\overline{b-a}^2}{64} \times c$.

Therefore $\frac{\overline{b-a}^2}{64} \times c - \frac{\overline{b-a}^2}{64x} \times c^2 = e$; hence $x = \frac{(\overline{b-a} \times c)^2}{\overline{b-a}^2 \times c - 64e} =$

$\frac{49 \times 144}{49 \times 12 - 3 \times 64} = \frac{196}{11} = 17^{\cdot} 8181$ feet, the anſwer.

The ſame anſwered by Mr. W. Marrat, *Boſton.*

Put $d = \frac{1}{2}$ the difference of the diameters of the piece of timber, $l =$ the leſs radius, and $x =$ LI, the length of the part remaining after 12 feet have been cut off. Then, by ſim. $\triangle$s, $d : x + 12 :: l : l(x + 12) \div d = Nn$, the length required to complete the cone, alſo, as $l(x + 12) \div d : 2 :: l(x + 12) \div d + 12 : (2l(x + 12) + 24d) \div l(x + 12)$, the circumference at EF; hence,

$\left(\frac{2l(x + 12) + 24d}{l(x + 12)} + 2\right) \div 2 =$ the mean girt of the part EC,

$\frac{1}{4}$th of which ſquared, and multiplied by the length $= 12$, we have

$\left(\frac{l(x+12) + 6d}{2l(x+12)}\right)^2 \times 12 =$ the cont. Again, $\left(\frac{2l(x+12)+24d}{l(x + 12)} + 9\right)$

$\div 2 = \frac{11l(x + 12) + 24d}{2l(x+12)} =$ the mean girt of the part BF;

conſequently

consequently its content is $= \left(\frac{(11 \div 4)l(x+12)+6d}{2l\,(x+12)}\right)^2 \times x$:

but the content of the whole piece is $(11 \div 8)^2 \times (x + 12)$.

Therefore $\left(\frac{l(x+12)+6d}{2l\,(x+12}\right)^2 \times 12 + \left(\frac{(11 \div 4)l(x+12)+6d}{2l\,(x+12)}\right)^2 \times$

$x = (11 \div 8)^2 \times (x + 12) + 3$. Hence $x = 17{\cdot}818$ nearly, answer.

Solutions to this question, agreeing in result with the above, were given by Messrs. Bosworth, Gregory, Johnston, Lowry, Peacock and Swale.

QUESTION 114, *answered by the Rev.* L. Evans, *the Proposer.*

The business in questions of this kind is, to find what each spirit is worth when reduced to proof, which is the standard of its worth. On this principle the answer is easily obtained. Thus,

$$\left.\begin{array}{l} 100 : 110 :: 18 : 19{\cdot}8\text{s.} \\ 100 : 118 :: 10{\cdot}5 : 12{\cdot}39\text{s.} \end{array}\right\} \text{per gallon proof.}$$

Difference 7·41s. B's cheaper than A's per gallon.

The same by Mr. Byerly, *Ripon.*

As $118 : 18 :: 1 : \frac{9}{59}$, and $1 - \frac{9}{59} = \frac{50}{59}$ the quantity of spirit in one gallon,

therefore $\frac{50}{59} : 10\frac{1}{2} :: 1 : 12{\cdot}39$s. the price per gallon of B's.

Again, $110 : 10 :: 1 : \frac{1}{11}$, and $1 - \frac{1}{11} = \frac{10}{11}$, the quantity of spirit in one gallon,

therefore $\frac{10}{11} : 18 :: 1 : 19{\cdot}8$s. the price per gallon of A's.

Hence $19{\cdot}8 - 12{\cdot}39 = 7{\cdot}41$s. B's cheaper than A's per gallon.

The ſame by Mr. William Marrat, *Teacher of the Mathematics, Boſton, Lincolnſhire.*

As 110 : 10 :: 4 : $\frac{4}{11}$ gallons of water, and $4 - \frac{4}{11} = \frac{40}{11}$ gallons of ſpirit; then reciprocally $4 : 18 :: \frac{40}{11} : 19{\cdot}8$s. the price per gallon of the brandy at 18s.

Again, As $118 : 18 :: 4 : \frac{36}{59}$ gallons of water, and $4 - \frac{36}{59} = \frac{200}{59}$ gallons of ſpirit, and reciprocally, $4 : 10\frac{1}{2} :: \frac{200}{59} : 12{\cdot}39$ the price per gallon of the brandy at 10s. 6d. Hence $19{\cdot}8 - 12{\cdot}39 = 7{\cdot}41$s. the anſwer required.

Anſwers to this queſtion were alſo given by Meſſrs. Boſworth, Gregory, Johnſton, Lowry, Swale, *and* Surtees.

VI. QUESTION 115, *anſwered by Mr.* J. Hartley, *Auditor's Office, Somerſet-Place, London.*

Fig. 281, pl. 18. Let D, E, F, be the three points the men ſtart from, whoſe diſtances are given, and likewiſe per queſtion DG : EG :: FG : EF, and the angle DGE = 2EGF. At the Lemma to Prob. 21, of *Simpſon's Algebra*, it is demonſtrated, "If a given right line DE be divided in any given ratio, at C, and the right line CF be taken to DC in the ratio of EC to DC — CE, and from F, as a centre at the diſtance FC, the circle CGO be deſcribed, and two right lines DG, EG be drawn from D and E, to meet any where in the periphery thereof; theſe lines will be to one another (every where) in the given ratio of DC to CE." Whence DG : EG :: DC : CE, from which two proportions it is eaſily proved that, DG = 2EG, and DC, CE having the ſame ratio to each other as DG to GE, it will appear, by Euclid VI. 3, that GC biſects the angle DGE, and that CGE = DGC = EGF; conſequently CG = FG, and GE perpendicular to DF; whence, by Euclid VI. 13, $\sqrt{CE \times EO} = GE = 173{\cdot}24$ yards, and DC = 346·48 yards; alſo by conſtruction GF = 200 yards.

The ſame anſwered by Mr. John Lowry, *Birmingham.*

Let D, E, F be the points the men ſtart from; then there is given

given DE = 300, EF = 100; alſo DG : EG :: FG : EF (not DF as printed), and the angle DGE = 2EGF. Make CE = EF, and with the diſtance FC and centres F and C deſcribe arcs to interſect in G, the point required. Join DG, CG and FG, join alſo GE, which will be perpendicular to DF. Now the △ FCG being equilateral, DC = CG = CF, and ∠CDG + ∠DGC = ∠FCG = ∠CGE + ∠EGF, hence it is evident that the △s DGF, EGF, are ſimilar; therefore, the angles DGC, CGE, EGF, are equal, and DGE = 2EGF, and alſo DG : GE :: GF : EF.

Calculation. $GE = \sqrt{GF^2 - EF^2} = \sqrt{200^2 - 100^2} = \sqrt{30000} = 173{\cdot}20508076$, $DG = \sqrt{DE^2 + EG^2} = \sqrt{300^2 + 30000} = \sqrt{120000} = 346{\cdot}41016151$, and FG = 200 yards, the diſtances required.

This Queſtion was alſo anſwered by Meſſrs. Boſworth, Gregory, *and* Sanderſon, *the Propoſer.*

VII. QUESTION 116, *anſwered by Mr.* Lowry.

Let CA, CB, fig. 282, pl. 18, be the aſymptotes to the given hyperbola VTX; deſcribe the oppoſite and conjugate hyperbolas QFK, mGn, and WEY. Suppoſe AB drawn as required, and draw the tangents BEH, AGI, IFH, alſo join FT, GE; the figure ABHI is a parallelogram, and GE = 2CE, is equal and parallel to AB (*Dr. Hutton on the Hyperbola, Pr.* 24. *Cor.* 4.), therefore CE is given. Hence this

Conſtruction. From the centre C, to the conjugate hyperbola WEY, apply CE = half the given line, and draw the tangent EB, meeting CB in B. From B, to AC, apply AB of the given length, and it will touch the curve at T.

N. B. The given line muſt not be leſs than the conjugate axe.

If PZ be a circle whoſe centre is C, and radius any line CP, leſs than the ſemitranſverſe axis of the hyperbola, a right line may eaſily be drawn to touch both the circle and hyperbola. For the triangle ACB is of a conſtant magnitude, being one fourth part of the parallelogram ABHI, that is, one fourth part of the rectangle under the axes, and CP is given, and perpendicular to AB, therefore, AB is given, and the conſtruction will be the ſame as above.

Other ingenious conſtructions nearly ſimilar to the above were given by Meſſrs. Swale *and* R. Wallace, *the propoſer.*

VIII. QUESTION 117, *anſwered by Mr.* Johnſton, *Birmingham.*

Analyſis. Let ACB (fig. 283, pl. 18.) be the △ to be conſtruct-

ed, CAD the given angle at the baſe, and CD the given biſecting line. Since the △ ACB is to be a maximum, the △ ACD, being the half thereof, muſt be a maximum; conſequently, by Theo. 6. Simpſon on the Max. et Min. AD is = AC. Hence the conſtruction is evident.

Similar to this was the anſwer by Mr. Gregory.

The Compoſition by Mr. R. Elliott, *Liverpool.*

On the given biſecting line, as a baſe, conſtruct the iſoſceles △ DCA, having the angle at A = the given one; make BD = AD, and join BC, ſo ſhall ACB be the △ required.

For CD biſects the baſe AB, the angle at A is = the given one, and the iſoſceles △ DCA is a maximum, therefore its double, or the △ ACB is alſo a max.

Mr Lewis Hill conſtructs the Problem generally, viz. When the △ is of a given magnitude, and the given line divides the baſe into ſegments having to each other a given ratio. His conſtruction is as follows:

On the given biſecting line CD (fig. 284, pl. 18.) let a ſegment of a circle be deſcribed, capable of containing the given angle. Make the rectangular parallelogram DEFC equal to twice the given area, and divide DE in the given ratio at Q; draw QB parallel to DC, and let it meet the circle in B. Join DB, BC, and continue BD ſo that AD may be to BD in the given ratio; join alſo AC, and ACB will be the triangle required.

Demon. Draw AP perpendicular to CD, and produce BQ to meet CF in G.

By ſim. △s, AP : QD :: AD : DB :: QE : QD, theref. AP = QE.

Whence, it is evident, that, the △ DCB is equal to half the parallelogram DCGQ, and the △ CDA equal to half the parallelogram QGFE, therefore the whole △ ACB is equal to half the parallelogram DEFC, that is, equal to the given magnitude.

The △ will evidently be a maximum when QB touches the circle at B, the △ CBD being then iſoſceles.

Conſtructions were alſo given by Meſſrs. Harris, Lowry, Peacock, Swale, *and* Thornoby.

IX. QUESTION 118, *anſwered by Mr.* Lowry.

Put a = 60 feet the length of the plane, b = 45 feet, the diſtance of the loweſt part of the plane from the ſurface of the water, i = fine 40°, to radius 1, and $g = 16\frac{1}{12}$ feet. Then, by Mecha-

nics,

nics, the force which accelerates the centre of gravity of the ball, while it *rolls* down the plane, is $\frac{5}{7}s$, and its velocity at the end of the plane is $= \sqrt{(20 \div 7) \times gsa}$, or the same as would be acquired by descending perpendicularly through the space $(5 \div 7) \times sa$, therefore the velocity with which the ball enters the fluid is $= 2\sqrt{g \times (5 \div 7 \times sa + b)} = 68{\cdot}26$ feet per second. Now put $N = 925$ the density of the ball (being oak, and not copper, as printed), $n = 1000$ that of the fluid, $d = 1$ the diameter of the globe, $m = 3n \div 8Nd = \frac{15}{37}$, $V = 68{\cdot}26$, the first velocity, $x =$ the space descended in any time t, and v the velocity at the end of that time.

Then, by Prob. 21, page 355, Vol. II. *Dr. Hutton's Course of Mathematics*, $mx =$ Hyp. Log. $\frac{V}{v}$, or $x = \frac{1}{m} \times$ Hyp. Log. $\frac{V}{v} = \frac{37}{15} \times 4{\cdot}22332 = 10{\cdot}417522$ feet the depth of the Reservoir.

N. B. The resistance which the ball meets with from the air in descending from the top of the plane to the surface of the water is not taken into consideration in the above solution; the effect being so very inconsiderable. The velocity, however, in that circumstance is easily determined from Prob. 22, page 350, Vol. II. of *Dr. Hutton's Course*; for, if s be put $= (5 \div 7)\, sa + b$, $c = 2{\cdot}71828182\&8$, $w = 2g \div m$, then the velocity is $= \sqrt{w - wc^{-2ms}}$.

Solutions similar to this, and agreeing with it in result, were received from Messrs. Bosworth, Byerly, Gregory, *and* Marrat.

The same by Mr. John Surtees, *Sunderland.*

Let $s =$ sine of the given angle to radius 1, $a = 60$ feet $=$ length of the inclined plane, $h = 45$ feet, and $g = 16\frac{1}{12}$: Then $2\sqrt{gsa} =$ the velocity of the ball at quitting the plane, that is, the velocity acquired by *falling* down the plane; and by the nature of projectiles, $2\sqrt{g \times (sa + h)} =$ its velocity just going into the water. Hence, by *Emerson's Fluxions*, page 384, the depth of the Reservoir,

Reſervoir, $z = \frac{44g \times s^2 \cdot 299565}{8\beta} \times \text{Log.} \frac{(s-\beta) \times 16\frac{1}{12}g - 2\beta^2}{(s-\beta) \times 16\frac{1}{12}g - 2\beta^2}$

where d = diameter of the globe of Oak = 1 foot, s = 925 its denſity or ſpecific gravity, β = 1000 the denſity of the fluid, g = $16\frac{1}{12}$ feet the ſpace through which a heavy body deſcends by gravity in one ſecond, t = time of deſcribing x, v = the velocity at the end of the time t, and b = the velocity with which the ball enters the fluid = $2\sqrt{g \times (ax + b)}$. Therefore t may readily be determined.

X. QUESTION 119, *anſwered by Mr.* Thomſhy.

Fig. 285, pl. 18. Let ABC be the required $\triangle$; put AB = a, the given ſide, d = the different of the ſegments of the baſe, and x = AD the greater ſegment; then $x - d$ = the leſs ſegment, and $BD^2 = a^2 - x^2$: but $AB^2 - BC^2 = AD^2 - DC^2 = 2xd - d^2$, therefore $BC^2 = a^2 - 2xd + d^2$; and by the queſtion AB × DC is to BC × BD in a given ration; ſuppoſe that of m to n. Hence $a^2 (x - d)^2 : (a^2 - 2xd + d^2) \cdot (a^2 - x^2) :: m^2 : n^2$, and by multiplying means and extremes, we have $2dm^2 x^3 - ((a^2 + d^2) m^2 - n^2 a^2) \times x^2 - (2da^2 (m^2 + n^2)) x = n^2 a^2 d^2 - m^2 a^4 - m^2 a^2 d^2$, a cubic equation, which ſhews that the Problem cannot be conſtructed geometrically, that is, by right lines and circles; but recourſe muſt be had to the conic ſections.

Meſſrs. Gregory, Lowry, *and* Johnſton *ſent obſervations of the ſame nature as the above.*

XI. QUESTION 120, *anſwered by Mr.* Lowry, *Birmingham.*

Let ADB (fig. 286, pl. 18.) repreſent the glaſs in its inclined poſition, PB the ſurface of the liquor; then there is given DE = 4, and $\angle BPC = 63°$ (not 96° as printed in the queſtion), alſo, by the queſtion, the ſolidity of the ſegment DPB muſt be equal to half the paraboloid ADB. Draw PC perpendicular to AB; then (by *Cor.* 4. *pa.* 379, *Dr. Hutton's Menſuration, new Edition*) ($BC^3 \div AB^2$) $\frac{1}{2}$ DE × ·785398, is equal to the content of the ſegment DPB, and $AB^2 \times \frac{1}{2}DE \times$ ·785398, is equal to the content of the paraboloid ABD; therefore $2BC^3 \div AB^2 = AB^2$; hence $AB = \sqrt[4]{2} \times BC$, and $AC = (\sqrt[4]{2} - 1) \times BC$. Put s, and c, for the ſine and coſine of 63°, then will $PC = \frac{c}{s} BC$, and, by *Prop.* II. *Cor. Dr. Hutton*

on

on the parabola, DE : PC :: AE^2 :: BC × AC, or DE : $\frac{c}{s}$ BC :: $\frac{1}{4}\sqrt{2} \times BC^2 : (\sqrt[4]{2} - 1) \times BC^2 :: \sqrt{\frac{2}{16}} : \sqrt[4]{2} - 1$; hence, BC = DE × $\frac{s}{c}$ × $(\sqrt[4]{2} - 1)$ × $\sqrt{8} = 4.293249$, AB = 5.117, and the content of the glaſs 41.126 ſolid inches.

W. W. R.

Ingenious ſolutions, to this queſtion, were alſo received from Meſſrs. Gregory, Johnſton, Marrat, *and* Thornoby.

XII. QUESTION 121, *anſwered by Mr.* Lowry.

Put $a = 4$lb. the weight of the ball, $f = 1000$lb. the elaſtic force of the ſpring, $b = 3$ inches $= \frac{1}{4}$ of a foot, the diſtance the ſpring is bent, when the velocity of the ſtriking body is deſtroyed, $g = 16\frac{1}{12}$ feet, $x =$ the ſpace the ſpring is bent at any time t, $v =$ the velocity at that time, and $z =$ the velocity of the ball when it firſt ſtrikes the ſpring. Now, by experiments made upon ſprings, it appears that the action of the ſpring, is as the compreſſing force, or in a conſtant ratio to the diſtance x; therefore $b : f :: x : fx \div b$, = the force of the ſpring when it is bent through the diſtance x: but, by the doctrine of motion, $-\dot{v} = 2gfx\dot{x} \div bav$, or $-v\dot{v} = 2gfx\dot{x} \div ba$, the fluent of which corrected by making $v = z$, is $z^2 - v^2 = 2gfx^2 \div ba$. Now, when $x = b$, v is $= 0$, therefore $z = \sqrt{2gfb \div a} = 44.83767$ the velocity of the ball when it firſt ſtrikes the ſpring, and the diſtance the ball muſt deſcend to acquire this velocity is $z^2 \div 4g = 31\frac{1}{4}$ feet, the height required.

Again, the time of deſcending the diſtance $z^2 \div 4g$ is $z \div 2g = 1.393917$ ſeconds; and to find the time of the ſprings bending 3 inches we have $\dot{t} = \dot{x} \div v$: but it is evident, from above, that $v = \sqrt{z^2 - (2gfx^2 \div ab}$, and $f = aa^2 \div 2gb$, therefore, $v = \sqrt{(z^2b^2 - z^2x^2) \div b^2}$; and $\dot{t}\,\dot{x} \div v = b\,\dot{x} \div z\sqrt{b^2 - x^2}$, and the correct fluent, when $x = b$, is $(b \div z) \times$ guadrantal arc of a circle, whoſe radius is unity, $= .008758$; hence, the whole time of deſcending is 1.402675 ſeconds.

The

The ſame anſwered by Mr. Surtees, *Sunderland.*

Let $w=4$lb. $c=1000$lb. $a=16\frac{1}{12}$ feet, $b=\frac{1}{4}$ (=3 inches), and $d=$ the height required. Then, by Prob. 21, page 405, Emerſon's Fluxions, $4ad=2a\,(cb\div w$; hence $d=b\times(c-2w)\div 2w=$ 31 feet = the height required. and $\sqrt{b\,(c-2w)\div 2aw}=1'39$ ſecond. = the time of deſcent.

According to one or other of theſe methods were the ſolutions given by Meſſrs. Byerly, Gregory, Johnſton, Marrat, Peacock, *and* Thornoby.

XIII. QUESTION 122, *anſwered by Mr.* Lowry.

Fig. 287, pl. 18. Let N be the North Pole; B, Berwick; P, Penzance; D, Dover; and B, Birmingham. Draw the great circles PN, PB′, B′N, B′B, BD, BN, and DN. Now in the ſpherical Δ B′NB, there are given, the $\angle$ B′NB = the diff. of longitude between Berwick and Birmingham = 0° 5′; NB = the colat. of Berwick = 34° 12′; and NB′ = the colat. of Birmingham = 37° 30; to find BB′. In the ſpherical Δ PNB′, there are given, the $\angle$ PNB′ = the diff. of longitudes between Penzance and Birmingham = 4° 10,; B′N = 37° 30′: and PN = the colat. of Penzance = 39° 52′; to find PB′. And in the ſpherical Δ B′ND, there are given the $\angle$ B′ND = the diff. of longitude between Birmingham and Dover = 3° 3′; B′N = 37° 30′; and DN = the colat. of Dover = 38° 53; to find BD.

Let O be the centre of the earth, draw OB′, OP, OB, and OD, and produce OB′ to *a*, ſo that B′*a* may be = 300 feet, the height of the telegraph at Birmingham. Alſo draw the tangents *aed*, *agc*, *afb*, to touch the circles at *e*, *g*, *f*, and to meet the radius of the earth, produced, at *d*, *c*, *b*; then are P*d*, B*c*, D*b*, the heights of the telegraphs at Penzance, Berwick, and Dover reſpectively. Join O*e*, which will be perpendicular to *ad*; then in the right angled plane Δ, there are given O*e* = the earth's radius, and O*a* = O*e* + 300, to find the $\angle$ *e*O*a*; now the arch B′P, or the meaſure of the $\angle$ *d*O*a*, being given, the $\angle$ *d*O*e* is eaſily found. Hence *d*O, and conſequently P*d*, the height of the telegraph at Penzance is readily had. And in the very ſame way are the heights of the telegraphs at Berwick and Dover found.

A method of ſolution ſimilar to the above was pointed out by ſeveral of our Correſpondents: but very few of them have attempted to work out their ſolutions; and of thoſe who have, no two agree in their concluſions. Not being able to rely upon any received, we have merely given the method pointed out by one of our correſpondents as above.

XIV.

XIV. QUESTION 123, *anſwered by Mr.* Thornoby.

Let AC (fig. 288, pl. 18), repreſent the candle, GFE, NMS, ſections of the given globes, and let lines be drawn as in the figure. Suppoſe A the point required, then it is evident that the ſhadows of the globes, upon the table, will be ellipſes, whoſe tranſverſe axes are BD, and LR and conjugates the ſhadows of chords drawn perpendicular to GF, MN, through the points H, V.

Since it will be extremely tedious to give a direct ſolution to this queſtion, we had better have recourſe to the method of approximation by trial and error. Thus, aſſume AE of any convenient length, then ES being given = 10 feet, AC = 13, OE = 6, and IS = 4 inches, AI becomes known, and ſo do PC, CO, CK, and AK, by ſim. △s and Eu. I. 47, alſo, by trigonometry, the angles AKC, BKC, OCG, OCF are known; hence, in the △s DCK, KCB, CK and all the angles are given to find DK and KB; then DK + KB, the tranſverſe axis, will be given. Again, by ſim. △s, GH, and CH, are eaſily found; hence, as CH : 2GH :: CK : to the conjugate axe. Therefore, the area of the ellipſis becomes known. And in the very ſame way may the area of the other ellipſis be found. Now, if the two areas happen to be equal, the queſtion is ſolved; if not, aſſume ſome other number for AE, and proceed as before, till you find the area of the ellipſis; note the reſults, and errors, and find the value of AE by the common method of trial and error.

A ſimilar method was pointed out by other Correſpondents.

XV. QUESTION 124, *anſwered by Mr.* Marrat, *Boſton.*

In fig. 289, pl. 18, EFI being a ſection of the equilateral cone, there are given HB = $1\frac{1}{2}$, CH = 4, and the content of the fruſtum EKLF is to be a maximum. Now this is equal to the difference between the cones EIF, and KIL, and, by the property of the equilateral cone, their ſolidities have a conſtant ratio, to the cubes of their ſides, or to the cubes of their altitudes; therefore it is obvious, that, when the fruſtum EKLF is a maximum, or when the quantity of water diſplaced is the greateſt poſſible, the difference of the cubes of IH, and IG, will be a max. This premiſed, put x = IG, then GF is = $x \div \sqrt{3}$, and by the property of the parabola, as $HG^2 = 2\frac{1}{4} : HC = 4 :: GF^2 = \frac{1}{3}x^2 : (16 \div 27)x^2 = CG$, hence $HI = x + (16 \div 27)x^2 - 4$; therefore $x^3 - (x + (16 \div 27)x^2 - 4)^3$ muſt be a maximum.

Which *put into fluxions and reduced* gives $x^5 + \frac{405}{96}x^4 - \frac{53946}{6912}x^3$

[illegible] Hence $x = \frac{5}{4}\sqrt{3}$ [illegible] ounces, averdupois.

[illegible] Mr. Harris, *Caermarthen*.

[illegible] and put $CH = \frac{1}{4}$ [illegible] $= a$, and $GF = \frac{1}{2}$, the side of [illegible] $= 1$. Then [illegible] of the equilateral Δ, [illegible] $CG = x^2 + f$; [illegible] $= x - \frac{1}{2}$, and $HI = x\sqrt{3 + x^2 + p - a}$. [illegible] $= x\sqrt{3 + x^2 + p} - a$ (IH) [illegible] $= x\sqrt{3 - x - a + x\sqrt{3}}$. Now, by mensuration, [illegible] $KLEF = EF^2 + KL^2 + EF \times KL$)

[illegible] $= \frac{1}{2} \times x\sqrt{3 - x^2 - x\sqrt{3}} + x\sqrt{3 + x^2 + 4x^2 +}$ [illegible] $x\sqrt{3 - x^2 - x\sqrt{3}} \times \frac{1}{2}x\sqrt{3 + x^2} \times (a - x^2 + f) \times$ [illegible] which reduced and reduced, gives $x^5 + \frac{5}{2}$ [illegible] $= \frac{5}{2}ax^4 - \frac{5}{2}x^3 - \frac{5}{2}px^2 + \frac{5}{2}x^2 + (a^2p^2 - 3ap^2)x +$ [illegible] $= 0$. Hence $x = 1.28$, &c.

[illegible] Mr's, **Boiworth, Gregory,** [illegible] Brown and Peacock.

XVII QUESTION [illegible] Mr. Gregory, *Cambridge*.

"When the globe sinks in water so as to conceal $\frac{2}{5}$ of its surface, $\frac{2}{5}$ of [illegible] will be the surfaces of spheric segments [illegible] $\frac{2}{5}$ of 15, or 6 inches or the globe's [illegible] in the water. Hence it is easy to find the solidity of [illegible] $=$ [illegible] and that of the segment immersed $=$ [illegible]. Now, it is evident from the principles of Hydrostatics, that the weight of the whole globe is equal to the weight of [illegible] the segment immersed: that is, as 1728: [illegible] ounces, weight of the globe. The [illegible] of the upper segment of the globe is $1767.15 - 622.0268 = 1145.1232$ [illegible]. The weight of its bulk of water is 662.68125 ounces, which, it is obvious, must be the weight of the ball of copper in water, necessary completely to immerse the given globe. Then by the nature of specific gravity, we have, as 8 (difference between the specific gravities of copper and water) : 9 (specific gravity of copper) :: $662.68125 : 745.5164$ ounces, weight of the globe of copper in air. Therefore, as $9000 : 1728 :: 745.5164 : 143.131916$ inches, content of the copper globe: whence $\sqrt[3]{143.131916 \div .5236} = 6.490134$ inches, its diameter, as required.

The

The ſame by Mr. William Peacock, *Birmingham.*

Becauſe the ſurfaces of ſegments of a ſphere are to each other, or to the ſurface of the whole ſphere, as their heights, $\frac{2}{5}$ths, or 6 inches, of the globes axis will be immerſed in the fluid, and by hydroſtatics, the weight of the globe is equal to the weight of water diſplaced by it, that is, equal to 22·4481b. Again, the weight of the copper in water, muſt evidently be equal to the weight of water diſplaced by the upper ſegment of the globe, that is, equal to 41·4175781b. Then, by the law of ſpecific gravities of bodies, as the difference between the ſpecific gravities of copper and water, is to the ſpecific gravities of copper, ſo is the weight of the copper in water, to its weight in air = 745·5164 ounces. Hence the diameter of the copper ball will be 6·49 inches.

Ingenious anſwers to this queſtion were alſo received from M.ſſrs. Boſworth, Byerly, Francis, *(the Propoſer)*, Johnſton, *and* Thornoby.

XVII. QUESTION 126, *anſwered by Mr.* Harris, *Caermarthen.*

Analyſis. Suppoſe ACB (fig. 290, pl. 18.) the $\triangle$ to be conſtructed, O the centre of the inſcribed circle GHI. Draw EF perpendicular to AB, biſecting it in L, and meeting a circle deſcribed through the points A, O, B, in E and F; draw alſo EP perpendicular to AO, and ON to EF, and join EO.

Since, OH (=OG = OI), and the $\angle$OCH, ($=\frac{1}{2}\angle$C,) are given, CO is given: but CO + AO + BO, is given; therefore AO + BO, and conſequently OP ($=\frac{1}{2}\times$ AO + BO), is given.

Again, the $\angle$AOB (= right $\angle + \frac{1}{2}\angle$C) is given, therefore the $\angle$POE ($=\frac{1}{2}\angle$AOB) is given, and therefore EP is given.

Hence the rect. EL × EN = (by Cor. 4. Pr. VI. Art. LII. Vol. I.) EP^2, and LN = OG is a given line; therefore EN is given, hence the diameter EF is given, and the conſtruction is evident.

The ſame anſwered by Mr. Lowry, *Birmingham.*

Since the radius of the inſcribed circle, and the vertical angle are given, the diſtance between the centre of that circle and the vertex will be given; therefore, the ſum of the diſtances from the centre of the inſcribed to the angular points at the baſe of the triangle is given. Hence this

Conſtruction.

Conſtruction. Fig. 291, Pl. 18. Make the $\angle AOB =$ half the vertical $\angle$ *plus* a right $\angle$, and take $OQ =$ half the ſum of the diſtances from the angular points at the baſe to the centre of the inſcribed circle. Drop the perpendicular QD and let it meet OD, drawn to biſect the $\angle AOB$ in D, and on OD demit the perpend. QP, and, from D, apply DF, to QP, ſo that $DF \times$ (DF + given radius) may be $=$ to the ſquare of QD. Through F, draw AFB perpend. to DF, and let it meet AO, OB in A and B, and draw AC ſo make the $\angle OAC = OAB$, and BC to make the $\angle OBC = OBA$; ſo ſhall ACB be the triangle required.

Demon. Continue DF to meet OE, drawn parallel to AB, in E, and AO to meet BC in I. Upon AB demit the perpend. OR. Becauſe the $\angle$s BAC, ABC, are biſected by the lines AO, BO, O is the centre, and OR the radius of a circle inſcribed in the Δ ABC. But, by the Corollary to Theo. 19, B. IV. Simpſon's Geometry, and ſimilar $\triangle$s $QD^2 = OD \cdot DP = FD \cdot (DF + FP) = FD \cdot (DF +$ given radius) by the conſtruction, therefore $OR = EF =$ the given radius.

Again $AO + OB - AQ = OQ =$ the given ſum of the diſtances from the angular points at the baſe to the centre of the inſcribed circle.

And the angle $AOB = OIB + OBI = ACB + IAC + OBI$.
But $\frac{1}{2}$ ACB + IAC + OBI = a right angle.
Therefore the $\angle ACB$ is $=$ the given one. *Q. E. D.*

The ſame by Mr. William Wallace, *Perth.*

Analyſis. Suppoſe ABC (fig. 292, pl. 18.) the triangle required, and that ABC is the given vertical angle. Let D be the centre of the inſcribed circle; join DA, DB, DC, and draw DE, DF, perpendicular to AB, BC, the ſides about the given angle. Becauſe, by hypotheſis, DE, or DF, the radius of the inſcribed circle is given, as alſo the angle ABC, it is evident that DB, which biſects the angle ABC, is given: now BD + AD + DC is given, by hypotheſis therefore AD + DC is given. Draw HK perpendicular to DB, meeting AB in L, and CD, produced in K. The angle KLA, or BLH is evidently half the ſum of the angles BAC, BCA, and the angle KDA is equal to the ſum of the angles DAC, DCA, that is, to half the ſum of the ſame angles BAC, BCA; therefore the angles KLA, KDA, are equal, and the points K, L, D, A are in the circumference of a circle, hence the angle DKH is equal to the angle DAE; now the angles at H and E are right angles, and DH $=$ DE, therefore DK $=$ DA, and CK is equal

to

to AD + DC, that is, to a given line: this problem is, therefore, reduced to the following well known problem, which has been enunciated, and ſolved, in various ways by moſt writers on geometry. Two ſtraight lines HL, BF, being given by poſition; it is required to apply between them a ſtraight line KC of a given length, which may paſs through D, a given point equally diſtant from theſe lines.

Conſtruction. With the given radius of the inſcribed circle, let a circle be deſcribed whoſe centre is D; draw the radii DE, DF, ſo that the angle EDF may be equal to the ſupplement of the given vertical angle: draw BA, BC touching the circle at the points E, F; thus the angle ABC will evidently be equal to the given vertical angle. Let *np* be the given ſum of the diſtances of the angles of the triangle from the centre of the inſcribed circle: join BD meeting the circle in H; draw HL perpendicular to BD; take *pq* = BD, and between the lines BF, HL apply KC equal to *nq*, ſo as to paſs through D, the centre of the circle.* Draw CA touching the circle at G and meeting BE at A, and ABC ſhall be the triangle required, as is evident from the Analyſis.

Remark. The preceding Analyſis contains the demonſtration of the following Theorem, which may, in certain caſes, be applied with advantage to the conſtruction of problems relating to triangles.

THEOREM.

If from D, the centre of a circle inſcribed in a triangle, ſtraight lines DA, DB, DC be drawn to the angles, and LO be drawn, touching the circle at H, the point where BD, one of theſe lines cuts the circle, and if AD, CD, the other two lines, be produced to meet the tangent in K and M: the lines CK, AM ſhall be equal to one another, and to the ſum of AD, DC.

Corollary. AL is equal to KO, and CO is equal to LM.

Meſſrs. Gregory, Johnſton, Peacock, *and* Thornoby *ſent ingenious conſtructions to this queſtion.*

XVIII. QUESTION 127, *anſwered by* Nobody.

XIX. QUESTION 128, *anſwered by* Cantabrigus.

In order to ſolve this curious queſtion, let us firſt endeavour to

 aſcertain

* *The manner of doing this is ſhewn in Prob.* 21, *Simſon's Geometry,* 3*rd Edit. alſo in Prop.* 32, *Lib. I. of D'Omerique's Geom. Anal. and elſe where.*

aſcertain how high a building may be raiſed *at the Equator* be
the ſtones are carried off its top, by reaſon of the centrifugal f
overcoming the centripetal. And here it ſeems neceſſary to
the velocity of a body revolving in a circle at any given diſt
from the Earth's centre, by means of its own gravity. Writers
central forces determine this in the following manner: Put $g =$
$\frac{1}{12}$ feet, the ſpace through which a body deſcends by the for
gravity in one ſecond at the ſurface of the earth, and $r =$ the r
of the earth; then the velocity with which a body muſt be proje
at the earth's ſurface to revolve round it, like a ſatellite, may
found by this theorem $\sqrt{2gr} =$ (in this caſe, ſuppoſing 25
miles the earth's circumference) 26000 feet per ſecond: [See
Hutton's Mathematical and Philoſophical Dictionary, Article CE
TRAL *Forces*; and *Thorp's Commentary on Newton's Princi*
Prop. 4. *Lib.* 1. *Pa.* 85.]. Then, ſince the force of gravity va
in the inverſe duplicate ratio of the diſtance, we have as $\sqrt{R} : \sqrt{}$

$r : 26000 \sqrt{\frac{r}{R}}$; the velocity of a body revolving round the ea

at any diſtance R. And $5078 \sqrt{\frac{R^3}{r^3}} = T$, the time of re

lution in the ſame: where $5078''$ is the periodic time of a b
revolving at the ſurface. From the nature of our preſent enquir
may be inferred, that when $23^h\ 56'$ (the periodic time of
earth's rotation on its axis) is ſubſtituted for T, in the laſt theor
if R denote the diſtance of the top of the building from the cen
of the earth, that theorem will enable us to diſcover the heigh
the building ſo that its top ſhall be carried round in the ſame ti
by the earth's diurnal rotation, as it would by the combinatio
the central forces. Thus, if $5078 \sqrt{\frac{R^3}{r^3}} = 86160$, then R

$\sqrt[3]{r^3 \times \frac{86160}{5078}^2} = 6{\cdot}6030\,1r$. Hence $R - r = 5{\cdot}60301$ ti
the earth's radius, the height of the building at the equator, ſo t
the ſtones will juſt remain on its top: for if they were laid
higher they would fall off, and be projected round the earth in
ſame manner as the moon.

SCHOLIUM.

Conſidering the earth as a ſphere, this problem may be reac
made general. Thus, in Fig. 293. Pl. 18, let P and p denote

poles of the earth, Pp the axis, EQ part of the equator, and EB = R in the foregoing theorems. It is required to find Ab the greateſt height to which a building can be raiſed in any latitude AQ. It is evident Cb is the radius of the circle deſcribed in the earth's revolution by b the top of the building; which muſt in all caſes be equal to EB = R. Hence, as ſine of CEb (= coſine of latitude) to (= R) :: radius (= ſine of bCE) : Eb, from which deduct the radius of the earth, and the remainder will be Ab = the height of the building.

Corollary. As the latitude AQ increaſes the height Ab will increaſe; becauſe the coſine of the latitude which is the firſt term in the proportion, decreaſes: and at the pole P, the height cannot be determined; becauſe the firſt term in the proportion will have vaniſhed.

The ſame anſwered by Mr. Surtees, *Sunderland.*

Let r = radius of the earth in feet = 3980 × 1760 × 3, g = $16\frac{1}{12}$, c = 3·1416, s = coſine of the latitude to radius 1, and T = 23h 56m = 86160 ſeconds = the time of one revolution, alſo R = height from the centre to the top of the building. Then, by Dr. *Hutton's Mathematical and Philoſophical Dictionary, Vol.* I. Pa. 261, when the centrifugal force is an exact balance for gravity $c\sqrt{\frac{2r}{g}} \times \sqrt{R^3 \div r^3} = T$, at the equator, and for any particular latitude $c\sqrt{\frac{2rR^3}{gr^3}} = T$, hence $R = \sqrt[3]{\frac{T^2r^2g}{2c^2}}$, and $R - r =$ the height required.

For London, s = ·6223 nearly, and $R - r = 141503016$ feet, =, nearly, 26800 miles = the height to which a building might be raiſed at London before the ſtones would leave the top by reaſon of the centrifugal force being an over balance for gravity.

XX. QUESTION 129, *anſwered by Mr.* William Wallace, *of Perth Academy.*

Figures 294, 295, 296, 297, 298, Plate XVIII. Let A, B, C be the three given points, and DEF the given circle, it is required to inſcribe in it a triangle, the ſides of which may paſs through the given points.

CASE I. Fig, 294. Firſt ſuppoſe the points to lie in a ſtraight line*.

Analyſis. Suppoſe that the triangle DEF is inſcribed in the circle, and that its ſides paſs through the given points A, B, C, as is required. Draw EG parallel to AB, meeting the circle again in G; and join FG, meeting AB in H: the angle AHF is equal to the angle EGF, or to ADF, therefore, the points A, D, H, F are in the circumference of a circle, and the rectangle AB·BH is equal to the rectangle DB·BF, now the point B, and the rectangle DB·BF are both given, therefore the rectangle AB·BH is given, and ſince the point A is given, the point H is alſo given. Draw EK touching the circle at E, and meeting AB in K: the angle KEF is equal to EGF, or to KHF, therefore, the points K, E, H, F are in the circumference of a circle, and the rectangle KC·CH is equal to EC·CF, now the point C, and the rectangle EC·CF are given, the point H has alſo been ſhewn to be given, therefore, the point K is given, and ſince EK is a tangent to the given circle, the point E is given; hence the following conſtruction:

Conſtruction. That the problem may be poſſible it is evident that either one of the given points, or all the three muſt lie without the given circle. Draw a ſtraight line through B, one of the given points, ſo as to meet the circle in L and M: in AB take the point H, ſo that the rectangle AB·BH may be equal to LB·BM, and ſo that the points A, H, may be on the ſame ſide, or on contrary ſides, of the point B, according as the points L, M are on the ſame ſide, or on contrary ſides of the point B. Let a ſtraight line paſſing through C meet the circle in L and M, and take K in AB, ſo that the rectangle HC·CK may be equal to LC·CM, and ſo that the points H, K may be on the ſame ſide, or on contrary ſides of C, according as the points L, M are on the ſame ſide, or on contrary ſides of C. Draw KE touching the circle at E†, join CE meeting the circle again at F, join BF meeting the circle again at D, and join AD; the ſtraight line AD ſhall paſs through E, and EDF ſhall be the triangle required.

Demonſtration, From the points E, F inflect EG, FG to the circumference of the circle. Becauſe the rectangle KC·CH is equal

* *This caſe is indeed excepted in the queſtion, but as it admits of a conſtruction conſiderably more ſimple than the moſt general caſe of the problem, it has been preſumed that a particular conſideration of the excepted caſe would be an improvement to the anſwer.*

† *This will always be poſſible, for it may be ſhewn that the point K is without the circle; whatever be the ſituation of the points, provided that either one only, or all the three be without the circle.*

equal to LC·CM, that is, to EC·CF, the points K, E, H, F are in the circumference of a circle, therefore, the angle AHF, or KHF is equal to KEF, that is, to EGF: again, becauſe the rectangle AB·BH is equal to LB·BM, or to DB·BF, the points A, D, H, F are in the circumference of a circle, therefore, the angle ADF is equal to AHF, that is, as has been ſhewn, to EGF: now the angles at D and G, being equal, muſt ſtand upon equal circumferences, therefore, AD will paſs through the point E, as was to be demonſtrated.

CASE II. Fig. 295. Next let A and B, two of the given points, be in LM the diameter of the given circle, and let C the the remaining point be without the diameter.

Analyſis. Suppoſe that DEF is the triangle required. Draw EG parallel to the diameter, meeting the circle again in G, and join FG, meeting the diameter in H: the angle AHF is equal to EGF, or to ADF, therefore, the points A, D, H, F are in the circumference of a circle, and the rectangle AB·BH is equal to FB·BD, that is, to a given ſpace, now the points A, B are given, therefore, the point H is alſo given. Draw FON through O the centre of the circle, and join NE meeting AB in K: the angle KHF is equal to EGF, or to KNF, therefore, the points K, N, H, F, are in the circumference of a circle, therefore, the rectangle KO·OH is equal to NO·OF, or to the ſquare of the radius, which is given; now the points O, H are given, therefore the point K is given; and becauſe NF is the diameter, the angle KEC is a right angle, and is, therefore, given: Join KC, and the point E will be in the circumference of a given circle, *viz.* that which has KC for its diameter, but it is alſo in the circumference of the given circle DEF, therefore, the point E is given.

Conſtruction. Take the point H in AB, ſo that the rectangle AB·BH may be equal to LB·BM, and ſo that the points A, H may be on the ſame ſide, or on contrary ſides of B, according as the points L, M lie on the ſame ſide, or on contrary ſides of B: Alſo take the point K ſo that the rectangle KO·OH may be equal to the ſquare of LO, the radius of the given circle, and ſo that the points K, H may be on different ſides of the centre O; join KC, and upon KC as a diameter deſcribe a circle, which, when the problem is poſſible, will cut the given circle in E; join CE meeting the circle again in F, and join BF meeting the circle again in D, join AD; the ſtraight line AD ſhall paſs through E, and DEF ſhall be the triangle required.

Demonſtration. Join KE which produce to N, and join FN; becauſe the angle KEF is a right angle, the line FN is the diameter, and ſince by conſtruction the rectangle KO·OH is equal to the ſquare of LO, or to NO·OF, the points K, N, H, F are in the

circumference of a circle, and the angle KNF, or ENF is equal to KHF, and becauſe the rectangle AB·BH is equal to LB·BM, or to DB·BF, the points A, D, H, F are in the circumference of a circle, and the angle ADF is equal to AHF, which has been ſhewn to be equal to ENF: becauſe the angles ADF, ENF are equal, they ſtand upon equal circumferences, therefore, AD muſt paſs through E, and the ſides of the triangle EDF which is inſcribed in the circle paſs through the given points as was required.

CASE III. Fig. 296. Let A and B, two of the given points, be in LM, the diameter of the circle, and ſo ſituated, that LA: AM = LB: BM.

In this caſe it is well known that if AD, BD be inflected to any point in the circumference of the circle, and produced to meet it again in E and F; the line which joins the points E and F ſhall be perpendicular to LM the diameter of the circle. Therefore, if through the point C (which muſt be within the circle) a perpendicular be drawn to the diameter, meeting the circumference in E and F, and FB be joined, meeting the circle in D, the ſtraight line which joins the points A, D ſhall paſs through E, and EDF ſhall be the triangle required.

Before proceeding to Caſe IV. it will be convenient to premiſe the following Lemma:

LEMMA. Figure 299, Plate XVIII.

Let HTR be a ſtraight line drawn from any point H in LM the diameter of a circle, meeting the circle in T and R, draw RV perpendicular to the diameter, meeting the circle in V, join HV meeting the circle in Q, and join RQ, meeting the diameter in K; MH ſhall be to HL as MK to KL. Alſo, let two points H, K be taken in LM, the diameter of a circle, ſo, that MH: HL = MK: KL: draw any ſtraight line through K, meeting the circle in R and Q, join HR and HQ meeting the circle in T and V: the arch LR ſhall be equal to LV, and LT ſhall be equal to LQ.

As the truth of both parts of this lemma is pretty generally known, we ſhall not add a demonſtration, but proceed to the fourth Caſe.

CASE IV. Fig. 297. Next let the three given points be any how ſituated.

Analyſis. Suppoſe that DEF is the triangle required, the ſides of which are to paſs through the given points A, B, C. Join AB, and draw EG parallel to AB, meeting the circle in G; join FG, meeting AB in H: it will appear as in the firſt two caſes, that the rectangle AB·BH is equal to DB·BF, that is, to a given ſpace, therefore,

therefore the point H is given. Let O be the centre of the given circle, join HO meeting the circle in L and M, the line HO is given by poſition, and the lines HL, HM are given. Draw FN perpendicular to LM, meeting the circle in N, and join NH, meeting the circle in P, join FP meeting OH in K; by the lemma, MH : HL = MK : KL, therefore, the point K is given. Join BK, meeting the circle in R and Q; the line BK, and the points Q, R are evidently given by poſition; join AR meeting the circle in S, and the point S will alſo be given. Join HQ meeting the circle in V and HR meeting the circle in T, and join SV. It has been ſhewn that the rectangle AB · BH is equal to DB·BF, that is, to RB · BQ; therefore, the points A, R, Q, H are in the circumference of a circle; hence the angle BHQ is equal to QRA, that is, to QVS, therefore, SV is parallel to AB, or EG, and the arch VG is equal to the arch SE. Now (by the Lemma) the arch LP is equal to LG, and LR to LV, therefore, the arch PR is equal to the arch GV; but GV was ſhewn to be equal to ES, therefore, PR is equal to ES, and the arch PE is equal to the arch RS. Now the points R, S, and, therefore, the arch RS were ſhewn to be given, therefore, the arch PE, and the angle PFE, or KFC are given in magnitude; and ſince the points C, K, are given, the point F is in the circumference of a given circle, which will paſs through the points C, K; but it is alſo in the circumference of the given circle DEF, therefore, the point F is given, and the line EF is given by poſition, therefore, alſo the ſtraight lines AED, BFD, are given by poſition.

Conſtruction. Let A and B be any two of the given points: join AB and draw any ſtraight line BR; meeting the circle in Q and R; take H in AB, ſo that the rectangle AB · BH may be equal to RB·BQ, and ſo that the points A, H may be on the ſame ſide, or on contrary ſides of B, according as the points Q, R are on the ſame ſide or contrary ſides of B. Let O be the centre of the given circle, join HO, meeting the circle in L and M, and take K in LM, ſo that MH : HL = MK : KL: join BK, meeting the circle in Q and R, and join AR, meeting the circle in S: join CK, and upon CK deſcribe a ſegment of a circle that may contain an angle equal to any angle, at the circumference of the given circle, which ſtands upon the arch RS, let this circle meet the given circle in F, join CF, meeting the given circle at E, join BF, meeting the circle at D: the ſtraight line which joins the points A, D, ſhall paſs through E, and the ſides of the triangle DEF, which is inſcribed in the given circle, ſhall paſs through the given points A, B, C, as is required.

Demonſtration. Join FK meeting the circle in P, draw HPN, HTR, HQV, HGF, and join EG and SV. Becauſe the rect-

angle

angle AB · BH is equal to RB · BQ, the points A, R, Q, H are in the circumference of a circle, therefore, the angle BHQ is equal to QRA, that is, to QVS, therefore, SV is parallel to AB; and becauſe MH : HL = MK : KL, the arch LR is equal to the arch LV; alſo LP is equal to LG (by the Lemma), therefore, PR is equal to GV; but, by conſtruction, the arch PE is equal to RS, therefore, PR is alſo equal to ES; wherefore, ES is equal to GV, and the ſtraight line EG is parallel to SV, or to AB. Becauſe, by conſtruction, the rectangle AB · BH is equal to RB · BQ, or to DB · BF, the points A, H, F, D are in the circumference of a circle, therefore, the angle FDA is equal to BHF, that is, (becauſe GE is parallel to AB,) to HGE, but the angle HGE is evidently equal to any angle at the circumference of the circle and ſtanding upon the circumference FGE, therefore, the angle FDA muſt ſtand upon the circumference FGE, therefore, DA paſſes through E, and the ſides of the triangle DEF, which is inſcribed in the given circle, paſs through the given points A, B, C, as required.

Beſides the four preceding caſes, in which it is eaſy to ſee that the points might be ſo ſituated as to render the problem impoſſible, there is yet a fifth caſe, in which the problem, inſtead of becoming impoſſible, may become unlimited, or admit of innumerable anſwers. This will be evident from the following Theorem, which, together with its demonſtration, will be found in Dr. *Stewart's Propoſitiones Geometricæ*, *Lib. I. Prop.* 33, and 34.

THEOREM. Fig. 298.

In LM the diameter of a circle, take a point H, and let HQ be perpendicular to the diameter; let A and B be two points in HQ, on the ſame ſide of H, if that point be within the circle, but on contrary ſides if it be without the circle, and let the rectangle AH · HB be equal to the rectangle LH · HM; from the points A, B inflect ſtraight lines AD, BD to D any point in the circumference, meeting the circle in E and F, join EF, meeting LM in the point K: MH ſhall be to HL as MK to KL.

Suppoſe now that A and B, two of the given points in our problem, are ſituated as required in the hypotheſis of the above propoſition, it is evident that the point K will be given, and ſince the remaining point C is ſuppoſed to be alſo given, the ſtraight line ECF will be given by poſition; hence the conſtruction is obvious.

But if we ſuppoſe the points C and K to coincide, then, it is evident that the problem would become indeterminate; for if any ſtraight line whatever be drawn through K, meeting the circle in E and F, and if AE and BF be joined, the lines AE and BF ſhall meet at D a point in the circumference of the given circle.

It

It ſeems proper to acknowledge, that the view here given of this curious geometrical problem was ſuggeſted by the following Poriſm, which appears to have been the laſt of the Third Book of *Euclid's Treatiſe on Poriſms.*

" If from two given points A, B, any two ſtraight lines AD, " BD be inflected to the circumference of a circle given by poſi- " tion, meeting it again in E and F, the ſtraight line which joins " E and F ſhall either contain a given angle with a ſtraight line " paſſing through a given point, or ſhall be parallel to a ſtraight " line given by poſition, or ſhall paſs through a given point."

A complete inveſtigation of this Poriſm may be ſeen in Dr. *Simſon's valuable work, entitled De Poriſmatibus.*

The ſame anſwered by Mr. Lowry, *Birmingham.*

A general ſolution to this problem may be derived from the converſe of *Lawſon's 7th Propoſition*, the demonſtration of which is given in the *Third Number of the Repoſitory.* Let D, E, F (fig. 122, pl. 19.) be the three given points, ABC the circle which is given in magnitude and poſition, S its centre, and ABC the required triangle. Through any two of the given points, as D, E, draw KDEL to interſect the circle in K and L, and meet BQ, drawn ſo as to make the $\angle$ BQE = ACB, in Q; then DE, KE, and EL, are given lines; and by reaſon of the equal angles, the triangles DCE, EBQ are ſimilar; therefore DE : CE :: EB : EQ, and by the circle KE : CE :: EB : EL, therefore, DE : KE :: EL : EQ, conſequently EQ is given, and therefore, Q is a given point. Through the given points Q, S, draw QGSH to meet the circle in G and H, then HQ, GQ, and HG are given lines, and if I be a point in HQ, ſuch, that HQ : GQ :: HI : GI (as in *Lawſon's Prop.*), then I is a given point. Now let IE, IB, be drawn to cut the circle in P, O, and R, and PQ, OD, to cut it in T and N reſpectively; join OT, NT, AM, and RM (M being the point where BQ interſects the circle); then by the *Propoſition above cited*, RM and OT are both perpendicular to HQ, conſequently they are parallel. Again, it is ſhewn above, that DE : KE :: EL : EQ, and by the circle KE · EL = OE · EP; therefore, DE : OE :: EP : EQ, conſequently the triangles EDO, EPQ, having the angles at E equal, are equiangular (Euclid. 6. VI.); therefore $\angle$TQK = PON = QTN, and ſince $\angle$AMB = ACB = BQE, it follows that TN and AM are each parallel to KL, and by reaſon of the parallels RM, OT, the $\angle$ABR = AMR = the ſupplement of the $\angle$NOT. Now I, E and D, being given points, O and N are alſo given points, and, therefore, the $\angle$NOT is given, and conſequently its ſupplement, or the $\angle$ABR is given; hence the

PORISM IV.

Let there be any two great circles given by poſition, and the great circle, and alſo a point, may be found, ſuch, that if through that point any great circle be drawn to interſect the given ones; and from the points of interſection, and likewiſe from the point found, great circles be drawn to the circle found; the ſum of the two arches ſo drawn will be equal to a given arch.

The ſame anſwered by Mr. Richard Elliott, *Liverpool.*

Fig. 304, plate 18. Let A and B be the given points, and AEB the ſtring; moreover let DF be the principal axis of the curve, and from any point E, let EC be drawn perpendicular to DF. Put the ſine of AD $= s$, and its coſine $= c$, the ſine of BD $= m$, and its coſine $= a$, the coſine of CD $= x$, and the coſine of CE $= y$. Then, by Trigonometry, $sy\sqrt{1-x^2}+cx =$ coſine of AC, and $my\sqrt{1-x^2}+max =$ coſine of BC. Hence, ſince the triangles AEC, BEC, are right-angled at C, we have the coſine AE $= sy\sqrt{1-x^2}+cxy$, and the coſine of BE $= my\sqrt{1-x^2}+axy$. Now, the Coſine of the ſum of the arches AE, BE, will be $(sy\sqrt{1-x^2}+axy)\times(my\sqrt{1-x^2}+axy)-\sqrt{1-(sy\sqrt{1-x^2}+cxy)^2}\times\sqrt{1-(my\sqrt{1-x^2}+axy)^2} = n$, the coſine of the length of the ſtring.

If the arch CD $= 0$, its coſine, x, will be $=$ unity, and the equation of the curve will become, by proper reduction, $(a^2+c^2-2acn)\times y^2 = 1-n^2$, hence, it appears that when the arch CD $= 0$, the arch CE is $=$ to a given quantity, and, therefore, that the curve will be convex at the point D, in all caſes except when $a^2+c^2-2acn = 1-n^2$, or $y = 1$.

If the two points A and B coincide, a will be equal to c, and s equal to m, and the equation in that caſe becomes $(sy\sqrt{1-x^2}+2acy)^2-1+(sy\sqrt{1-x^2}+cxy)^2 = n$, or $sy\sqrt{1-x^2}+cxy = \sqrt{\frac{1+n}{2}}$, as it ought, being the equation to a circle on a ſphere.

The ſame anſwered by Mr. Gompertz, *London.*

Fig. 300, pl. 18. Put s and $c =$ ſine and coſine of AB, m and $a =$

s = fine and cofine of AF + FB, x = fine of PB, and y = fine of PF, all to radius 1. By trigonometry, cofine AP = $sx + c\sqrt{1-x^2}$, cofine of BF = $\sqrt{1-y^2} \times \sqrt{1-x^2}$, and cofine of AF = $m\sqrt{1-(1-y^2)\times(1-x^2)} + n\sqrt{1-y^2} \times \sqrt{1-x^2}$.

Alſo, by trigonometry, as cof. AP : rad. :: cof. AF : cof. PF, that is, as $sx + c\sqrt{1-x^2} : 1 :: m\sqrt{1-(1-y^2)\times(1-x^2)} + n\sqrt{1-y^2} \times \sqrt{1-x^2} : \sqrt{1-y^2}$.

Hence $\frac{sx}{\sqrt{1-x^2}} + c = \frac{m\sqrt{1-(1-1^2)\times(1-1^2)}}{\sqrt{1-y^2} \times \sqrt{1-x^2}} + n$, the equation required.

And thus it was anſwered by Meſſrs. Furnaſs, Harris, Surtees, *and* Thornoby.

The Medal for ſolving the Mathematical Prize Queſtion is decided in favour of Mr. *Lowry*, who will pleaſe to ſend for it to Mr. *Glendinning's*, by whom it will be delivered, free from expence, to any part in London.

ARTICLE XXVIII.

MATHEMATICAL QUESTIONS.

To be anſwered in Number X.

I. QUESTION 157, *by Mr.* Olinthus Gregory.

THE young Algebraiſt is requeſted to determine the dimenſions and area of a right angled triangle, whoſe baſe, perpendicular, and hypothenuſe are x^{5x}, x^{6x}, and x^{7x} reſpectively.

II. QUESTION 158, *by Mr.* John Byerly, *Ripon.*

It was obſerved, on the top of one of the high mountains in Italy, that the atmoſphere was only half as denſe as it was at the bottom. Query the height of the mountain?

III. QUESTION 159, *by Mr.* W. Burdon, *Acaſter-Malbis.*

Required a theorem for determining the diameter of an air balloon,

iron, to raiſe a given weight, to a given height above the earth's ſurface, the weight of each ſquare yard of the ſubſtance compoſing it, and the ſpecific gravity of the air with which it is filled, being given?

IV. QUESTION 160, *by Mr.* R. Simpſon, *Croxdale, Durham.*

The weight of a cylindrical pillar of marble is two tons. Query its dimenſions, when it juſt ſupports itſelf from falling, its ſide making an angle of 60° with the horizon?

V. QUESTION 161, *by Mr.* Gregory, *Cambridge.*

A ſurveyor is requeſted to lay out 5A. 1R. 24P. of land in the form of a right angled triangle, of ſuch dimenſions, that the diſtance from one extremity of the hypothenuſe to the point where the inſcribed circle touches it, ſhall be 6 chains. Being unable to find the dimenſions of the triangle, as required, he begs the favour of ſome ingenious gentleman to tell him the length of each ſide thereof?

VI. QUESTION 162, *by Mr.* W. Marrat, *Boſton.*

Being deſirous to know what quantity of water was contained in a glaſs ſphere, whoſe diameter is 1 foot, and thickneſs of the glaſs 1-20th of an inch, I immerſed it in a veſſel of water, and found it ſunk, till the ſurface of the water in the ſphere, was on a level with the ſurface of the water in the veſſel. How much water was in the ſphere, the ſpecific gravity of glaſs and water being 2600 and 1000?

VII. QUESTION 163, *by Mr.* Thomas Milner, *Lartington.*

Why does Mr. Emerſon in the inveſtigation of Prop. X. B. III. Principia, uſe AF × SA : AB × SF inſtead of $Aa - Ff : Aa - Bb$?

VIII. QUESTION 164, *by Mr.* Hewitt, *Teacher of the Mathematics, Bunhill-Row, London.*

A tower 70 feet high caſt a ſhadow 81 feet long, weſt 8° ſoutherly, at 24 min. paſt 8 in the morning. Required from hence the day of the month (in the ſpring 1798) and the latitude of the place?

IX.

IX. QUESTION 165, *by Mr.* George Brown, *Teacher of the Mathematics, Howdon-Pans, North-Shields.*

One morning in May 1798, I obſerved the ſun to riſe exactly at 4 o'clock, and at 6 o'clock, the ſame morning, I found his altitude was 15° 35′. Required the latitude of the place and the day of the month, by a ſimple equation?

X. QUESTION 166, *by Mr.* Samuel Thornoby.

If a veſſel in the form of a fruſtum of a cone, whoſe top diameter is 16, bottom diameter 4, and depth 20 inches, be filled with water to the depth of 10 inches; it is required to determine how high the water will riſe if a globe of 12 inches diameter be let fall into the ſaid veſſel.

XI. QUESTION 167, *by Mr.* Thomas Bulmer, *Teacher of the Mathematics, Sunderland, Durham.*

At what diſtance from the greater end of a piece of fir in form of a fruſtum of a ſquare pyramid, muſt a prop be placed, ſo as it may juſt hang in equilibrio, when a weight of 30lb. is hung at the greater end, and one of 10lb. at the leſs; the ſide of the greater end being nine inches, that of the leſs ſix inches, and the length ſix feet.

XII. QUESTION 168, *by Mr.* John Surtees, *Sunderland.*

Being at ſea on June 21, 1798, in a briſk gale, at nine knots or miles (60 to a degree) per hour, at 4h. 30m. A. M. per watch, I obſerved the true altitude of the ſun's centre = 22° 28′, then ſtanding away between ſouth and eaſt, at 9h. 50m. 45. A. M. per watch, (the watch being ſuppoſed to keep true time between the obſervations.) I found the true altitude of the ſun's centre = 33° 18′, and his bearing due ſouth. Required the true latitudes of the two places of obſervation, their difference of longitude, and the courſe the ſhip ſteered, admitting the earth to be a perfect ſphere?

XIII. QUESTION 169, *by Mr.* John Blackwell, *Hungerford.*

On May the 12th 1799, in latitude 51° 23′ north, at nine o'clock, I obſerved an elliptical picture frame, which I ſuſpended in the ſun ſhine, by a thread connected at the extremity of the

conjugate axis, to make ſeveral revolutions about the ſaid conjugate axis, the ſhadow of which formed ſeveral variable ſpaces on the plane of the horizon, but the greateſt of which was 172·8 ſquare inches, from whence it is required to find the area of the frame; the tranſverſe to the conjugate axis being as 3 to 2?

XIV. QUESTION 170, *by Mr.* Peacock, *Birmingham.*

Required the area of the greateſt ellipſis that can be inſcribed in a parabola, whoſe baſe is 5 and abſciſſa 4, when the tranſverſe axis is parallel to the baſe of the parabola; and alſo the area of the greateſt parabola that can be inſcribed in an ellipſis whoſe axes are 5 and 4?

XV. QUESTION 171, *by Mr.* Gregory, *Cambridge.*

During how long an interval will it be impoſſible that there ſhould be any primary Rainbow, on July 1, 1800, in latitude 52°: and during how long an interval, on the ſame day, in the ſame place, can there be no ſecondary bow?

XVI. QUESTION 172, *by Mr.* Burdon, *Acaſter Malbis, York.*

Given the radii of the inſcribed and circumſcribed circles, to determine the $\triangle$ when the ſides are in geometrical progreſſion.

XVII. QUESTION 173, *by Mr.* Fletcher, *Hollinwood, near Mancheſter.*

In a given circle ACBD to inſcribe a Trapezium ACBD ſuch, that AD = DB; and if from the angular points A, B, C, D; perpendiculars *Ae*, *Bg*, *Ch*, and *Di* be drawn to the diagonals CD and AB, and the points *e*, *i*, *g*, *h*, *e*, be joined, the continual product of the ſides of the trapezium *eighe* and thoſe of the trapezium ACBDA; or $AC \times CB \times BD \times DA \times ie \times ig \times gh \times he$, may be a maximum.

XVIII. QUESTION 174, *by Mr.* Geo. Sanderſon, *London.*

In *Euler's* Theory of Ships, tranſlated by Watſon, art. 31, page 104. If there be three tangents in geometrical progreſſion, the firſt of which is given equal $\frac{b}{a}$; the difference of the other two

arches

arches will be a maximum, when the ſine of twice the greater arch is equal to half the ſine of thrice the mean of leſs of the two unknown arches; required the inveſtigation?

XIX. QUESTION 175, *by Mr.* J. H. Swale, *Cheſter.*

Given the baſe, and one of the angles at the baſe, to conſtruct the plane △, when the ſum, or difference of the ſquares of the ſides, together with the ſquare of the perpendicular, is equal to a given ſpace.

XX. QUESTION 176, *by Mr.* Johnſton, *Birmingham.*

Given the ratio of the ſides, the radius of the circumſcribing circle, and the prolongation of the perpendicular, from the vertical angle upon the baſe, to the periphery of the circumſcribing circle, to conſtruct the plane △.

XXI. QUESTION 177, *by Mr.* Lowry, *Birmingham.*

Given the radius of the circumſcribing circle, and the difference of the ſquares of the ſides, to conſtruct the plane △, when the area is a maximum.

XXII. QUESTION 178, *by Mr.* Lowry.

Given the baſe and the two points therein, where the perpendicular and the diameter of the circumſcribing circle, drawn from the vertex, interſect it, to conſtruct the plane △.

XXIII. QUESTION 179, *by Mr.* Louis Hill.

Given the difference of the ſegments of the baſe made by the perpendicular, the difference of the ſegments of the baſe made by the line biſecting the vertical angle, and the difference of the cubes of the ſides, to conſtruct the plane △.

XXIV. QUESTION 180, *by Mr.* John Lowry.

Two circles being given in magnitude and poſition, and alſo a point in the line joining their centres; it is required to draw a line, through that point, to interſect the circles, ſo that the intercepted chords may have a given ſum or difference.

XXV. QUESTION 181, *by Mr.* Wallace, *Perth.*

Let AB, AC be two ſtraight lines given by poſition, let B and C be given points in theſe lines, and let P be a given point without them.

them. It is required to draw two ſtraight lines PD, PE, cutting the given lines in D and E, ſo that the angle DPE may be of a given magnitude, and ſo that the rectangle BD × CE may be equal to a given ſpace?

XXVI. PRIZE QUESTION 182, *by Mr.* S. Thornsby.

I have a cylindrical veſſel, the diameter of the baſe of which is 2 feet, and its depth 1 foot. Now ſuppoſing this veſſel were full of water, and my eye placed perpendicularly above the centre, at 6 inches above the ſurface, the bottom would appear, not flat, but concave: if the bottom of the veſſel were in the ſame ſhape and poſition as it appears to my eye in that ſituation, how much leſs water would it then contain than it does in reality?

ARTICLE XXIX.

HINTS

RELATIVE TO

FRICTION IN MECHANICS.

By Mr. REUBEN BURROW.

HYPOTHESIS.

IN the following eſtimation of friction, the weight or force neceſſary to overcome the reſiſtance, &c. is ſuppoſed to be proportional to the preſſure.

OF FRICTION IN THE INCLINED PLANE.

Let AB be an inclined plane, (Fig. 305, Pl. 19.) and let PR repreſent a weight ſuſtained on it by any force R*m*, acting in the direction R*m*; and draw PD perpendicular to AB, and let R*m* meet PD in *n*: Now as R*n* repreſents the force that would be neceſſary to ſuſtain the body, excluſive of friction, and P*n* repreſents the preſſure againſt the plane, if *mt* be drawn perpendicular to PD meeting it in *t*, then will *nm* be the force neceſſary to overcome the friction in that direction, and P*t* the real preſſure againſt the plane AB, when the whole force R*m*, neceſſary to overcome both the weight and the friction, acts in the direction R*m*; and as the force *nm* is equivalent to *nt* and *tm*, and *nt* has no other effect than to alter the preſſure, therefore *tm* is the only force which overcomes the reſiſtance of friction; and as this force is as the preſſure, therefore *tm* is proportional to P*t*, and hence the locus of all the P*t*'s a right line.

Again,

Again, ſuppoſe the body, inſtead of being drawn along, to be ſuſtained at reſt only upon the plane; this, it is evident, will require a leſs force than the other, becauſe the friction prevents the body in part from deſcending. (Fig. 306, Pl. 19.) Let R*m* be the force required, and let the ſame conſtruction be made as before; then becauſe R*n* is the force that would be neceſſary if there was no friction, *mn* is the effect of the friction itſelf; but *mn* is equivalent to the forces *mt* and *tn*; and as P*n* would be the preſſure excluſive of friction, P*t* is the preſſure inclusive; and as the force loſt is as the friction, and *mt* is as the force loſt, therefore *mt* is as P*t*, for the friction is as the preſſure; conſequently the locus of all the points *m* is a right line paſſing through P, and making the ſame angle as DPQ in the former caſe, and only differing by being drawn on the contrary ſide of PD.

SCHOLIUM.

In what follows, the force requiſite to ſuſtain any body is conſidered under three different diſtinctions. Firſt, when it is juſt barely ſufficient to overcome the weight and reſiſtance ariſing from friction, and the body is conſidered as juſt beginning to move in the direction of the force applied, and the force in this caſe is called the *moving force:* ſecondly, when this force is diminiſhed till the body would begin to move or deſcend in a contrary direction if the force was diminiſhed farther; this laſt I call the *ſuſpending force*; and it is plain that whatever force is applied to the body leſs than the moving, and greater than the ſuſpending force, the body will remain at reſt: laſtly, it is manifeſt that there is an intermediate ſtate, in which ſuch a degree of force may be applied, that the friction will have no effect either way; and this force is the ſame as would keep the body in equilibrio if there was no friction, becauſe the effect or tendency of friction is to keep the body at reſt, or prevent it from moving either way; this being premiſed, there will be little difficulty in the following.

PROBLEM I.

Having given the weight of the body to be ſuſtained, the inclination of the plane, and the ratio of the friction to the preſſure; to find the force requiſite to ſuſtain the weight in a given direction.

In the foregoing figures, draw PR and PD at right angles to the horizon and plane reſpectively, PR repreſenting the weight; take PD to DQ as the preſſure to the friction, and let DQ be taken upwards or downwards as the requiſite force is motive or ſuſpenſive; join PQ and draw the line R*n* in the given direction meeting PQ in *n*; then R*n* is the force required.

Corollary

Corollary 1. If the friction be the n part of the preſſure, and W be the weight, s and c the ſine and coſine of the plane's elevation, then the moving force parallel to the plane will be $W(s + c : n)$, and the ſuſpending force $W(s - c : n)$.

Corollary 2. If the direction of the force be parallel to the horizon, and t be the tangent of the plane's elevation, then $W(tn + 1) : (n - t)$ will be the moving force, and $W(tn - 1) : (n + t)$ the ſuſpending force, and Wt the force excluding friction.

Example. If the weight be a ton, the friction $\frac{1}{3}$ of the preſſure; $AB = 5$, $BC = 3$, and $AC = 4$, then the moving force will be 3235 pounds, the ſuſpending force 747 pounds, and the force excluding friction 1680 pounds; nearly.

PROBLEM II.

Given the weight of the body, the inclination of the plane, and the ratio of the friction to the preſſure; to find the direction ſo that the ſuſtaining force may be a given quantity, or the leaſt poſſible.

Draw DQ and QP as before, and let PR be to Rm as the weight to the given force; then from the centre R, with a diſtance equal to Rn, interſect PQ in m; then Rm is the required direction when the force is given; but to have it the leaſt poſſible, draw Rm to right angles to PQ, then Rm is the direction required.

Corollary 1. An expreſſion for the ſuſtaining force when the leaſt poſſible, may be found as follows: In the triangles PDQ, RQm, the angle Q is common, therefore PQ : PD :: RQ : Rn; but PD is a fourth proportional to AB, AC, and PR, and DQ is to PD as 1 to n, ſuppoſing this the given ratio; alſo RD is a fourth proportional to AB, BC, and PR, conſequently RQ is equal to DQ either added to or ſubtracted from DR, as it is the firſt or ſecond caſe; and becauſe PQ : PD :: $\sqrt{(nn + 1)} : n$ :: RQ : Rm, therefore $Rm \pm PR\ (n.BC \pm AC) : AB\sqrt{(nn + 1)}$ or $(ns \pm c)\ W : (\sqrt{nn + 1})$ by ſubſtituting s and c for the natural ſine and coſine of the plane's elevation, and uſing the negative or affirmative ſign as the force required, is the moving or ſuſpending one reſpectively.

Example. If $AB = 5$, $BC = 3$, and $AC = 4$, and the the weight 1 ton, then the leaſt moving and ſuſtaining forces will be 1825 and 702 pounds reſpectively.

Corollary 2. Becauſe the triangles PDQ and RQm are ſimilar, and the ratio of PD to DQ conſtant to each fixed value of n, therefore the angle QRm being equal to DPQ, will alſo be conſtant, whether the inclination of the plane be variable or not; and hence the angles of the direction with the plane for the draught to [illegible] with the greateſt advantage, are found for different values [illegible] lows:

n	QRm	n	QRm	n	QRm	n	QRm	n	QRm	n	QRm
	° ′		° ′		° ′		° ′		° ′		° ′
1	45.0	2	26.34	3	18.26	4	14. 2	5	11.10	6	9.0
1¼	38.40	2¼	23.58	3¼	16.54	4¼	13.15	5¼	10.47	7	8. 8
1½	33.41	2½	21.48	3½	15.57	4½	12.32	5½	10.1	8	7. 8
1¾	29.43	2¾	19.59	3¾	14.56	4¾	11.53	5¾	9.5	9	6.2

N. B. The direction, or angle QRm, is to be taken below the plane for the ſuſpending, and above the plane for the moving force.

Scholium. Though at firſt ſight the former part of the above Problem, which ſhews the beſt method of applying an active force, ſeems ſuperior to the other, yet, on farther conſideration, the other appears of equal conſequence, and particularly in building and faſtening walls, banks of earth, fortifications, &c. and the application of what are called *land-ties*, &c. Thus if a weight, for inſtance, is to be drawn along the plane RB, and the friction be ¼ of the preſſure, the beſt direction is when Rm makes an angle of 18°. 26′ *above* the plane; but if the weight is a quantity of earth or ſtone, or any thing to be ſuſpended, as in the caſe of land-ties, the beſt angle (on the foregoing ſuppoſition) muſt be 18°. 26′ *below* the plane.

SCHOLIUM.

In thoſe propoſitions the friction is eſtimated according to the moſt generally received opinion, that the reſiſtance is proportional to the whole preſſure compounded of the weight of the body, and the additional force neceſſary to overcome the friction; but it has been aſſerted, that there may be caſes where the friction is not proportional to the whole preſſure, but to that which would ariſe if the body was ſuſtained in a given direction, excluſive of friction; and that there might alſo be caſes, where the reſiſtance, ariſing from tenacity or cohеſion, might be as the relative preſſure againſt the plane, and the force to overcome it the ſame in every direction; ſomething ſimilar to a globe ſtuck faſt in wet tenacious clay: I ſhall therefore give ſolutions to both caſes.

In the firſt caſe, (Fig. 307, Pl. 19.) the force requiſite to ſuſtain the body in direction RV, excluſive of friction, is Rn; and as Rn is equivalent to RD and Dn, therefore Pn is the preſſure excluſive of

of friction; and as the friction is the n part of the preſſure, the force acting parallel to AB to overcome it, is the n part of Pn; but the force which acting in direction Rn will be equivalent to the n part of Pn in the direction Rn, is a fourth proportional to n times RD, Pn, and Rn; but becauſe DQ is the n part of DP, therefore fn is the n part of Pn, and the fourth proportional aforeſaid will be nz; conſequently the ſum or difference of Rn and nz muſt be a given quantity, or the leaſt poſſible: the Problem therefore is reduced (Fig. 308, Pl. 19) to drawing a line Rn from the given point R, meeting the two lines PD and PQ given in poſition in n and z, ſo that nz added to or taken from Rn, the ſum or difference may be a given quantity, or the leaſt poſſible. To do this, let DS be taken equal to DR, and draw Sr parallel to PD meeting PQ in M; then becauſe Rn is equal to rn, the ſum or difference of the quantities aforeſaid is rz; and when rz is required to be a given quantity, the queſtion is reduced to that particular caſe of the inclinations of Apollonius, in ſolids, which has been reſolved by Newton and Barrow: the limits of the Problem, or the mode of drawing the line Rr, ſo that the intercepted part rz may be the leaſt poſſible, may be inveſtigated as follows:

(Fig. 310, Pl. 19.) Suppoſe it done, and Rrz the poſition required, and let Rrw be indefinitely near to Rz, and Mh perpendicular to Rz; then by applying the analyſis of the ancients to the *Newtonian* doctrine of prime and ultimate ratios, mn is equal to zr; and if from the centre R, with the diſtances Rz and Rn, the arcs zv and nt be ſuppoſed to be deſcribed, vn is equal to zt, and conſequently tr equal to mv; but $rt : tn :: rh : Mh$, and $tn : zv :: Rr : Rz$, and $zv : vm :: Mh : hz$, whence by compounding the proportions, $tr : vm :: Rr \cdot rh : Rz \cdot zh$, and as the two firſt terms are equal, the two laſt are equal, and conſequently $Rr : Rz :: zh : rh$, and dividing $Rr : rz :: zh : rz$, therefore Rr is equal to zh, and conſequently the point h is in an hyperbola, whoſe aſymptotes are QM and SM produced: but becauſe the angle MhR is a right angle, the point h is alſo in the circumference of a circle; therefore a line drawn from R to h, the point where the hyperbola and circle interſect, is the poſition required.

In the other caſe, where the reſiſtance ariſing from tenacity or coheſion is ſuppoſed to be as the relative preſſure againſt the plane, and the force to overcome it the ſame in each direction, we have Rn for the ſuſtaining force, excluſive of friction; and the n part of Pn for the friction; and conſequently the ſum or difference of theſe is the expreſſion for the whole force; and the Problem may be thus conſtructed. Take PD to DQ as the preſſure to the friction, and join PQ; on PD deſcribe a circle, in which take Dv equal to DQ; join Pv, and draw RV perpendicular to it: then RV will repreſent

reſent the direction and meaſure of the whole force when it is leaſt poſſible.

For DQ and Dv are equal, and conſequently nf is equal to Vn; t DQ is the n part of DP, therefore nf or Vn is the n part of ; and conſequently RV is equal to the ſum or difference of Rn, d the n part of Pn: but RV is the leaſt poſſible by conſtruction, d therefore the other is a minimum alſo. For draw any other e Rk meeting PV in k and PD in m; and draw mq, mt, parallel DQ and Dv; then the ſum or difference of Rm and mt is equal the ſum or difference of Rm and mq; but the ſum or difference Rm and mt is greater than RV, and therefore the ſum or difrence of Rn and the n part of Pn is the leaſt poſſible.

PROBLEM III.

Given the weight of the body, the inclination of the plane, and he force ſuſtaining the body in a given direction: to find the ratio f the friction to the preſſure.

Take PR as before, (ſee Fig. 305, 306, Pl. 19.) draw Rm in the given direction, and take PR to Rn as the weight of the body to he force ſuſtaining it; draw Pm meeting AB in Q, and PD perpendicular to AB; then PD is to DQ as the preſſure to the friction.

PROBLEM IV.

If AkqN be the ſegment of an equilateral triangle, which, by moving parallel to itſelf and the horizon, generates a ſolid, upon which a figure hmGEHKpqk moves, touching the former in hm and qp; required the effect of the friction; ſtill ſuppoſing it the n part of the preſſure.

Let P be the centre of gravity of half the body (Fig. 311, Pl. 19.) and PR its weight as before; then the body by means of its inflexibility is kept together in the ſame manner as if it was actuated by a force parallel to the horizon; but if PDn be perpendicular to Ak, and Rn parallel to the horizontal line AC, meeting PD in n, Pn will be the preſſure againſt the ſide Ak, and the friction is the n part of Pn; but PR : Pn :: AC : AB; therefore if AC repreſent the weight of half the body, the n part of AB will expreſs the weight requiſite to overcome the friction for that half; and by doubling the expreſſion they ſerve for the whole. Wherefore let W repreſent the weight of the body, f the ſecant of the angle BAC; then Wf will be the preſſure againſt the plane AD; and the n part of Wf the force neceſſary to overcome the friction; and as this laſt is the force neceſſary to draw the body along

along a horizontal plane, therefore the force neceſſary to draw the body along a horizontal plane, is to that neceſſary to draw it along the body whoſe ſection is $AhqN$, as AC to AB, or as 1 to f.

Becauſe when the angle CAB is given, the ratio of PR to Pa is conſtant: therefore when the ſolid whoſe ſection is $AhqN$ is elevated, making an angle with the horizon, ſo that its baſe forms an inclined plane; PR in that caſe repreſents the preſſure in a normal direction to that plane, and Pa the preſſure againſt the ſolid; and as the friction is increaſed in the ratio of the preſſure, therefore if the preſſure which the body would have on the inclined plane be increaſed in the ratio of AC to AB, or radius to the ſecant of the angle CAB, then the preſſure on the angular plane or body, whoſe perpendicular ſection is $AhqN$, will be had, and conſequently its n part, or the friction. Hence this conſtruction (Fig. 309, Pl. 19.); let PR repreſent the weight; then PD at right angles to AB repreſents the preſſure that the body would exert againſt the common inclined plane; take DK to DP as AB in the foregoing figure to AC, or as the ſecant of the inclination of the angular plane with its baſe to radius; let Dq be the n part of DK, and join Kq; then RM drawn any how to meet Kq in M, gives RM for the meaſure of the whole force in that direction; and it is the moving or ſuſpending force, according as DQ is taken upwards or downwards in the line AB.

It is evident that Kq is parallel to PQ, and therefore though the leaſt force (which is perpendicular to Kq) differ from that in the former caſes; yet the directions for having the greateſt effect are ſtill the ſame as in the foregoing table; the demonſtration is in effect the ſame as the firſt.

Corollary. By ſuppoſing f to be the ſecant of the angle (Fig. 312, Pl. 19.) that the ſides of the angular plane make with the baſe, proceeding as Corollary 2d of Problem 1ſt, and putting t for the natural tangent of the plane's inclination, and W for PR the weight, we have $W\ (tn+f) : (n - t)$ for the moving: and $W\ (tn - f) : (n + t)$ for the ſuſpending force, neceſſary to draw the body along the angular inclined plane by a force acting parallel to the baſe of the plane.

Example. Let AB, BC, and AC, be 5, 3, and 4, reſpectively, and let the inclination of the ſides be 45°; the weight of a ton and the friction one third of the preſſure; then 3648 pounds is the moving, and 499 the ſuſpending force.

SCHOLIUM.

In this propoſition, thoſe parts of the plane on which the body moves are ſuppoſed rectilinear, as moſtly happens in practice; but, if

he friction is eaſily eſtimated in curvilinear ſurfaces, and may be found generally as follows:

Let AMP (Fig. 313, Pl. 19.) be half the ſection perpendicular to the horizon, and to the axis of the ſolid which forms the curvilineal plane on which the body is moved; AP the axis: PM the ordinate, and MS a tangent to the curve at the point M; alſo let RM repreſent the weight or preſſure in a direction perpendicular to the horizon at the point M; and let RF be perpendicular to MS meeting MP in F: alſo let PN be taken equal to MR, and PQ equal to RF; and ſuppoſe the ſame conſtruction to be made for every point of the curve, and let HN be the locus of all the points N, and GQ the locus of all the points Q, then will the friction, when drawn along the horizontal plane, be to the friction of the ſame body when drawn along the curvilinear plane in the ſame direction, as the area APNH to the area APQG.

For the friction on the horizontal plane being as the ſum of the preſſures, is as the ſum of all the elementary lines MR or PN; that is, as the area AHNP; and the friction on the curvilinear plane is for the ſame reaſon as the ſum of all the RF or PQ, namely, as the area APQG; hence the truth of the propoſition is manifeſt.

Corollary 1. Becauſe M*n* or the fluxion of *y* is to M*m* the fluxion of the curve, as MR or PN to RF or PQ, therefore if PN be a function of AP, PQ will be a fourth proportional to the fluxion of the ordinate, the fluxion of the curve AM, and this function; wherefore if the curves HN and AM be given, the nature of the curve GQ will be known, and its area may be found by the common methods of quadratures.

Corollary 2. It is evident that when the planes are inclined to the horizon, the frictions of the right and curvilinear planes are ſtill in the ſame ratio as in the preceding caſes, and conſequently may be found by the ſame mode of proceeding.

Corollary 3. It is alſo evident, that the above method holds good whether the parts of the body are connected together or not, with reſpect to their motion in the direction RM, ſo long as each elementary part MR may be conſidered as ſuſtained at the point M by a force parallel to MP; but when the body is rigid or inflexible, the caſe becomes more ſimple, for MR is then conſtant, and APNH becomes a parallelogram.

Corollary 4. By ſuppoſing given properties to exiſt in any two of the curves AM, HN, or GQ, the nature of the third will be known; and hence a number of problems relative to friction may be propoſed and reſolved by a proper application of the direct and inverſe methods of fluxions.

diſtance will have the ſame effect, &c.

This is two evident to need a demonſtration.

OF FRICTION IN THE SCREW.

As any force acting perpendicular to the direction c body does not affect the motion of the body in that the force acting perpendicular to the axis of the ſcrew on the motion of a body raiſed thereby, excluſive of therefore requires the ſame force to raiſe a body by ſcrew, as to raiſe the ſame body in equal time along plane of the ſame elevation, as the threads of the ſcre of a force acting parallel to the baſe of the inclined p if we ſuppoſe the weight ſo contracted or condenſ capable of being placed on one of the threads of the faſtened to an imaginary lever always perpendicular to i it is evident this lever will have no effect but to chang tion of the weight, and keep it in the midſt of the th ſcrew; and if a force be applied at the weight always pe to this lever, ſo as to ſuſtain or draw it along, this fo determined exactly the ſame as was done before in plane: but the rigidity of the parts of the " female ſc axactly the ſame purpoſe as this imaginary lever, and weight act upon the threads like a body ſuſtained on plane by a force parallel to its baſe; and as the force t both the weight and the friction is reciprocally as the d the centre of the axis, therefore the diſtance of from the centre of the axis is to the diſtance the ſame c

ame things as before, and the upper ſign is for the
the lower for the ſuſpending force. N. B. t is the
nt of the angle made by a line touching one of the
plane at right angles to the axis of the ſcrew; or it
he diſtance of the reſpective edges of two threads,
he circumference of the cylinder, out of which the

1. When lines drawn from the center of the axis of
coincide with the threads, are at right angles to the
e expreſſion becomes $Wd\ (tn \pm 1) : (n \mp t)\ D$, for
lius or unity.
2. When n is equal to t, the moving force will be
the ſuſpending force will be nothing when t is the n
d when $Wd\ (tn - f)$; $(n + t)\ D$, becomes negative,
e quantity of force, which muſt act in a contrary di-
uce the body juſt to a ſtate of ſuſpenſion.

SCHOLIUM.

e needleſs to make any allowance for the curvilinear
e threads of ſcrews, as they ſeldom differ much from
going forms; neither is it of much conſequence to
ir parts being at different diſtances from the axis, as
feldom bears any conſiderable ratio to the length of the
ich they act; but the caſe is different when large bodies
ach other, and therefore it will be neceſſary to ſhew
proceeding in ſuch caſes.
AQ be a convex ſolid, generated by the revolution of
AQ about its axis perpendicular to the horizon, and
ncave body exactly fitting it; then if this laſt body be
ut the axis AP by means of the lever Pf, the force
overcome the friction of one body turning upon the
e found as follows. Suppoſe the revolving body di-
n infinite number of concentric tubes, that may deſcend
of each other, and preſs freely againſt the body on
revolve, and yet be ſo connected that the lever Pf may
e angular velocity at the ſame time to each; alſo let the
N of the curve HN repreſent the weight or preſſure
on perpendicular to the horizon) of each of the indefi-
parts Mk, or elementary lines of the body at the
from the axis, and let e be the circumference of a
: radius is unity: then becauſe the friction of each of
ary tubes MRSQ is as its preſſure, and the preſſure is
er of lines Mk, and the preſſure of each; therefore as
is as PM.M$n.c$, we have the n part of this expreſſion

for the force which, acting at M, would overcome the f
the cylindrical tube if moved round upon a horizontal pl
as the preſſure of each elementary part is increaſed in the
Mn to M*m*, when moved on the ſolid MAQ, the real
be (PM.*c*.M*m*.PN) : *n* ; alſo P*f* : PM :: (PM:*c*.M*m*.P
the ſmall elementary force which will overcome the laſt fc
acting at *f*; conſequently the whole force will be equal to
of (PM*n*.PN.M*m*.*c*) : (*n*,P*f*.)

Corollary. By means of the curves AM, HN, &c. c
may be drawn ſimilar to thoſe in the Corollaries to the
of the fourth Propoſition.

OF FRICTION IN THE LEVER.

It has been already obſerved, that a force acting perper
the direction of a body in motion, does not alter the bod
in that direction; therefore if (Fig. 315, Pl. 19.) we fu
to be an upright cylinder, and AB a body touching it in a
the figure, and retained cloſe to it by an imaginary force
it perpendicular towards the axis; then if a force CP b
to C, the center of gravity of AB, and be always ſuppo
perpendicularly to the radius CN, drawn from the cen
axis to the point C, the friction will be the ſame in di
body round the cylinder, as in drawing it along a horizc
with an equal preſſure; and if it be moved round by a fo
at a greater diſtance, the force will be reciprocally as th
on the contrary, if the body AB be fixed, and the cylin
round about its axis, the friction will be the ſame as if th
was fixed, and the body drawn round it by CP, as befc
wiſe the friction is the ſame, whether the cylinder be fixe
body AB moved round the axis MR by a force Q*c* apj
or whether the point *c* be fixed with AB faſtened to C
cylinder be revolved in a circle whoſe centre is *c*, ſo as
retain its parallelism with reſpect to any fixt object; and
caſe obtains in the axletrees of carriages, ſince every po
wheel's contact with the ground may be conſidered as th
motion for that inſtant, therefore the effect of the reſiſta
from the friction of the concave part of the nave upor
tree, is to the effect that would ariſe from drawing the ſa
over a horizontal plane of the ſame kind, as the parts tha
other, as the radius of the axis to the radius of the wheel
be obſerved, that this is not the only friction to which ca
ſubject; for there is another part, ariſing from the cohef
wheel and the ground at their contact, which is to be
allowed for by the three firſt Propoſitions.

In the above the preſſure and friction have been ſuppoſed to be as the weight, as it is on a horizontal plane; but by the Scholium to the fourth Propoſition, it is plain that the preſſure is greater than the weight, and may be ſo in any proportion: however, as it appears by calculation, that the preſſure on an arc of ninety degrees is to that on its chord, only as 1,183 to 1, when both the concave and convex parts have exactly the ſame curvature, the difference will be ſo trifling when the cylinders have different curvatures as uſual, as to require very ſeldom to be allowed for.

This being premiſed, let M (Fig. 316, 317, 318, Pl. 19.) be a weight placed at the point A of a lever, moveable about an axis whoſe centre is d and radius dn; and let N be the ſuſtaining force acting at B: now it is evident that the preſſure on the axis differs ſo little from the weight, that it may be ſafely taken for it without any conſiderable error, except in ſome remarkable caſes, which may be allowed for from what has been ſaid already; and therefore the friction which ought in ſtrictneſs to be taken as the n part of the preſſure, will here be taken as the n part of the weight upon the axis. Now if N be taken for the force which, acting at B, would be juſt ſufficient to keep the weight M at A in equilibro, excluſive of friction, and if W be the additional force to be added to N ſo as to overcome the friction; then will $M + P$, $M - P$, and $P - M$, be the weight upon the axis at d in the firſt, ſecond, and third figures reſpectively, (ſuppoſing the ſum of M and N to be equal to P). Now as the friction is the n part of each of theſe quantities, and its effect is to keep the lever in a ſtate of reſt, therefore in whatever direction the force at N endeavours to draw the lever by acting at B, the friction tends to counteract that force by keeping the lever ſteady, or acting in a contrary direction at n; and as the effect of the friction, and the additional force W, are in equilibrio, and the friction acts by means of the leaver dn, and the force W by the lever dB; therefore Bd is to dn as the ſum or difference of the n part of $N + W$ and M is to W; conſequently $W = dn\ (M + N) : (n.Bd - dn,)$ in the firſt figure; $W = dn\ (M - N) : (n.Bd + dn,)$ in the ſecond figure; and in the third figure, $W = dn\ (N - M) : (n.Bd - dn.)$ All theſe are the expreſſions for the moving forces.

To find the ſuſpending forces, or the forces which, acting at N, ſhall be juſt ſufficient to prevent the weight M from deſcending: Let M and N be the ſame as before, and let w be the force which, taken from N, will leave a force juſt ſufficient to prevent M from deſcending; then the weight upon d in the firſt figure will be $M + N - w$; in the ſecond figure, the weight will be $M - N + w$; and in the third figure, $N - M - w$; and by proceeding

as before, the values of w in the ſuſpending forces are $dn\ (M+N) : (n.Bd + dn) : dn\ (M - N) : (n.Bd - dn)$, and $dn\ (N - M) : (n.Bd + dn,)$ in the firſt, ſecond, and third figures reſpectively.

Becauſe $Bd : dA :: M : N$, therefore if this value of N be ſubſtituted in each of the above expreſſions for the friction, the whole force capable of ſuſtaining the friction and weight M will be had. Thus, for example, the moving force to overcome the friction and weight M in the firſt figure, will be $M\ (n.dA + dn) : (n.Bd - dn,)$ and the ſuſpending force $M\ (n.dA - dn) : (n.Bd + dn;)$ in the ſecond figure, the moving force will be $M\ (n.dA + dn) : (n.Bd + dn,)$ and the ſuſpending force $M\ (n.dA - dn) : (n.Bd - dn;)$ and in the third figure, the moving force will be $M\ (n.dA - dn) : n.Bd - dn,)$ and the ſuſpending force will be $M\ (n.dA + dn) : (n.Bd + dn)$.

The method of finding n from each of the above equations is evident, and conſequently the ratio of the friction to the preſſure by experiments.

OF FRICTION IN THE WEDGE.

Let AC (Fig. 319, Pl. 19.) be the force neceſſary to ſuſtain the wedge QPB in the direction aB perpendicular to QP, friction included; and let AB be the force excluſive of friction: draw AN and AH perpendicular to BQ and BP; CG parallel to AN, and CF parallel to AH: Now GA and AF, the forces of the wood againſt the ſides of the wedge, in thoſe directions, compound a force equivalent to the diagonal CA in the direction CA, and therefore a force repreſented by AC in that direction, muſt be applied to the head of the wedge at a to overcome theſe forces. Let gr be the n part of Ag, and let the lines Ar be drawn, and alſo GK and FZ perpendicular to AG and AF, meeting the lines Ar in K and Z; then will GK and FZ repreſent the friction againſt the ſides BP and BQ, being each the n part of AG and AF, the preſſure againſt each ſide reſpectively; wherefore if Be be taken in PB, and Bn in BQ, equal to GK and FZ reſpectively, the forces Be and Bn in thoſe directions muſt compound a force to which the force BC in the direction BC muſt be equivalent; and conſequently if Bm be the force compounded of Be and Bn, and Cm be joined, Cm muſt be perpendicular to mB; ſince Be or GK is the force of friction ariſing from the preſſure againſt BP, which tends to prevent the wedge from moving either in the direction BP or PB; and Bn or FZ has a ſimilar effect with reſpect to the direction in the line BQ; and by hypotheſis BC is juſt ſufficient to balance theſe forces. It is alſo evident from what was ſaid concerning the inclined plane, that Be and Bn muſt be taken in the directions PB and QB

QB for the moving force, but in the directions BP and BQ for the ſuſpending force.

The method of calculation is evident; for as aB, AG, and AF, are perpendicular to QP, BP, and BQ, the triangles QPB and CAG are ſimilar, and the parallelogram $Bnme$ ſimilar to FAGC; whence by ſuppoſing certain parts given, the reſt may be found, &c.

Corollary. When the wedge is iſoſceles the point m falls on C, and Be is equal to Bn, and therefore Be or GK is equal to (AB + BC) PB) : (n.QP); but PB : Ba :: 2Be : BC, and therefore BC = 2Ba (AB + BC) : (n.QP) or qual to (2Ba.BA) : (n.QP — 2Ba), and therefore AC = (n.QP.AB) : (n.QP — 2Ba); and by following the ſame method for the ſuſpending force, we find BC = (2Ba.AB) : (n.QP + 2Ba,) and conſequently AC is equal to (n.QP.AB) : (n.QP + 2Ba).

SCHOLIUM.

By proceeding in a ſimilar method, the forces of the arch-ſtones of bridges may be determined; for let QbbP be a ſtone ſuſtained by the parts of the arch preſſing againſt Pb and Qb, and let A be its centre of gravity, and AB perpendicular to the horizon; alſo let AB and AC be the ſame as before; then becauſe the body is in equilibrio, the force in direction AC will be equivalent to the force in a contrary direction, ariſing from the preſſures againſt the body in the directions GA and KA, together with the force of friction; and becauſe the preſſures are AG and AK, if Be, (the e part of AC) be drawn parallel to PB; and Bn (the n part of AF) be drawn parallel to Qb; and the parallelogram $Bnre$ be compleated, and Cn joined; Bm will be the force ariſing from friction, and the angle BnC a right angle. The adjacent figure (Fig. 320, Pl. 19.) is for the moving force; but the method is ſimilar for the ſuſpenſive force; and it is evident that the one conſtruction is of uſe to determine the force which tends to break an arch by preſſing it downwards, and the other the force that tends to break it upwards.

But as that excellent mathematician, *P. Friſi*, in his *Inſtituzioni di Meccanica*, has objected to the diviſion of the force AB into the forces AN and AH, and thence concluded *Belidor* and *Couplet* to have been miſtaken on that account in their writings upon bridges; I ſhall, therefore, prove that the common method is really in conſequence of what that gentleman himſelf allows, and that his objections are not well founded. In the firſt place, he allows the force AB to be equivalent to the forces AV and AD or VB; now (excluding friction) if that part of the arch which touches Pb was removed, it is evident QbbP would immediately begin to de-

ſcend

ſecond alone $Q\delta$ with a force repreſented by VB or AD; but this preſſure is counteracted by that part of the arch which touches $P\delta$; and therefore the force of the arch, in the direction HA, muſt be ſuch as to be equivalent to DA in the direction DA or BV; but the parallelogram of forces HA will be equivalent to DA in the direction DA, and therefore HA is the real preſſure or force againſt $P\delta$. Again, HD is the preſſure in a perpendicular direction to $Q\delta$ ariſing from the force AB, therefore AV, together with HD, is the whole preſſure againſt $Q\delta$ in the direction AV; but becauſe the body is in equilibrio, and conſequently the action or force in the direction AV equal to the reſiſtance in a contrary direction; therefore AV+ HD or AN becauſe NV is equal to HD by the property of the parallelogram repreſents the preſſure againſt $Q\delta$, and AH the preſſure againſt $P\delta$; which is contrary to what *P. Friſi* aſſerts, and agreeable to the uſual method.

The ſame learned Author has made another very material miſtake, from a ſimilar cauſe, at page 57 of the aforeſaid Treatiſe, relative to the tenſion of ropes; which cannot be attributed to haſte or inadvertency, as he expreſsly aſſerts the holders of the common opinion to be miſtaken, in conſequence of their uſing the theory of compoſition of forces without ſufficient precaution: I ſhall, therefore, after giving his own words, take the liberty of ſhewing where I apprehend he is miſtaken.

" Parleremo più a lungo delle altre richerche matematiche, alle quali ha dato occaſione la controverſia inſorta intorno alla cupola di S. Pietro. Coll' occaſione che ſi è diſcorſo in Milano di munire la Fabbrica del Duomo di un Conduttore elettrico, che dalla cima dell' aguglia ſi diriga diſ. e ſcendelle per differenti parti del tetto, ſi è ancora parlato dell' azione, che i fili del Conduttore potrebbero eſercitare contra l'aguglia, e ſi ſono propoſti varj Problemi intorno alle tenſioni delle ſteſſe. Io qui aggiugnerò le ſoluzioni, che ho ritrovati, e incominciero palla prima riſoluzione delle forze tendenti, la quale ſiccome è interamente differente da quella che hanno ſeguitato altri Autori, così non ſarà meravaglia che porti dei riſultati interamente differenti da quelli che ſono ſtati ſinora publicati. Pendan (Fig. 9 e 11. Pl. 10,) filo QVR, dai punti Q, *ed* R, e vi ſi attacchi in V il peſo P. Si produca la verticale PV in A; ſi eſprima il peſo P colla forza AV, e dal punto A; ſi tirino ſopra QV, RV le perpendicolari AM, AN. Sara MV l'intera forza eſercitata ſecondo QV, ed NV ſará quella che ſi eſercitera ſecondo RV.

" La ſteſſa coſa ſi dedurrebbe riſolvedo la forza AV nelle due A*q*, A*r* paralliele ai fili QV, RV, e poi riſolvendo di nuovo la forza A*q* nelle due AN, N*q*, e ſimilmente la A*r* in due altre AM,

Mr.

Mr. Mentre conquelle riſoluzioni é manifeſto che la forza totale ecitata nel tendere il filo QV dev' eſſere $Aq - Mr = rV - Mr = MV$, e la tenſione del fizo $RV = Vb - Nq = NV$.

" S'ingannerebbe chi miſuraſſe ſeparatamente la tenſione del filo QV dalla forza Aq, oſſia rV, e la tenſione di RV da Ar, oppure da qV Egli è vero, che le due tenſioni equivalgono inſieme, come alla ſua forza AV, così ancora alle due Ar, Aq, oppure alle quattro inſieme AN, Nq, AM, Mr. ma nel prendere le tenſioni ſeparate bisogna in oltre avvertire, che quando l' angolo QVR non è retto, una porzione di Aq agiſce ſecondo RV, ed una porzione di Ar ſecondo QV: e ſeparando le azioni ſara MV, la tenſione del filo QV, ed NV quella di RV."

In the firſt place, I ſhall demonſtrate the truth of the eſtabliſhed method from principles that FRISI has himſelf allowed; and, ſecondly, point out the abſurdity of his concluſions.

1. Let Vn and Sr be parallel to AN; then becauſe NVn is a right angle, and the force VA may be reſolved into VN and Vn, in thoſe directions, therefore, if RV and VP were to remain in the ſame poſition, and the force which now keeps the body ſuſpended by acting in the direction VQ, was to act in the direction Vn with a force expreſſed by Vn, it is then granted that the equilibrium would ſtill be maintained, and the tenſions would be as Vn and VN; and therefore as no force VS whatever acting at V in the direction RV, can have any effect in the direction Vn perpendicular to RV, it neceſſarily follows, that the force in any other direction VQ muſt be ſuch as to be equivalent to Vn in the direction Vn; but it is likewiſe granted, that no other force but Vr in the direction VQ can be equivalent to Vn in the direction Vn; and as the force Vr is equivalent to Vn and VS, and as VS, or its equal, qN, only gives an additional tenſion to NV, the tenſion which the chord RV was ſuppoſed to have before, which whole tenſion is equal to the reaction of the tack R; therefore qV is the tenſion of the chord RV, and Vr that of Qv.

2. Let the points Q and R coincide, and RV, QV, and VP, will then be perpendicular to the horizon; and if VQ or VR be aſſumed to expreſs the weight P, then will the point A, R, Q, M and N coincide: and according to *Friſi's* principle, the tenſions of RV, VQ, and VP, will be equal; but, from the well known principle of the pulley, each chord VQ and VR bears but half the weight P, and therefore this abſurdity follows, that a chord is as much ſtretched with half the weight as it would with the whole.

Again, if the points R, V and Q be ſuppoſed horizontal, it follows, from the common theory, that the tenſion of the rope RVQ would be infinite; but VN and VM vaniſh when RVQ is horizontal; and therefore, by *Friſi's* principle, the tenſion in that caſe

eaſe would be nothing at all; but it is well known from the moſt common experiments to be very conſiderable, even when RQV is but nearly horizontal; and therefore the new theory of this great mathematician is indefenſible.

Remark. All the foregoing, except the laſt Scholium, was written in 1775, before the author had ſeen any thing to ſpeak of on the ſubject. He had deſigned and executed great part of an extenſive treatiſe on friction, according to different hypotheſes; but as no body would be at the riſk of publiſhing it, and he could not afford it himſelf, the moſt of it was accidentally loſt. What is here given is an extract only of ſome of the firſt part, where velocity was not taken into the account, and where there were no complicated algebraic or fluxional expreſſions, which would be difficult to print in this country.

N. B. This curious paper is taken from the firſt vol. of Aſiatic Reſearches.

29

ARTICLE XXVIII.

FOUR PROPOSITIONS, &c.

SHEWING NOT ONLY THAT THE DISTANCE OF THE

SUN,

As attempted to be determined from

THE THEORY OF GRAVITY,

BY A LATE AUTHOR,

Is, upon his Own Principles, Erroneous, but alſo, that it is more than probable

THIS IMPORTANT QUESTION

CAN NEVER BE SATISFACTORILY ANSWERED BY ANY CALCULUS OF THE KIND.

SEVERAL of our Readers, it may be preſumed, will recognize the following Propoſitions, and the circumſtances from which they originated; as there are others who will not, we ſubjoin, for their information, the following particulars.

In the year 1761, Dr. MATTHEW STEWART, Profeſſor of Mathematics in the Univerſity of Edinburgh, publiſhed a Volume of

of Tracts, *Mathematical and Phyſical*, in which he propoſed to determine the Sun's diſtance from the Earth, by a calculation founded on the Theory of Gravity. Two years afterwards the calculation appeared, in his Eſſay on *The Sun's Diſtance*, and the reſult ſo far exceeded all former computations, as to produce attentive enquiries into the principles he had adopted. His method though highly ingenious, and reflecting honour on his Talents, was at length found to be liable to many inconveniences, and inadequate to the purpoſe. To point out ſome inſtances of its failure, theſe PROPOSITIONS were publiſhed, anonymouſly, in 1768, by Mr. DAWSON, then Surveyor, at Sudbury, in Yorkſhire, in which his candour and abilities are equally conſpicuous. The Work is now ſo ſcarce as to be almoſt unattainable; and, as the ſubject ſtill retains its importance, we are happy in being able, through the medium of ſome Mathematical Friends, and the permiſſion of the Author, to lay its contents before our Readers.

PROPOSITION I. *Fig.* 278, *Pl.* 18.

Suppoſing the orbit of the moon to be a circle, and to coincide with the plane of the ecliptic; it is required to determine the forces with which the ſun diſturbs the motion of the moon round the earth.

Let M repreſent the moon in its orbit ABCD, E the earth, and S the ſun. Draw SM and produce it to Q, ſo that SE may be to SQ as SM^2 to SE^2. Then, if SE repreſent the gravity of the earth to the ſun, SQ will repreſent the gravity of the moon to the ſun when at M. From the point Q draw QN parallel to ME, meeting the line SE produced in N. Likewiſe draw ML parallel to EN, LE from the point L to the centre E, and LO perpendicular to ME. The force of the moon to the ſun, repreſented by QS, may be reſolved into the two forces QN, NS, and in thoſe directions. But the line SE repreſents the force of the earth to the ſun, which if taken out of the line SN leaves NE. Hence QN and NE are the two forces which diſturb the moon

in

in its progreſs round the earth; the joint effect of which in increaſing or decreaſing the gravity of the moon towards the earth in any part of its orbit, as M, is what we are here enquiring after.

From M draw MG perpendicular to EC, and put SE $= a$, CE or ME $= r$, EG or MF $= y$; then, from the above proportion, SQ $= \frac{SE^2 \times SE}{SM^2} =$ (by ſubſtitution) $\frac{a^3}{a^2 - 2ay + r^2}$.

From SQ take SM $= \sqrt{a^2 - 2ay + r^2} = a - y$ nearly, and there remains MQ $= \frac{a^3}{a^2 - 2ay + r^2} - a - y = \frac{3a^2y - 2ay^2 - ar^2 + r^2y}{a^2 - 2ay^2 + r^2}$. If r^2y be rejected in the numerator, and r^2 in the denominator, the value of the fraction will be very near the ſame as before, (both numerator and denominator being decreaſed at the ſame time, and the quantities rejected being much leſs than the reſt) that is, MQ will then be $= \frac{3ay - 2y^2 - r^2}{a - 2y}$ nearly; which expreſſion let us put $= z$,

The triangles SME, MQL, are ſimilar; hence SM $= \sqrt{a^2 - ay + r^2} = a - y$: SE $= a$ :: MQ $= z$: ML $= \frac{az}{a - y}$; and SE $= a$: ME $= r$ :: ML $= \frac{az}{a - y}$: QL $= \frac{rz}{a - y}$. That part of the diſturbing force repreſented by QL acts in the direction QL or ME from M towards E, and conſequently increaſes the gravity of the moon towards the earth.—The MQ may be reſolved into the two forces QL and

and LM, and in thoſe directions. QL acts directly towards the earth, and therefore need not be reſolved into any other forces. The force ML and the remaining part LN are reduced into LE, and in that direction. This, again, is reſolved into LO, OE, and in theſe directions. Now the force LO is exactly balanced when the moon is at a point equidiſtant from C on the other ſide of its orbit, and therefore, taking a whole revolution together, does not diſturb its motion round the earth. The force OE therefore is what we are to eſtimate.

From the ſimilar triangles LOM, EFM, we have

$MO = \frac{ML \times MF}{ME} = \frac{ayz}{r \times a - y}$, therefore, OE

$= \frac{ayz}{r \times a - y} - r =$ (by ſubſtituting for z its value)

$\frac{3a^2y^2 - 2ay^3 - ayr^2}{r \times a^2 - 3ay + 2y^2} - r =$ (by reduction) to

$\frac{3a^2y^2 - 2ay^3 - a^2r^2 + 2ar^2y - 2r^2y^2}{r \times a^2 - 3ay + 2y^2}$.

This force is in the direction EM, and conſequently leſſens the gravity of the moon to the earth. Hence

if the force $QL = \frac{rz}{a - y} =$ (by ſubſtitution)

$\frac{3r^2ay - 2r^2y^2 - r^4}{r \times a^2 - 3ay + 2y^2}$ be taken from the above, there

will then remain $\frac{3a^2y^2 - a^2r^2 - 2ay^3 - r^2ay + r^4}{r \times a^2 - 3ay + 2y^2}$;

an expreſſion for the force that muſt be deducted from the gravity of the moon to the earth, in order to obtain the true force with which the moon is im-

 pelled

pelled towards the earth when at M. If r^4 in
merator, and $2y^2$ in the denominator be n
and a ſtruck out of every term, v

$\frac{3ay^2 - ar^2 - 2y^3 - r^2y}{r \times a - 3y}$ = the true diſturbi

nearly. (For beſides the terms neglected
but a very ſmall proportion to the reſt, botl
rator and denominator are diminiſhed at t
time, conſequently the value of the fraction
be very little altered). Again, if $2y^3$ and r^2y
wiſe neglected, the expreſſion for the di

force becomes $\frac{3ay^2 - ar^2}{r \times a - 3y}$; which, though
curate than the above, is much more ſimp
turns out the very ſame that profeſſor *Stew*
found prop. 6th, tract 4th, of his Tracts Phyſ
Mathematical, as will eaſily appear, by intr
our ſymbols into his concluſions, &c.

Corollary I.

When the moon comes to K, the other en
diameter MK, the diſturbing force will

$\frac{3ay^2 - ar^2 + 2y^3 + r^2y}{r \times a + 3y}$; y being here nega

Hence half the ſum of theſe two forces w
mean ſolar one, and $= \frac{3a^2y^2 - a^2r^2 - 6y^4 -}{r \times a^2 - 9y^2}$

And if Dr. *Stewart's* expreſſion, found abc

r

ceed in like manner, we ſhall have $\frac{a^2}{r} \times$ $\frac{\quad - r^2}{\quad - 9y^2}$, for the mean force; which is the ſame as he has determined prop. 8th of the before-mentioned tract.

COROLLARY II.

The proportion between the mean ſolar force determined above, and the force of the moon to the is eaſily determined. Let p repreſent the odic time of the moon round the earth, P the periodic time of the earth about the ſun; then, the of the earth to the ſun, is to the mean ſolar. is to $\frac{8a^2y^2 - a^2r^2 - 6y^4 - 3r^2y^2}{r \times a^2 - 9y^2}$. And the laws of central forces) the force of the moon the earth, is to the force of the earth to the ſun, $\frac{r}{p^2}$ to $\frac{a}{P^2}$: Hence the force of the moon to the th, is to the mean ſolar force, as $\frac{r}{p^2}$ is to $\frac{1}{P^2} \times$ $\frac{8y^2 - a^2r^2 - 6y^4 - 3r^2y^2}{r \times a^2 - 9y^2}$, or, as r to $\frac{p^2}{P^2} \times$ $\frac{8y^2 - a^2r^2 - 6y^4 - 3r^2y^2}{r \times a^2 - 9y^2}$.

PROPOSITION II. *Fig.* 278, *Pl.* 18.

required to determine the proportion of the variation of the ſun's diſtance from the earth, to a corre-

ponding

ponding variation of the disturbing force of the s upon the moon at any point M in its orbit.

It is proved by the writers on this part of physic astronomy, that the distance of the moon from t earth increases and decreases as the distance of t sun decreases and increases; but an indefinite small variation of the sun's distance, will scarce all effect the distance of the moon. Therefore, in t expression for the disturbing force, viz.* $\frac{\rho^2}{P^2}$ $\frac{3ay^2 - ar^2 - 2y^3 - r^2y}{r \times a - 3y}$, r may be looked upon as a constant quantity, and consequently, y will likewi be given for any given point M in its orbit. T therefore, the expression for the disturbing force t thrown into fluxions, supposing only a variable, w shall have the variation of the disturbing force = $\frac{\rho^2}{P^2} \times \frac{\dot{a} \times -y}{a-3y^2} \times \frac{7y^2 - 4r^2}{r}$: But the variation the sun's distance is $\dot{a}$; hence the variation of t disturbing force is to the variation of the sun's d tance, as $\frac{\rho^2}{P^2} \times -y \times \frac{7y^2 - 4r^2}{r}$ to $\overline{a-3y}^2$.

If $\frac{\rho^2}{P^2} \times \frac{n}{r \times a - 3y}$ be neglected from the disturb ing force, n being supposed an indefinitely small neg tive or positive quantity) the variation in the sun distance, occasioned thereby, will be found by th proportion, viz. as $\frac{\rho^2}{P^2} \times -y \times \overline{7y^2 - 4r^2}$ is to r $\overline{a-3y}$

* This will easily appear from Cor. 2. last Prop.

$\overline{a-3y}^2$, ſo is $\frac{p^2}{P^2} \times \frac{n}{r \times a-3y}$ to $\overline{a-3y} \times$

$\frac{n}{-y \times 7y^2-4r^2} =$ the variation in the ſun's diſtance, conſequent upon neglecting an indefinitely ſmall quantity (n) in the numerator of the fraction, expreſſing the diſturbing force.

Corollary I.

For any given point in the orbit, the variation in the ſun's diſtance is as $\overline{a-3y} \times n$; that is, as the ſun's diſtance multiplied into the part neglected in the numerator, nearly. This is evident from what is gone before. If n be ſuppoſed conſtant, the variation will then be as $\overline{a-3y}$, that is, directly as the diſtance, nearly; for $3y$ is very ſmall compared with a.

Corollary II.

Let us now ſuppoſe $\frac{p^2}{P^2} \times n$ to be any quantity neglected from the expreſſion for the diſturbing force; then proportioning, as above, there comes out

$\overline{a-3y}^2 \times \frac{rn}{-y \times 7y^2-4r^2}$ for a correſponding variation of the ſun's diſtance; which is as the quantity neglected multiplied into the ſquare of the diſtance, nearly; but if n be conſtant, it is as the ſquare of the diſtance directly; for $3y$, as was obſerved before, bears only a very ſmall proportion to a.

SCHOLIUM.

From the laſt propoſition and its corollaries it will plainly appear, that the variation of the diſturbing force is extremely ſmall, when compared with a correſponding variation of the ſun's diſtance, and this ſtill leſs as the diſtance of the ſun is greater; *a concluſion directly contrary to the opinion of Dr.* STEWART. For, in the preface to Tracts Phyſical and Mathematical, we have the following paragraph: "It is well known that the various methods hitherto "attempted to ſolve this curious problem, have failed "in a great meaſure, on account of the vaſt diſtance "of the ſun from the earth; but the method hinted "at here, will give the ſolution the more accurately, "the more diſtant the ſun is from the earth; for it "proceeds on the ſuppoſition that the diſtance of the "ſun from the earth is great."

Upon a ſlight view of the matter this aſſertion looks probable. For, if any quantity (n) be neglected in the numerator of the fraction expreſſing the diſturbing force, its value in the fraction is as $\frac{n}{r \times a - 3y}$, which is evidently leſs as a, or the diſtance of the ſun from the earth, is greater: And, therefore, it would ſeem to follow, that it would leſs affect the diſtance of the ſun, determined by this method. But this, as we have found above, is juſt the reverſe. And here let us conſider, whether the ſingle truth juſt demonſtrated, does not of itſelf ſufficiently prove that the diſtance of the ſun can ſcarcely ever be truly aſcertained by the *theory of gravity*.

PROPOSITION III. *Fig.* 323, *Pl.* 20.

The mean ſolar force for any point of the orbit, as M, being

being $\frac{p^2}{P^2} \times \frac{3a^2y^2 - a^2r^2 - 6y^4 - 3r^2y^2}{r \times a^2 - 9y^2}$, (*cor.* 2, *prop.* 1); *it is required to find the ſum of all the mean forces for a quadrant of the orbit* BC.

Draw the line *bc*, equal to the arch BC, and let *m*, in the line, correſpond to M, in the arch. At *m* erect a perpendicular, and therein take *mr* equal to the mean diſturbing force at M $= \frac{p^2}{P^2} \times \frac{3a^2y^2 - a^2r^2 - 6y^4 - 3r^2y^2}{r \times a^2 - 9y^2}$. Let this be done for every point of the line *bc*, and draw the curve *qrsno* bounding theſe perpendiculars.

From this conſtruction it may eaſily be ſeen, that the curve will cut the axis *bc* in ſome as *n*, that is, when $3a^2y^2 = a^2r^2 + 6y^2 + 3r^2y^2$; for then the ordinate $mr = 0$. From *n* to *b* the ordinates are all negative, and from *n* to *c* all poſitive.

Suppoſe *y* to be a flowing quantity, and all the reſt conſtant, the fluxion of the arch BM, or line $bm = \frac{r\dot{y}}{\sqrt{r^2 - y^2}} = \frac{\dot{y}}{\sqrt{1 - y^2}}$, (putting $r = 1$); then $\frac{\dot{y}}{\sqrt{1 - y^2}} \times \frac{p^2}{P^2} \times \frac{3a^2y^2 - a^2 - 6y^4 - 3y^2}{1 \times a^2 - 9y^2} =$ fluxion of the area of the curve lined ſpace *obcq*. The expreſſion for the ordinate being reduced into a ſeries, we have

$$nr = \frac{p^2}{P^2} \times \frac{}{-1 + \frac{3a^2 - 12}{a^2} y^2 + \frac{21a^2 - 108}{a^4} y^4 + \frac{189}{a^4} y^6 \text{ \&c.}}$$

Therefore,

Therefore, the fluxion of the ſpace $= \frac{\dot{y}}{\sqrt{1-y^2}} \times$

$$\frac{p^2}{P^2} \times -1 + \frac{3a^2-12}{a^2} y^2 + \frac{21a^2-108}{a^4} y^4 + \frac{189}{a^4} y^6 \text{ &c.}$$

and the fluent, when $y = 1$, becomes $\frac{p^2}{P^2} \times$

$$-c + \frac{c}{2} \times \frac{3a^2-12}{a^2} + \frac{3c}{8} \times \frac{21a^2-108}{a^4} + \frac{5c}{16} \times \frac{189}{a^4} \text{ &c.}$$

c being put $=$ line bc or arch $BC = \frac{3.14159}{2}$. This expreſſion is equal to the difference of the two areas obn, and cnq, or $= cnq - obn$; that is, equal the ſum of the poſitive forces, that have acted upon the body in paſſing from B to C, made leſs by the negative forces.

Corollary I.

Draw the line tp parallel to bc, and at ſuch a diſtance from it, that the area $tbcp$ may be $=$ the area $cnq - obn$; then will tb or $cp = \frac{p^2}{P^2} \times$

$$-1 + \frac{1}{2} \times \frac{3a^2-12}{a^2} + \frac{3}{8} \times \frac{21a^2-108}{a^4} + \frac{5}{16} \times \frac{189}{a^4} \text{ &c.}$$

For the baſe c, multiplied into this perpendicular, gives the area of the parallelogram $tbcp = \frac{p^2}{P^2} \times$

$$-c + \frac{c}{2} \times \frac{3a^2-12}{a^2} \text{ &c.} = \text{the curve lined area}$$

found

found above. This perpendicular is therefore a *mean ſolar force*, which, when reduced, becomes $=$ $\frac{p^2}{P^2} \times \frac{8a^4+30a^2+297}{16a^4}$, r being all along ſuppoſed equal 1.

Corollary II.

If the mean diſturbing force for the point M, made uſe of by *Profeſſor Stewart*, be taken equal the ordinate mr, and the curve lined area, found and reduced in the ſame manner as above, we ſhall have the mean ſolar force $= \frac{p^2}{P^2} \times \frac{8a^4+90a^2+729}{16a^4}$.

Scholium.

As Dr. *Stewart* makes uſe of the mean ſolar force found above, for determining the ſun's diſtance, it will not be amiſs to examine what proportion a ſmall variation in the ſun's diſtance bears to a correſponding one in the mean ſolar force; and likewiſe, whether a mean force can properly be made uſe of in the enquiry at all, or not.

Let the variation of the ſun's diſtance be repreſented by $\dot{a}$, then the variation of the mean ſolar force, viz. $\frac{p^2}{P^2} \times \frac{8a^4+30a^2+297}{16a^4}$, will be found, when thrown into fluxions, $= \frac{p^2}{P^2} \times \frac{-15a^2\dot{a}-297\dot{a}}{4a^5}$.

Therefore, the variation of the ſun's diſtance is to the variation of the mean ſolar force, as $4a^5 \times P^2$ to $p^2 \times -15a^2+297$, or (by neglecting 297) as $4a^3$

$4a^3 \times P^2$ to $-15p^2$. Hence, it is plain that, when the mean ſolar force increaſes, the diſtance of the ſun decreaſes, and *e contra*, and that in proportion as the cube of the diſtance directly.

Here again we cannot help obſerving, that the greater the value of a (or the diſtance of the ſun) is, the leſs accurate will this method of calculation be. For, ſuppoſing the variation of the mean diſturbing force to be conſtant, the variation of the ſun's diſtance will be as the cube of the diſtance, very nearly. It will likewiſe evidently appear, that, to get the ſun's diſtance to any tolerable exactneſs, the mean diſturbing force muſt be perfectly true; for the variation in this is to a correſponding variation in the ſun's diſtance, as 1 to $\frac{4a^3P^2}{15p^2}$. Taking a at the loweſt, ſuppoſe only equal to 300 times the moon's diſtance, then the proportion of the variation juſt mentioned, is as 1 to $\frac{4 \times 27000000}{15} \times \frac{p^2}{P^2}$, or $7200000 \times 178{\cdot}725 \left(\frac{p^2}{P^2} \text{ being} = 178{\cdot}725.\right)$ It will likewiſe eaſily appear, that the mean diſturbing force bears a much greater proportion to an indefinitely ſmall variation therein, than the diſtance of the ſun to a correſponding variation in the diſtance; that is, $\frac{p^2}{P^2} \times \frac{8a^4+30a^2+297}{16a^4}$ bears a greater proportion to $\frac{p^2}{P^2} \times \frac{-15a^2\dot{a}-297\dot{a}}{4a^5}$, than a to $\dot{a}$. For, dividing the antecedents by the conſequents, the meaſures of the

the two ratios are $\frac{a}{\div 4a} \times \frac{8a^4+30a^2+297}{-15a^2-297}$ and $\frac{a}{\div a}$;

which are to each other as $8a^4 + 30a^2 + 297$ to $-4 \times 15a^2 + 297$; or (by neglecting 297) as $2a^2 + 7\frac{1}{2}$ (or $2a^2$, nearly) to -15. It will therefore by no means appear ſtrange, that the diſtance of the ſun, found by making uſe of the mean force, according to Dr. *Stewart*, ſhould be much different from that which comes out from making uſe of the true mean force, as will appear farther on.

It will be neceſſary here to remark, that we call the mean force, determined above, the true *mean force*, only to diſtinguiſh it from Dr. *Stewart's* ; for though it be much more accurate than his, there are too many quantities neglected to make it deſerving of its title, otherwiſe than by way of compariſon.*

* Upon the republication of this ſmall pamphlet, the author wiſhes to obſerve, that he was a very young mathematician at the time it was firſt drawn up, in conſequence of which, ſome inaccuracies may be found, which a longer experience would probably have prevented. But they are not of that nature or magnitude as any way to affect the truth of the concluſions thence deduced. He was *then* fully ſenſible that the ſeveral algebraical expreſſions for the various forces there inveſtigated, were not correct, but that they were nearer the truth than Doctor Stewart's, which was all that was neceſſary to prove what the pamphlet aſſerted.

To correct the above-mentioned inaccuracies, beſides the labour it would require, would now be wholly uſeleſs; as the author believes it is *generally* allowed, ſince the publication of the *Four Propoſitions*, and particularly of Mr. Landen's *Animadverſions*, that the diſtance of the ſun will ſcarcely ever be determined by the Theory of Gravity.

The intention of the writer of this pamphlet was, not only to prove what has juſt been mentioned, but likewiſe to point out *how* an error of ſuch magnitude originated, and *how* it ſhould eſcape the notice of ſo exquiſite a Geometrician as the late Doctor Stewart: in *this* he hopes he has completely ſucceeded.

But

But to proceed with another objection. Since the perpendicular *cp* (fig. 323, pl. 20) is plainly an arithmetical mean among all the perpendiculars which may be conceived erected upon the line *bc*, the mean disturbing force, which it represents, must be an *arithmetical mean* among all the forces, which have acted in the description of the quadrant. Now, were the motion of the apses directly in the simple ratio of the disturbing force, then if any number of disturbing forces, all different from one another, were to act each an equal time, it is evident they would occasion the same motion in the apses, as if a force, which was an arithmetical mean among these forces, were to act for the whole time. But it will presently be seen, that the motion of the apses is not in the direct simple ratio of the disturbing force: hence it must be manifest, that a mean solar force, of the nature of the above, can never rightly answer the purpose for which it is intended, were it ever so accurately determined.

(To be continued.)

ARTICLE XXXI.

USEFUL PROPOSITIONS IN GEOMETRY.

By *Mr.* M. A. HARRISON.

(Continued from page 25.)

PROP. XIII. THEO. Fig. 257, Pl. 17.

IF RS be produced to meet the circle at *m*, and points C,*m*; F,*m*; be joined; C*m*, will pass the centre O, and F*m*, will cut IB at right an L.

Demon.

Demon. Since the points C, R, *m*, are in the periphery, and the angle C R *m*, a right one, it is evident that C *m* is the diameter, and conſequently paſſes through the centre O.

Again, ſince the arch C F is equal to the arch E *m*; and the points E, and F, being in the periphery, the angles F E C, E F *m*, ſtanding on equal arches, will be equal, and therefore F *m* will be parallel to E C; but it has been ſhewn that C E is perpendicular to I B, therefore F *m* is perpendicular to I B.

Q. E. D.

PROP. XIV. THEO.

Let K′ be the point where a perpendicular to L T, from L, meets T U. Then if L′ K′ be joined, it will paſs through L, the middle of the baſe.

Demon. Join L′L, K′L, and let FH, drawn ⊥ to AC, meet CE in V, and join LH. It has been ſhewn,

that FI, PH; and FC, IG, are reſpectively = and paral.
therefore FH is parallel and equal to IP;
hence LT or PG is equal to HC.
Now, ſince LT is equal and parallel to HC, the lines TC, LH, (joining their extremes) will be = and ∥:
but TC, L′F, are equal and parallel;
therefore LH, L′F, are equal and parallel,
and therefore HF, L′L, are equal and parallel.
Hence L′L is perpendicular to LT, that is, the points L′, L, K′, are in the ſame ſtraight line. *Q. E. D.*

Cor. The triangles, FCV, L′TK′; FCH, L′TL; VCH, K′TL, are reſpectively equal in all reſpects.

PROP. XV. THEO.

Lines being drawn as before, FC (= ER = E*m*,) will be equal to the diameter of the circle paſſing through the points L, T, D.

Demon. Produce L′K′ to meet the circle ULTD in T′, and join TT′. Since TLT′ is a right angle, and the points T, L, T′, in the periphery, it is evident that TT′ is the diameter of the circle, whose centre is *o*.

Again, since L*o* is ‖ to L′T, the ∠s T′L*o*, T′L′T will be equal: but L*o* is = *o*T′; hence, the ∠ LT′*o* = T′L*o* = T′LT; therefore the △ T′L′T is isosceles, and conseq. L′T = TT′: but L′T = FC; therefore FC = TT′, the diameter.

Q. E. D.

Otherwise. Because *o*L is parallel to TL′,

T′*o* : T′T :: *o*L : TL′:

but the antecedents T′*o*, *o*L are equal;

therefore the consequents T′T, TL′ are equal:

but TL′ = FC; therefore T′T = FC. *Q. E. D.*

Cor. 1. L′L = LT′, and L′K′ — K′T′ = 2 LK′.

Cor. 2. T′L, produced, will meet AC at right ∠s in P.

PROP. XVI. THEO.

Things remaining as before, I say the perpendicular *on*, will be equal to half NF, or half SE.

Demon. L*o* will evidently meet the circle in the point C′, where the circle cuts CDR.

Since LC′ (TT′) = FC, and LD = NC; DC′ will be = NF, because the angles at N and D are right ones:

but L*n* = *n*D, and *no* is parallel to DC′;

therefore, *no* = ½ DC′ = ½ NF, or ½ SE. *Q. E. D.*

Cor. 1. Join D*o*, and produce it to meet the circle in A′, and the points L, A′, E, will be in a right line.

Cor. 2. LA′ is equal to DC′.

PROP. XVII. THEO.

Things remaining still as before, I say the triangles ABC, TDU, will be to one another in the constant FE to FN.

Demon.

Demon. Since the △s ACB, TDU, are naturally equiangular, their areas will be to one another as the ſquares of their homologus ſides, that is, as △ ACB : △ TDU :: $AB^2 : TU^2$ :: $LB^2 : TQ^2$ (QU^2) :: EB^2 (EF·EL) : LT^2 (ES·EL=NF·EL, by Prop. IV.) :: FE : FN. *Q. E. D.*

Cor. When EF is a given line, and the △ TDU, either a given ſpace, or a maximum, the ſolid FNL·LB will either be a given ſolid, or a maximum.

ARTICLE XXXII.

ATWOOD's INVESTIGATIONS ON WATCH BALANCES.

(Continued from page 136.)

BUT if the points of quieſcence of the balance and auxiliary ſprings, inſtead of coinciding, according to the principle of Mr. MUDGE's conſtruction, ſhould deviate from this adjuſtment by a ſmall arc OQ=ON (fig. 82 and 83, pl. 5.); to what extent the daily rate of the time-keeper may be affected by this alteration in the poſition of the quieſcent points, remains unknown, unleſs it be inveſtigated by aſſuming the deviation of the points of quieſcence, as one of the conditions on which the time of vibration depends. This condition is included in the preceding inveſtigations, which may be now applied to the ſolution of ſome caſes, which are ſuggeſted by conſidering the conſtruction of Mr. MUDGE's time-keeper.

Any minute or particular deſcription of this ingenious invention would be foreign to the ſubject of theoretic inveſtigation; an outline only of the conſtruction on which the action of the ſeveral ſprings

depends will be neceſſary, to render the applicati
of the preceding theorems ſufficiently intelligible.

ONEBQ (fig. 86, pl. 6.) is the circumference
the balance, vibrating by the action of a ſpiral ſpr
on an axis CADH, paſſing through the centre C; t
axis is diſcontinued from A to D to make room
the other parts of the work. CA and DH are c
nected by means of a branch or crank AXYD, wh
is fixed to the axis CADH, and always vibrates w
the balance on the ſaid axis.*

LM, ZW, are two rods affixed to the crank at
points L and Z parallel to XY; which rods alſo
brate with the balance. *c*, *d*, *e*, *f*, *g*, *r*, *s*, are fi
parts of the machine. TR is an axis in the ſame ri
line with CADH carrying an arm GO at right ang
to it † (or nearly ſo), and a ſmall auxiliary ſpring
which is wound up whenever the arm GO is turn
round the axis TR in the direction of the arc O*h*;
is a curved pallet fixed to the axis TR, which receiv
the tooth of the balance-wheel near the axis; t
tooth proceeding along the curved ſurface by t
force of the main ſpring, turns the axis and the a
nexed arm GO in the direction of the arc O*h*, and
the ſame time winds up the auxiliary ſpring *u*.
ſmall projection at the extremity of the curved ſu
face of the pallet *p* prevents further progreſs of t
tooth, when the arm OG has been turned throu
an arc O*h* ‡ of about 27°; conſequently the ſpring
has then been wound through the ſame arc or ang
OG*h* = 27°.

FS is another axis in the ſame right line w

* The additional weight affixed to the balance at *t* counterpo
the weight of the crank or branch AXYD, ſo as to bring the cer
of gravity of the whole into the axis of motion.

† In the machine GO and IO are not exactly at right angle
TR and FS; but are ſo repreſented in the figure, in order to m
the different poſitions of the arms GO and IO the more diſtinct

‡ *Vide infra* the note in page 240.

CAD

CADH, exactly ſimilar to that which has been deſcribed. This axis FS carries with it the arm IO; the auxiliary ſpring *v* is annexed to this axis, and is wound up when the axis is turned in the direction of the arc O*k*. *q* is a pallet ſimilar to the former pallet *p*, and is placed ſo as to receive the tooth of the balance-wheel, which by its action on the pallet winds up the ſpring *v*, and carries the arm OI through an angle O*Ik*, = 27°, further motion being prevented by a ſmall projection at the extremity of the pallet *q*, ſimilar to that which has been already mentioned. *lm* repreſents the balance-wheel, the upper tooth of which acts on the pallet *p*, and the lower tooth on the pallet *q*, alternately winding up the auxiliary ſprings *u* and *v*, in the manner deſcribed in the ſubſequent page: the axis of the balance-wheel *no* is parallel to the line CO or GO.

The ſeveral arcs expreſſed on the circumference of the balance, *i. e.* OQ, O*h*, ON, O*k*, are equal to the reſpective arcs denoted by the ſame letters in the circumference of the circles deſcribed by the extremities of the arms GO and IO.

Suppoſe that, when the balance is quieſcent, the main-ſpring being unwound, the branch or crank AXYD is in the poſition repreſented in fig. 86, AX being parallel to CO; if the quieſcent points of the auxiliary ſprings coincide with that of the balance ſpring, the arm GO will juſt touch the rod LM; and in like manner the arm IO will juſt touch the rod WZ; the two arms GO and IO, in this poſition, are parallel to the line CO. This poſition of the balance and auxiliary ſprings remains as long as the main-ſpring of the machine continues unwound; but whenever the action of the main-ſpring ſets the balance-wheel in motion, a tooth thereof meeting with one or other of the pallets *p* or *q*, will wind up one of the auxiliary ſprings; ſuppoſe it ſhould be the auxiliary ſpring *u*: the arm GO being carried into the poſition

G*h*, by the force of the balance-wheel a
pallet *p*, remains in that position as long
of the balance-wheel continues locked by
tion at the extremity of the pallet *p*: and
itself, not being at all affected by the m
arm GO, nor by the winding up of t
remains in its quiescent position; conse
vibration can take place except by the
some external force to set the machine
Suppose an impulse to be given to the b
cient to carry it through the semi-arc O
about * 135°, according to Mr. MU
struction.

The balance during this motion carrie
erank AXYD, and the affixed rods LM.
the balance has described an angle of abo
the angle OC*h* or OG*h*, the rod LM m
arm G*h*, and by turning the axis TR, a
p, in the direction of the arc O*h*, releases
the balance-wheel from the projection a
mity of the pallet *p*; the balance-wheel
revolves, and the lower tooth meeting wi
q, winds up the auxiliary spring *v*, and
arm IO with a circular motion throug
OI*h* about 27°; in which position the
mains as long as the tooth of the balan
locked by the pallet *q*. While the sprin
ing up through the arc O*h*, the balance
remaining part of the semi-arc *h*B, and
motion the rod LM carries round the arr
ing it to describe an angle *h*CB, or *h*GB
measured by the arc *h*B = ‡ 108°; when

* *Vide infra* the note in page 240.

† The angle *h*GB is not expressed in the figure fo
but is easily imagined, being precisely equal to the
presented on the balance.

‡ This magnitude of the arc *h*B has been inferre
lowing circumstances, communicated by Mr. MUD

has arrived at the extremity of the ſemi-arc OB = 135°, the auxiliary ſpring *u* will have been wound up through the ſame angle = 135°, *i. e.* 27°, by the force of the main-ſpring acting on the pallet *p*, and 108° by the balance itſelf carrying along with it the arm GO, or G*k*, while it deſcribes the arc *k*B. The balance thererfore returns through the arc BO by the joint action of the balance ſpring, and the auxiliary ſpring *u*; the acceleration of both ſprings ceaſing the inſtant the balance arrives at the quieſcent point O; when the balance has proceeded in its vibration about 27° beyond the point O to the poſition C*k*, the rod ZW meets with the arm I*k*, and by carrying it forward, releaſes the tooth of the balance-wheel from the pallet *q*; the balance-wheel accordingly revolves, and the upper tooth meeting with the pallet *p*, winds up the auxiliary ſpring *u* as before. The balance, with the crank, proceeding to deſcribe the remaining part of the ſemi-arc *k*E, winds up the ſpring *v* through the further angle *k*CE = 108°; the balance returns through the ſemi-arc EO, by the joint action of the balance-ſpring and the auxiliary ſpring *v*, both of which ceaſe to accelerate the balance the inſtant it has arrived at O.

It is remarkable, according to this conſtruction, that no force or impulſe whatever is communicated to the balance from the main-ſpring, and yet the vibrations are continued of their due length: on further conſideration it appears, that the maintaining power of the machine, inſtead of communicating any force

whole arc of vibration is about $\frac{3}{4}$ of a circle, or 270°, conſequently the ſemi-arc of vibration is 135°. 2dly. The action of the balance, by carrying round the arm OG from *k* to B, winds up the auxiliary ſpring through an angle about four times greater than the angle O*k*, through which it is wound up by the action of the balance-wheel on the pallet." Wherefore, if OB, or 135°, is divided into five parts, one of them will be O*k* = 27°, and the other four parts, or *k*B = 108°.

or impulſe, acts by removing a part of the force which retards the balance while it is deſcribing the latter ſemi-arc of each vibration.

In the preceding account it has been ſhewn, that the balance deſcribes the ſemi-arc from B to O by the joint action of the two ſprings; now for a moment let it be ſuppoſed that the balance vibrated through the entire arc BOE (fig. 85, pl. 5, and fig. 86, pl. 6.) by the joint action of the two ſprings, in the ſame manner as if one balance-ſpring only was applied of the ſame ſtrength with both; in this caſe, the balance commencing its vibration at the extremity of the arc B, after having paſſed the ſemi-arc BO with an accelerated motion, would deſcribe an equal arc OE on the other ſide of O, by retarded motion, provided it was not obſtructed by friction, or other irregular reſiſtances; but ſuch reſiſtances taking place will cauſe the latter ſemi-arc, which is deſcribed by retarded motion, to fall ſhort of the arc OE by ſome ſmall difference ES. There are two modes by which this latter ſemi-arc may be reſtored to its due length OE; either by communicating an impulſe to the balance from the main-ſpring, or by removing a part of the force which retards the aſcent of the balance while it deſcribes the latter ſemi-arc OE. Mr. MUDGE has diſcovered and applied to his time-keeper the latter mode of ſupplying the power which is required to continue the balance in motion. During the progreſs of the balance through the ſemi-arc BO, it is accelerated by the joint action of the balance ſpring and auxiliary ſpring *a*; but while it deſcribes the latter ſemi-arc OE (fig. 85 and 86), it is retarded by the joint actions of the balance ſpring and auxiliary ſpring, only while it deſcribes a part of the ſemiarc from *k* to E, the retardation of the auxiliary ſpring being removed while it deſcribes the firſt 27° of this ſemiarc fr

It

It is evident, according to Mr. MUDGE's conſtruction, that the diminution of retardation, which is equivalent to the ſupply of power to the balance, muſt always be of the ſame magnitude, ſo far as regards the influence of the main ſpring, provided it has ſufficient force to wind up the auxiliary ſprings through the conſtant angle $OGk = OCk$ (fig. 86.); and the effects of heat and cold on the auxiliary ſprings are neceſſarily included in the compenſation which is applied for heat and cold to the balance ſpring. The maintaining power, therefore, by which the motion of the balance is continued, muſt be always uniformly the ſame; this is an object uſually held to be of material conſequence in the conſtruction of watches, and though often attempted by ingenious perſons, has probably been accompliſhed in its full extent, for the firſt time, by Mr. MUDGE.

This conſtruction poſſeſſes the further advantage of having the balance perfectly detached from the wheel work of the machine; the only communication* between the balance and the balance wheel is that which ſubſiſts while the pallet is diſengaged from the tooth; an inſtant of time in a practical ſenſe is almoſt evaneſcent: it ſhould also be remarked, that the preſſure

* The exact times in which the balance deſcribes any portions of the arc of vibration may be readily obtained by having recourſe to the theorem inveſtigated in page 162, vol. 1. If we can aſcertain the arc deſcribed by the balance while the tooth of the balance wheel is releaſed from the projection of the pallet, the portion of time in which the balance is connected with the wheel-work of the machine will be known. By an experiment made on a very exact model of Mr. MUDGE's conſtruction, it appeared that the balance deſcribed an arc of about 8°, while the tooth was releaſed from the pallet. And ſince the balance describes 108° of its ſemi-vibration before the pallet begins to be moved, it will have deſcribed 116° before the tooth is releaſed; we are therefore to find the times in which the balance deſcribes ſeverally the two arcs 108° and 116°: the difference of theſe times will be the time in which the balance wheel is releaſed from the pallet. By the theorem referred to,

The

* From the inveſtigation in page 164, vol. 1. it appea
F is the accelerative force on the circumference of the b
the angular diſtance from quieſcence c°, $p = 3{\cdot}14159$, &c.
radius of the balance expreſſed in inches, $l = 193$ inches;

of a ſemi-vibration will be $= \sqrt{p^2 rc^{\circ} \div (8lF \times 180^{\circ})}$. I
ſent caſe $c^{\circ} = 90^{\circ}$, $r = 1$ inch, $F = f + \frac{1}{2\,0} f = 1{\cdot}0040892$

fore the time of a ſemi-vibration $= \sqrt{p^2 rc^{\circ} \div (8lF \times 180$
part of a ſecond preciſely.

If the balance ſhould vibrate by the force of the auxiliar
only, the force of acceleration on the circumference at th
90° from quieſcence is $\frac{1}{2\,0} f = 0{\cdot}0478138$; wherefore $\frac{1}{2\,0}$

ſubſtituted for F in the quantity $\sqrt{p^2 rc^{\circ} \div (8lF \times 180^{\circ})}$,
values remaining the ſame as in the former caſe, the time c

Pts

vibration will now be $\sqrt{p^2 rc^{\circ} \div (8l \times \frac{1}{2\,0} f \times 180^{\circ})} =$
and the time ſhewn by the watch in any portion of mean ti
$= t \div 4{\cdot}5825 = t \times {\cdot}21822$. Thus if t ſhould be tal
one minute, or 60 ſeconds of time, the time ſhewn by the
60 ſeconds will be $= 60 \times {\cdot}21822 = 13$ ſeconds nearly.

This would be the reſult if the forces of the balance and
ſprings were preciſely in the proportion of 20 to 1. Mr.
mentioned this proportion by eſtimation only, not having
morandum of experiments made to aſcertain the exact prop

If, therefore, the actual rate of the watch ſhould be

ARTICLE XXXIII.

Anſwers to the Mathematical Queſtions propoſed in ARTICLE XVIII. NO. VII.

I. QUESTION 131, *anſwered by Mr.* Burdon, *Acaſter-Malbis.*

LET x be the required number of days; then by the queſtion, $\frac{x-8}{15}$, $\frac{x-11}{21}$, $\frac{x-5}{27}$, and $\frac{x-18}{29}$ are whole numbers.

Suppoſe $\frac{x-8}{15} = p$, then $x = 15p + 8$. Therefore, $\frac{x-11}{21} = \frac{5p-1}{7}$ = a whole number; let it be taken from $\frac{7p}{7}$, a whole number, and the remainder $\frac{2p+1}{7}$ is a whole number: Multiply this by 4, then $\frac{8p+4}{7} = p + \frac{p+4}{7}$, a whole number; and ſo is the remainder $\frac{p+4}{7}$ a whole number; let it be $= q$, then $p = 7q - 4$, and $x = 105q - 52$. This value being ſubſtituted for x in the third condition of the queſtion, we have $\frac{x-5}{27} = \frac{35q-19}{9} = 3q - 2 + \frac{8q-1}{9}$ = a whole number; and ſo is $\frac{8q-1}{9}$ a whole number; ſubtract

it from $\frac{9q}{9}$, and the remainder $\frac{q+1}{9}$ = a whole number

hence $q = 9r - 1$, and $x = 945r - 157$; then in t

condition of the question, we have $\frac{x-18}{29} = \frac{945r-}{29}$

$32r - 6 + \frac{17r-1}{29}$ = a whole number; let the remai

multiplied by 2, and $\frac{34r-2}{29} = r + \frac{5r-2}{29}$ = a whol

ber; multiply the remainder by 6, then $\frac{30r-12}{29}$ a who

ber, that is $r + \frac{r-12}{29}$, or $\frac{r-12}{29}$ a whole number

hence $r = 29s + 12$, and $x = 27405s + 11183$. If then $x = 11183$ days, and consequently Mr. Milner is 31st year of his age.

The same by Mr. J. Collins, *School-Master, Kensington*

Let the number which answers the two first conditions question be denoted by $15x + 8$, then will $\frac{15x+8-}{21}$

$\frac{5x-1}{7}$ be a whole number, in which the least value of

therefore $15x + 8 = 53$. Now the least common mul 15 and 21 being 105, it is evident that the number answe three first conditions may be represented by $105y + 5$

$\frac{105y+53-5}{27}$, or $\frac{8y+16}{9}$ is a positive whole num

which the least value of y (being a whole number) is 7 $105y + 53 = 788$.

Laſtly, the leaſt common multiple of 15, 21, and 27 being 945, we may aſſume $945z + 788$ as the number ſought, and then $\frac{945z + 788 - 18}{29}$, or $\frac{17z + 770}{29}$ is a whole number, and the leaſt value of z is 11: wherefore $945z + 788 = 11183$ days, or $32\frac{3}{5}$ years, nearly.

The ſame, by Mr. Swale, *Chester.*

Let x = Mr. Milner's age in days, then $\frac{x - 8}{15}$, $\frac{x - 11}{21}$, $\frac{x - 5}{27}$, and $\frac{x - 18}{29}$ muſt be whole numbers; now put theſe expreſſions equal to a, c, m, and n reſpectively, and then $x = 15a + 8 = 21c + 11 = 27m + 5 = 29n + 18$; from equating the two firſt values of x, we get $a = \frac{21c + 3}{15} = c + \frac{6c + 3}{15} = c + \frac{2c + 1}{5} = c + s$, putting $2c + 1 = 5s$; but, ſince $c = \frac{5s - 1}{2} = 2s + \frac{s - 1}{2} = 2s + t$, where $2t = s - 1$, we have $a = 3s + t$, and $x = 45s + 15t + 8$. Again, by equating the third and fourth values of x, we get $m = \frac{29n + 13}{27} = n + \frac{2n + 13}{27} = n + r$, by aſſuming $27r = 2n + 13$: but we have alſo $n = \frac{27r - 13}{2} = 13r - 6 + \frac{r - 1}{2} = 13r + v - 6$, where $v = r - 1$; hence $m = n + r = 14r + v - 6$, and conſequently $x = 27m + 5 = 378r + 27v - 157$. Moreover, ſince $2t = s - 1$, and $2v = r - 1$, then $s = 2t$

$+ 1$, by assuming $35w = 16v + 21$: but $v =$

$= 2w - 1 + \frac{3w - 5}{16} = 2w + d - 1$, where $16d$

5; now $w = \frac{16d + 5}{3} = 5d + 1 + \frac{d + 2}{3} = 5d$

putting $3h = d + 2$, or $d = 3h - 2$. Conseq. $w =$
$v = 35h - 21$, $t = 261h - 155$, and $x = 105t$
$27405h - 16222$, where h may be any integer wh
$h = 1$, $x = 11183$.

According to one or other of these methods the questi
answered by Messrs. Bosworth, Evans, Gregory, Ha
Johnson, Lowry, M'Doneld, Marrat, Miss Susan Ma
Minor, Milner, Reed, *and* Thornoby.

II. QUESTION 132, *answered by Mr.* William Fi
cher of the Mathematics at Hamstead, Middlesex

Suppose the rectangle's breadth $= 2$, the circle's di
$ba = 10$; also $2x(10^2 - x^2) = 99.6612 =$ the

this for another ſuppoſition, we find 2·834 for the breadth of the rectangle, very near; hence its length is 23·32, and the circle's diameter 9·166; alſo the area of each = 66 nearly.

The ſame anſwered by Merones Minor.

Fig. 324, Pl. 20. Let ABC be the given ſemicircle, DEFG the inſcribed parallelogram, and BKHL the circle inſcribed in the remaining ſegment. Put a = 12 = rad. = AI = BI, p = ·7854, and x = IH = DE = FG; then $a - x$ = BH, $\sqrt{(a^2 - x^2)}$ = HE, and the area of the parallelogram is $2x\sqrt{(a^2 - x^2)} = (a - x)^2 \times p$, the area of the circle. Whence by *Ex Alg. Rule* 5, *Pa.* 283, I find x = 2·8311, very near, and BH = 9·1689; alſo EF = $2\sqrt{(14·8311 \times 9·1689)}$ = 23·3224.

The ſame, by Mr. John Blackwell, *Hungerford.*

Put a = 12, the ſemidiameter of the ſemicircle, b = ·7854, and x = the diameter of the circle required, then will $a - x$ = the br. of the parollelogram and $2\sqrt{a^2 - (a - x)^2}$ its length; theref. $(a - x) \times 2\sqrt{a^2 - (a - x)^2}$ = the area of the parallelogram, and bx^2 = the area of the circle. Hence $(a - x) \times 2\sqrt{a^2 - (a - x)^2} = bx^2$. This equation reduced, produces a cubic, wherein x = 9·1689. Whence the length of the parallelogram is eaſily found.

The ſame, by Mr. J. Hartley, *Auditor's Office, Somerſet Place, London.*

Make the rad. of the ſemicircle = 12 = a, ·7854 = c, BH = x, and EF = y, then will $a - x$ = HI, and, per queſtion, EF × HI = area of the circle BLHK, *i. e.*, $cx^2 = ay = xy$; alſo, by Eu. I. 47, $\sqrt{a^2 - (a - x)^2} = \frac{1}{2}y$, or $y = \sqrt{(8ax - 4x^2)}$. Whence $\sqrt{(8ax - 4x^2)} = y = cx^2 \div (a - x)$. Hence, from the reſolution of a cubic equation, x = 9·16 &c, $a - x$ = 2·83 = HI, and $y = cx^2 \div (a - x)$ = 23·32 &c = EF.

This queſtion was alſo anſwerd by Meſſrs. Boſworth, Bverley, Cotes, Evans, Gregory, Harris, Hill, Johnſon, Lowry, Marrat, Miſs May, Reed, Swale, *and* Thornoby.

III. QUESTION 138, *anſwered by Miſs* Suſan May, *Birmingham.*

Three numbers anſwering the conditions of the queſtion may be found by the following ſimple proceſs. Let a^2 be the given ſquare, and put $\frac{x}{2} + a$, $\frac{x}{2} - a$, and $2x$ for the numbers required. The three firſt conditions are evidently fulfilled, $\left(\frac{x}{2} + a\right).\left(\frac{x}{2} - a\right) + a^2 = \left(\frac{x}{2}\right)^2$, $\left(\frac{x}{2} + a\right).\ 2x + a^2 = (x + a)^2$, and $\left(\frac{x}{2} - a\right).\ 2x + a^2 = (x - a)^2$, being all ſquares. We have, therefore, only to make the product of the three numbers when added to the given ſquare, a ſquare number, that is, $\left(\frac{x}{2} + a\right)\cdot\left(\frac{x}{2} - a\right)\cdot 2x + a^2 = \frac{x^3}{2} - 2xa^2 + a^2$, muſt be a ſquare; aſſume its ſide $= xa - a$, then $\frac{x^3}{2} - 2xa^2 + a^2 = x^2a^2 - 2xa^2 + a^2$; hence $x = 2a^2$, and the three numbers are $a^2 + a$, $a^2 - a$, and $4a^2$.

If $a = 2$, the numbers are 6, 2, and 16, and the four ſquares 4^2, 6^2, 10^2, and 14^2.

The ſame anſwered by Mr. George Reed, *pupil to Mr.* Thoma Bulmer, *Sunderland.*

Let x, y, and z be the three numbers ſought, and a^2 the give ſquare. Then by the queſtion,

$xy +$

$$\left.\begin{array}{l} xy + a^2 = \square = r^2, \text{ or } xy = r^2 - a^2 \\ xz + a^2 = \square = s^2, \\ yz + a^2 = \square = m^2, \\ xyz + a^2 = \square = v^2, \text{ or } xy = \dfrac{v^2 - a^2}{z}; \end{array}\right\} \text{Hence } r^2 - a^2 = \frac{v^2 - a^2}{z}$$

or $z = \dfrac{v^2 - a^2}{r^2 - a^2}$, and, by ſubſtitution, $\dfrac{v^2x - a^2x}{r^2 - a^2} + a^2 = n^2$, or

$x = \dfrac{r^2n^2 - n^2a^2 + a^4 - a^2r^2}{v^2 - a^2}$; and, again, by ſubſtitution,

$\dfrac{v^2y - a^2y}{r^2 - a^2} + a^2 = m^2$, or $y = \dfrac{m^2r^2 - m^2a^2 + a^4 - a^2r^2}{v^2 - a^2}$.

And if $a = 2$, $r = 3$, $v = 8$, $n = 4$, and $m = 8$; then $x = 1$, $y = 5$, and $z = 12$, the three numbers required.

Thus the queſtion was anſwered by Meſſrs. Boſworth, Evans, Gregory, Hill, Johnſon, Lowry, Simpſon *the propoſer, and* Thornoby.

The ſame otherwiſe by Mr. I. H. Swale.

Let x, y, and z be the three required numbers, and m^2 the given ſquare. Then muſt xy, xz, yz, and $xyz + m^2$ be tranſformed into ſquares. Now ſince xy and xz are ſquares, their product x^2yz will be a ſquare, and conſequently yz will be a ſquare, in all caſes wherein xy and xz are ſuch. Hence, we have only to make $\sqrt{xy}$, $\sqrt{xz}$, and $\sqrt{m^2 + xyz}$, rational numbers, whole or fractional. Aſſume $xy = r^2$, and $xz = v^2$; then, $x = \dfrac{r^2}{y} = \dfrac{v^2}{z}$, $y = \dfrac{r^2z}{v^2}$, and $xyz = zr^2$; alſo the formula $m + xyz$ will become,

by

by ſubſtitution, $m^2 + sv^2$, a ſquare, ſuppoſe it equal to $m^2 + 2ms + s^2$. Whence $z = \frac{s^2 + 2ms}{v^2}$, $y = \frac{r^2 z}{v^2} = \frac{s^2 + 2ms}{v^2}$, and $x = \frac{r^2 v^2}{s^2 + 2ms}$, where r, s, v, and m may be taken at pleaſure.

If $r = 2$, $s = 3$, $v = 4$ and $m = 5$, we ſhall have $x = \frac{64}{39}$, $y = \frac{39}{16}$, and $z = \frac{39}{4}$; alſo, $xy = 4$, $xz = 16$, $yz = \frac{1521}{64}$, and $m^2 + xyz = 64$; all ſquares, the ſides of which are 2, 4, $\frac{39}{8}$, and 8, reſpectively.

And thus the queſtion was alſo anſwered by Meſſrs. Collins Hartley, M'Doneld, *and* Merones Minor.

IV. QUESTION 134, *anſwered by Mr.* T. Barber, *St. John's College, Cambridge.*

Analyſis. Fig. 325, Pl. 20. Let F be the given point, and BG the required chord. Join F with C the centre, and produce FC both ways to the points H and D, taking DF : FH :: GF : FB; join alſo HB and DG, then the △s HBF and GFD are ſimilar. Alſo take DA : HE :: DF : FH and join BE, AG, then ſince DF : FH :: DA : HE, DF : FH :: AF : FE :: GF : FB, theref. the △s AGF, BFE are ſimilar, and the ∠AGF = FBE, theref. the whole ∠HBE = the whole ∠AGD, that is to a right angle: Whence this conſtruction.

Through the given point F draw the diameter DA which produce to H, taking DF : FH in the given ratio, take alſo DA : HE in the ſa[illegible]n HE as a diameter deſcribe a ſemicircle cutting

ing the propoſed circle in the point B, join BF and produce it till it meets the circumference at G and BG will be the required chord.

The ſame anſwered by Mr. Louis Hill.

Let $m : n$ be the given ratio. Through the given point P (fig. 326, pl. 20.) draw the diameter APV, let O be the centre of the given circle. On PA produced take DP : PO $: 2m : n - m$, and on DP as a diameter deſcribe a ſemicircle to interſect the given circle in E, draw EPF and it is the line required.

Demon. Join DE, and upon EF demit the $\perp$ OI; thereby ſim. $\triangle$s, EP : PI :: DP : PO :: $2m : n - m$, by conſ. therefore by compoſition, ſince EI = IF, EP : PF :: $m : n$, *i.e.* EP, PF are in the given ratio. *Q. E. D.*

Remark. Since the ſemicircle may cut the given circle in two points, there will be two lines anſwering the problem.

The ſame, by Mr. Johnſon.

Through the given point P (fig. 326, pl. 20.) draw the diameter APV, and take AP to PB in the given ratio, from P apply PE ſuch that its ſquare may = the rect. VP·PB, produce EP to meet the circle in F, and EF is the chord required. For by conſtruction,

VP : EP :: EP : PB, and by the circle VP : EP :: FP : AP; therefore FP : AP :: EP : PB, or FP : EP :: AP : PB, *i.e.* in the given ratio.

The ſame by Merones Minor.

Analyſis. Fig. 326, Pl. 20. Let P be the given point and $m : n$ to the given ratio, n being the greater, and let EF be drawn as required. Through P and the centre O let the diameter AV be drawn. Then, by *Em. Geo. Cor.* 2. *to Prop.* xx. B. IV, AP × PV = EP × PF = $(n \div m) \times EP^2$, and conſq. EP = $\sqrt{(AP \times \overline{n \div n} \times PV)}$: Hence this conſtruction:

Find PG a 4th proportional to n, m, and PV, and upon AG let a ſemicircle be deſcribed; make PH $\perp$ to AG, and from P let PE = PH be applied to the given circle, and draw EPF the chord required.

Anſwers equally ingenious were given by Meſſrs. Burdon, Boſworth, Byerly, Collins, Evans, Gregory, Harris, Lowry, M'Doneld, Matheſis, *and* Swale.

V. QUES-

V. QUESTION 135, *answered by Mr.* J. Johnson.

Let ACB (fig. 327, pl. 20.) be the $\triangle$ required, and suppose CE to be drawn from the vertex to the middle of the base AB. Put $a^2 = \frac{1}{2}$ the sum, $d^2 = \frac{1}{2}$ the diff. of the squares of the given sides, b the sum of the base and $\perp$, and $x =$ the base; then $ED = d^2 \div x$, $CD = b - x$, and $EC^2 = (d^2 \div x)^2 + (b - x)^2$; hence $AE^2 + EC^2 = x^2 \div 4 + (b - x)^2 + (d^2 \div x)^2 = a^2$, and by reduction we get $5x^4 - 8bx^3 + (4b^2 - 4a^2)x^2 + 4d^4 = 0$, from which equation x may be determined.

The same, by Merones Minor.

Put $s =$ the sum of the sides, $d =$ their diff. $x =$ the base, and $a - x = \perp$. Then by *Eu. Geo.* 38. II. *Cor.* 2, $\frac{1}{4}(s^2 - x^2) \times \frac{1}{4}(x^2 - d^2) = (\text{area})^2 = (a - x)^2 \times \frac{1}{4}x^2$, consequently $5x^4 - 8a^2x^3 + (4a^2 - s^2 - d^2)x^2 + s^2d^2 = 0$. Whence x may be determined.

Ingenious answers were also given by Messrs. Bosworth, Gregory, Hartley, Hill, Lowry, Swale, *and* Thornoby.

VI. QUESTION 136, *answered by Miss* Susan May.

Let ACB (fig. 328, pl. 20.) be the $\triangle$, then since the $\angle$ ACB is 36°, or 1-10th part of the circle, AB is the side of a decagon inscribed in the circle whose radius is AC or CB. Now by *Eu. Geo. B.* iv. *Prop.* 47, *Cor.* 2, AC : AB :: 2 : $\sqrt{5} - 1$, hence, by hyp. $\triangle$s,

As $4 + \sqrt{5} - 1 : 2 :: 16 + 8\sqrt{5} : 4 + 4\sqrt{5} = AC = BC$ = the sides,

and $4 + \sqrt{5} - 1 : \sqrt{5} - 1 :: 16 + 8\sqrt{5} : 8 = AB =$ the base.

The same, by Mr. Bosworth, *Cambridge.*

Let the required $\triangle$ be represented by ABD, Fig. 329, Pl. 20. The sides of this $\triangle$ may be determined, without referring to trigonometrical tables or any other known values of lines and tangents in the following manner. Since the $\triangle$ is an isosceles one, and the vertical $\angle = 36°$, each $\angle$ at the base $= (180° - 36°) \div 2 = 72°$, which is double that at the vertex. On AB set off AC equal to BD, and draw CD; then, as appears from the demonstration of Eu. iv. 10. $BA \times BC = BD^2$. Let BC be denoted by x, and suppose $BD = 6$; then, $BA = 6 + x$, and $BA \times BC = 6x + x^2$,

$+ x^2$, which by the above property $= BD^2 = 36$. This equation reduced gives $x = 3\sqrt{5} - 3$; whence AB or AD is eaſily found $= 3 + 3\sqrt{5}$, and the perimeter of this ſuppoſed $\triangle$, which is ſimilar to the given one $= 6 + \overline{3 + 3\sqrt{5}} + \overline{3 + 3\sqrt{5}} = 12 + 5\sqrt{5}$. But the perimeter of the given $\triangle$ is $16 + 8\sqrt{5}$, therefore by ſim. $\triangle$s,

$$12 + 6\sqrt{5} : \left\{\begin{matrix} 6 \\ 3 + 3\sqrt{5} \\ 3 + 3\sqrt{5} \end{matrix}\right\} :: 16 + 8\sqrt{5} : \left\{\begin{matrix} 8 = \ \ = BD \\ 4 + 4\sqrt{5} = AB \\ 4 + 4\sqrt{5} = AD. \end{matrix}\right.$$

Mr. Gregory, *the propoſer, deduced a ſolution, nearly ſimilar, from the ſame propoſition in* Euclid.

The ſame, anſwered by Mr. J. Hartley, *London.*

The vertical $\angle$ being 36 (1-10th of a circle), it is plain, the baſe of the $\triangle$ is the ſide of a decagon, and each of its ſides the radius of the circumſcribing circle. By *Em. Geo. B.* iv. *Pr.* 47. *Cor.* 2, it is as rad. : ſide of the decagon :: 2 : $\sqrt{5} - 1$. Make x = rad. of the circle, a = the given perimeter, then will $a - 2x$ = the baſe of the iſoſceles $\triangle$. Therefore, by the above proportion, $x : a - 2x :: 2 : \sqrt{5} - 1$; hence $x = 2a \div (\sqrt{5} + 3) = (32 + 16\sqrt{5}) \div (3 + \sqrt{5}) = 12{\cdot}94427 =$ one of the equal ſides of the $\triangle$, and $a - 2x = 8 =$ its baſe.

The ſame, by Mr. J. T. M'Doneld, *Holland, near Wigan.*

Let ABC (fig. 328, pl. 20.) be the iſoſceles $\triangle$ whoſe perimeter is $16 + 8\sqrt{5}$, and vertical 36. Then to determine the ſides, produce CA, CB, till CE, CF are each = to half the perimeter, and through the points C, E, F let a circle be deſcribed, whoſe centre O, will be in CD $\perp$ to AB. Biſect the arch FC in G, and draw the right lines OE, OF, EF, DF, DG, and CG. Now by Eu. III. 20, $\angle$ EOF = 2ECF = 72 = 1-5th of the circumference, and arch DF = 1-5th arch DC. Put the chord DF = x, and $CF = 8 + 4\sqrt{5} = a$, then $CD = \sqrt{(a^2 + x^2)}$, $2FH = 2ax \div \sqrt{(a^2 + x^2)} = EF = CG$, and $DG = \sqrt{[a^2 + x^2 - (4a^2x^2 \div \overline{a^2 + x^2})]}$. Alſo by *Fm. Trig. B.* i. *Pr.* 39, *Cor.* 2, $DG \times DF = OD^2$, that is, $\sqrt{[a^2x^2 + x^4 - (4a^2x^4 \div \overline{a^2 + x^2})]} = (a^2 + x^2) \div 4$: Whence $x = 5{\cdot}505 = DF = DI$; $FH = 5{\cdot}2355$; $CH = 16{\cdot}115$; and $CI = 12{\cdot}311$.

Hence

Hence, by ſim. $\triangle$s, CH : CI :: EF : AB ($= 8$) :: CF or CB or CA ($= 4 + 4\sqrt{5}$).

The ſame, by Mr. W. Marrat, *Teacher of the Mathematcs at the Grammar School, Boſton, Lincolnſhire.*

An iſoceles $\triangle$ ABC (fig. 328, pl. 20.) whoſe vertical $\angle$ is 36°, is $=$ to one of the $\triangle$s of a decagon. Now the area of a decagon whoſe ſide is unity, is, by page 114, *Hutton's Mens.* 7·694288, 1-10th of which, or 7·694288 is the area of one $\triangle$, which put $= m$. Alſo put $16 + 8\sqrt{5} = s$, and AB $= x$,

and $1^2 : m :: x^2 : mx^2 =$ its area $=$ CI $\times \frac{1}{2}$ AB $= \frac{x}{2}\sqrt{\left(\frac{s-x}{2}\right)^2 - \frac{x^2}{4}}$. Hence $x = 8 =$ AB, and AC $=$ BC $= 12{\cdot}944$.

The ſame otherwiſe by Mr. W. Burdon, *Acaſter Malbis.*

The ſine of 36° to rad. 1, is $\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}$, and that of 72° is $\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{2}}$. And in every plane $\triangle$ it is, as the ſum of the ſines of the three $\angle$s, is to its perimeter, ſo is the ſine of any one of its $\angle$s, to its oppoſite ſide. This proportion gives each of the equal ſides of the $\triangle = 4 + 4\sqrt{5}$, and the other ſide $= 8$.

Meſſrs. Blackwall, Byerley, Evans, Frances, Harris, Hill, Johnſon, Lowry, Merones Minor, *and* Peacock, *gave neat ſolutions to this queſtion.*

VII. QUESTION 137, *anſwered by Mr.* J. T. M'Donell.

Let $x =$ the radius of the cone's baſe, $p = 3{\cdot}1416$, and s its convex ſurface, then will the altitude be $= \sqrt{s^2 \div p^2x^2 - x^2}$; alſo by *Emerſon's Fluxions, pa.* 319, the diſtance between the cent

of oſcillation and ſuſpenſion, will be $= (4s^6 - 3p^6x^4)$ $(p^6s^6x^6 - p^6x^6)$, which, that the cone may vibrate the poſſible, muſt be a minimum; but $(4s^6 - 3p^6x^4) \div \sqrt{}$ $p^6x^6)$ is alſo a min. This expreſſion being fluxed and the equation reduced, we get $x^8 + (s^6 \div p^6).x^4 = 4s^6 \div$

Hence $x = \sqrt{\frac{s^6}{p^6}}\sqrt{\frac{19}{12} - \frac{s^6}{2p^6}}$.

$s = 100$ ſquare inches, then $x = 5{\cdot}265$, the altitude 23, and the diſtance from the vertex to the centre of $n = 4{\cdot}24296$. Hence the centre of oſcillation falls inches below the cone's baſe; and when the radius of the ual to the altitude, the centre of oſcillation falls exactly ſe's centre.

not be improper to obſerve here, that, the inveſtigations uthors, who make the diſtance of the centre of oſcilla- n ſuſpended by its vertex, to be *conſtantly four-fifths* of le, are erroneous; as they conſider all parts of the cone's nove with equal velocity, which is evidently not the

us the queſtion was anſwered by the propoſer, Mr. W. Paſs- *cher of the Mathematics at Hull, who alſo favoured us ollowing different ſolution.*

: OI, the ſemibaſe of the cone, $x = AO$ its altitude, 16. Then the diſtance from the centre of motion to

ſcillation, by *Simpſon's Fluxions, pa.* 225, is $= \frac{4x}{5} +$

h muſt be a *minimum* that the cone may vibrate the

oſſible, or in this caſe $4v + y^6 \div x$ is a *min.* Then $2cy$ ery of the cone's baſe, and $cy\sqrt{x^2 + y^2} = s$, the convex ı ſquare inches; and $y\sqrt{x^2 + y^2} = s \div c$, therefore

$y^6 = s^6 \div c^6 = n$; or $y^6 = -\frac{x^6}{2} + \sqrt{n + \frac{x^{12}}{4}}$;

now ſubſtituting for y^2, in $4x + y^2 \div x$, we have $\frac{7x}{2} +$

$\sqrt{\frac{n}{x^2} + \frac{x^2}{4}}$, a *min.* This being put into fluxions and reduced

gives $x = \sqrt{-\frac{17n}{8} + \sqrt{\frac{n^2}{3} + \left|\frac{17}{8}\right|^2 \cdot n^2}}$. If the convex

ſurface in ſquare inches be 100, then $x = 2{\cdot}97239$, $y = 5{\cdot}26566$, ſlant ſide $= 6{\cdot}046$, and the diſtance from the centre of oſcillation from A $= 4{\cdot}24$ nearly.

Schelium. The reſult of this ſolution militates againſt what has been aſſerted, on this ſubject, by ſome great Mathematicians: *Stone* in his fluxions, pa. 178, makes the centre of oſcillation to be 4-5ths of the altitude of the cone; but this anſwer is evidently erroneous, for he ſuppoſes that every particle of matter in the baſe of the cone moves with the ſame velocity, which is clearly not the caſe. And even Mr. *Emerſon* in his Mechanics, avers that the centre of oſcillation of a cone is 4-5ths its axis nearly. Now it appears from what is done above that it can only be ſo in one particular caſe; and that the centre of oſcillation cannot fall within the cone at all unleſs the altitude be greater than the ſemidiameter of the cone's baſe; and when the altitude and ſemibaſe are equal, the centre of the baſe is the centre of oſcillation, but when the ſemidiameter of the baſe exceeds the altitude it always falls below the baſe.

Other anſwers were given by Meſſrs. Barber, Boſworth, Byerly, Gregory, Hill, Johnſon, Lowry, Merones Minor, *and* Thornoby.

VIII. QUESTION 138, *anſwered by Mr.* Gregory, *Cambridge.*

To find the angle of the mountain's elevation ſuppoſing it to be conical of a regular form, the following method may be uſed: Let AIH (Fig. 331, Pl. 20.) repreſent the mountain, from the vertex of which meaſure the right lines AB, AC, and proceed to meaſure on the plane *towards the ſame point of the compaſs* any convenient diſtance, as ſuppoſe to E and F. Upon the line BF erect perpendiculars as BC, DE, (which may be done various ways with the *chain* only) and continue them until they meet CE in C and E. Then (FB and CE being ſuppoſed produced till they meet in G, in the baſe of the mountain, directly under the vertex) the $\triangle$s CKE and GBC being ſimilar, as EK (= DE — BC) : KC (= BD)

(= BD) :: BC : BG. Whence, in the △ AGB, right angled at G, AB, and BG are known, and consequently the angle ABG, the elevation of the mountain may be determined by a well known proportion.

The ſame anſwered by Mr. Francis.

Fig. 331, Pl. 20. Let A repreſent an object on the ſummit of the mountain, and ABD, ACE two right lines carried down the ſide of the mountain to its edge at B and C, and continued from thence along the plane to D and E. Set off BC and DE at right angles to BD, (which, I preſume, every ſurveyor can do with the help of the chain only); meaſure the diſtances DE, BD, BC, and BA. Then as DE — BC : BD :: BC : the horizontal diſtance between B and the point immediately under the mountain's top, or object A. We have, therefore, two ſides of a right angled △ to find the perpendicular height of the mountain, which being known the area of the mountain's baſe, which is to be delineated inſtead of its ſuperficies in the map, is readily obtained.

Similar anſwers were alſo given by Meſſrs. Boſworth, Burdon, Byerley, Lowry, Merones Minor, *and* Peacock.

IX. QUESTION 139, *anſwered by the propoſer*, Mr. John Harris, *Teacher of the Mathematics and Land Surveyor, Carmarthen.*

By *Emerſon's Dialling, Sect.* I. *Pr.* 10. If the dial be ſet up in any place, in a ſituation quite parallel to its firſt ſituation, it will always ſhew the time of the day in its original place: but, before that can be done, the latitudes and difference of longitudes of the two places muſt be known; and theſe being given, new requiſites muſt be found to ſet it up by. Therefore, let BPH (Fig. 332, Pl. 20.) be the meridian, DHC the horizon, and B the zenith of the place it was made for; NS the meridian, and NE SW the horizon of the place A, it is to be ſet up at, and let P be the pole of the world and join AB with a great circle. Then in the ſpherical △ ABP there are given, BP, the colat. of the place B, and AP = that of the place A, and the ∠APB = their difference of longitude, to find AB = ∠NDO = the inclination of the two horizons, or the co-reclination of the dial plane, at the place A. Alſo, to find the ∠BAP, = the ſupplement of its declination there.

Now, to find the elevation of the ſubſtile, or hour line of 12 on the dial, above the horizon = HD, we have given in the rt. ∠d ſpherical △ POH, HP = the ſtile's height, and the ∠ P =

the diff. of long. of the places to find HO, and in the rt. ∠d △ OND there are given DN = the co-decl. of the plane at A (found above,) and the ∠ODN = the inclination of the horizons; to find the arch DO. Then HO + DO = DH the elevation required. And according to theſe new requiſites, when found, the dial muſt be ſet up.

The ſame, by the Rev. L. Evans, *Royal Academy, Weſtwick.*

The plane of the gnomon of the given dial muſt make an ∠ with the plane of the meridian of the place, it is to be ſet up at, equal to the difference of longitude of the two places, ſuppoſing, as the queſtion does, the latitudes of thoſe places to be the ſame. If otherwiſe, calculations will be neceſſary to determine other requiſites.

The ſame, by Merones Minor.

By *Emerſon's Dialling*, 26. II. *and Cor. ibid.* put the dial in a poſition parallel to its former poſition, and the thing is done. That is for the horizontal dial note the diff. of lat. if any, between the places given, then raiſe the north ſide of the dial, if it be removed northward, or the contrary, ſo many degrees as that diff. And for the diff. of long. raiſe the eaſt ſide of the dial if it be removed eaſtward, or the contrary, ſo many degrees as that diff. and there fix it: taking care that the edge of the ſtile be parallel to earth's axis.

This queſtion was alſo anſwered by Meſſrs. Blackwell, Boſworth, Byerley, Gregory, *and* Lowry.

X. QUESTION 140, *anſwered by Mr.* O. G. Gregory, *Teacher of Mathematics, Cambridge.*

If the propoſer intended the pendulum to make 50399 vibrations in five hours, *at the top of the rock*, and not at the *top of the light-houſe*; in which caſe I underſtand him, the ſolution will be thus. From *Simpſon's Fluxions*, Vol. I. Pa. 225, the diſtance of the centre of oſcillation from the point of ſuſpenſion, will be found $= 4{\cdot}971 + \frac{2 \times {\cdot}6^2}{5 \times 4{\cdot}971} = 4{\cdot}99996 +$ or 5 very nearly. Then, by the nature of pendulums, as $\sqrt{5} : \sqrt{39{\cdot}2} :: 60 : 60\sqrt{39{\cdot}2} \div 5 =$

168 vibrations in a minute at the ſurface of the earth: whence $168 \times 60 \times 5 = 50400$ the number of vibrations in 5 hours. Now, as the forces of gravity, at different diſtances from the earth's centre (above the ſurface) are as the ſquares of the number of vibrations of the ſame pendulum, in equal times, at thoſe diſtances; and the ſaid forces being alſo inverſely as the ſquares of thoſe diſtances; it follows, that the numbers of vibrations are inverſely as the diſtances from the centre. Hence, as $50399 : 50400 :: 3979 : 3979{\cdot}07894997$, which leſſened by 3979 leaves $\cdot07894997 = 138{\cdot}9819472$ yards the height of the rock; $\frac{2}{3}$ of which, *i. e.* $92{\cdot}65463$ yards, is the height of the light-houſe.

The diſtance of the privateer from the foot of the rock, may be readily found, as follows: the pendulum performs 20 vibrations at the ſurface of the earth in $7\frac{1}{7}$ ſeconds (for, as $168'' : 60'' :: 20'' : 7''\frac{1}{7}$): therefore, as $1'' : 1142$ feet (velo. of ſound per ſec.) $:: 7''\frac{1}{7} : 8157\frac{1}{7}$ feet, or 1 mile, and 959 yards, the diſtance of the privateer.

The ſame, by Mr. T. Barber, *St. John's College, Cambridge.*

In this queſtion it will be neceſſary to find the centre of oſcillation of the pendulum, ſince it is very ſhort, and therefore the calculation will be materially affected by a ſmall error committed. Now the diſtance of the centre of oſcillation from that of ſuſpenſion (*by Vince's Fluxions*, p. 110) is $= d + \frac{2r^2}{5d} = 4{\cdot}971 + \frac{{\cdot}72}{4{\cdot}855} = 4{\cdot}971 + {\cdot}02896 = 4{\cdot}99996$. If 50399, the number of vibra. performed in 5 hours at the top of the rock be divided by 18000, the number of ſeconds in 5 hours, the quotient, $2{\cdot}799944$, will be the number of vibra. performed in $1''$ at the top of the rock by the pendulum whoſe len. is $4{\cdot}99996$, and, therefore, $\frac{1}{2{\cdot}799944} = {\cdot}35715$ will be the time of one vibra.; and ſince the number of vibra. performed in a given time varies inverſely as the ſquare root of the length of the pendulum, $\sqrt{4{\cdot}99996} : \sqrt{39{\cdot}2} :: 1 : \sqrt{39{\cdot}2} \div 4{\cdot}99996$, the number of vibra. performed in $1''$ by the ſhorter pendulum at the ſurface of

the earth, therefore $\sqrt{4.99996 \div 39^2} = .35714142$ will be the time of one vibra. of the ſhorter pendulum at the earth's ſurface. Now, ſince the ſquare of the time varies inverſely as the force on the ſame pendulum, and the force inverſely as the ſquare of the diſtance from the centre of the earth, the time varies directly as the diſtance; therefore $.35714142 : .35715 :: 3979 : 3979.095591$ the diſtance of the top of the rock from the earth's centre, and therefore $.095591$ miles $= 168.24$ yards, is the height of the rock, $\frac{2}{3}$ of which, 112.16 yards is that of the light-houſe.

The diſtance of the privateer from the foot of the rock may be found by this proportion, $1'' : 7.1428$ (time of 20 vibra.) $::$ 1142 (velo. of ſound per ſec.) : 8157.0776 feet, or 2719.0255 yards, the diſtance of the privateer.

Upon this principle is the anſwer given by Meſſrs. Boſworth, Burdon, Byerley, Lowry, *and* Merones Minor.

The ſame by the Rev. L. Evans.

By *Emerſon's Mechanics* the diſtance of the centre of oſcillation of the pendulum from the point of ſuſpenſion will be $4.971 \times \frac{2 \times 6^2}{5 \times 4.971} = 4.999$ or 5 nearly. Then as $\sqrt{5} : \sqrt{39^2} :: 60 :$ 168 $=$ vib. in a minute, and $168 \times 60 \times 5 = 50400$ vib. in 5 hours on the ſurface of the earth. And $50400 - 50399 = 1$ vib. loſt, by the diff. of gravity at the *top of the light-houſe*, and ſurface of the earth. And ſince the number of vib. in a given time is reciprocally as the diſtance from the earth's centre, we have $50400 : 3979 :: 50399 : 3979.078949$, therefore $3979.078949 - 3979 = .078949$ or 138.95024 yards the height of the rock and light-houſe, $\frac{2}{3}$ of which is 55.580096 yards the height of the light-houſe. The diſtance of the privateer is eaſily found.

And thus the queſtion is anſwered by Meſſrs. Blackwell, *and* Francis, *the propoſer.*

XI. QUESTION 141, *anſwered by* Merones Minor.

CASE I. ANALYSIS. Fig. 333, *Pl.* 20.

Suppoſe it true. Through F let PFQ be drawn $\perp$ to CD meeting CD and AB in P and Q, and join BF, CF.

Since

Since AB+CD:AD::AB:DF, by altern. AB+CD:AB::AD:DF, and by diviſion CD: AB :: AF: DF: but AF: DF :: FQ: FP; theref. by eqality CD : AB :: FQ : FP;
wherefore the $\triangle$ BAF = $\triangle$ CDF:
but the $\triangle$ BEF = $\triangle$ CEF;
wherefore the ſpace ABEF = ſpace DCEF;
therefore the trapezoid ABCD is divided into two equal parts by the line EF drawn from the middle of BC. *Q. Q. V.*

CASE II. ANALYSIS. Fig. 333, *Pl.* 20.

Suppoſe it true. Join AE, DE.
Since AB+CD:BC::AB:CE, by altern. AB + CD:AB::BC:BE, and by diviſion CD: AB :: BE : CE; hence the $\triangle$ ABE = $\triangle$ DCE; but the $\triangle$ AEF = $\triangle$ DEF; theref. the ſpace ABEF = ſpace DCEF; wherefore the trapezoid ABCD is divided into two equal parts by the line EF drawn from the middle of AD. *Q. Q. V.*

The ſame, by Mr. Harris, *Carmarthen.*

Produce BA till AG = AB, and join AE, DE, BF, CF, and FG.

When BC is biſected the $\triangle$ BFE = CFE, by Eu. 38. I. theref. the $\triangle$ CFD = FAB = FAG, and the $\angle$ CDF = FAG, by parallels; conſequently CD·DF = FA·AG = FA·AB. Therefore AB : CD :: DF : FA, and by compoſition AB + CD : AD (DF + FA) :: AB : DF.

And when AD is biſected, the $\triangle$ AEF = FED, but the trapez. ABEF = trapez. CDEF; theref. the $\triangle$s ABE, DCE are equal, and, by hyp. the $\angle$s at B and C are right; conſequently AB·BE = DC·CE, theref. AB : CD :: CE : BE; theref. by compoſ. AB + CD : BC (CE + BE) :: AB : CE. *Q. E. D.*

The ſame, by Mr. Swale.

Produce DA, CB to meet in P'. Join BF, CF, AE, DE, and demit upon DP' the perpendiculars BG', CH.

1. Since BE = EC, by hyp. the $\triangle$s CFE, BFE, are =, Eu. 38. 1.
conſequently the $\triangle$ CFD = $\triangle$ BAF, and conſeq FD·CH = AF·BG'.
Hence, DF : FA :: BG' : CH :: P'B : P'C :: AB : CD;
wherefore componendo &c. AB + CD : AD :: AB : DF.

2. Since AF = FD, by hyp.; the $\triangle$s DEF, AEF are equal,
conſequently, the $\triangle$ ECD = $\triangle$ EBA,
and conſeq. EC·CD = EB·BA.

Hence

Hence, AB : CD :: CE : EB,
therefore, by comp. AB + CD : BC :: AB : CE.

Ingenious demonſtrations were alſo given by Meſſrs. Boſworth, Burdon, Elliott, Evans *the propoſer*, Gregory, Hill, Johnſon, Lowry, *and* M'Donald.

XII. QUESTION 142, *anſwered by Mr.* Lowry, *Birmingham.*

This queſtion may be ſolved by various methods, a few of which are here exhibited. There is, however no method of deſcribing the △, upon the ſurface of a ſphere, with the given data, that I am yet acquainted with, except in ſome particular caſes. The propoſer, no doubt, intended this queſtion for an illuſtration of the uſe of *Prob.* 17, *Book I. of Conſtructions* of his ſpherical Geometry, but whoever will examine the *conſtruction* there given will find inſurmountable difficulties in the application of it. The lines neceſſary to be drawn muſt paſs through a *ſolid* ſphere, and even ſuppoſing it poſſible to draw them in that manner, yet ſtill as great a difficulty would remain in drawing a tangent from an *imaginary* point to touch the ſphere at the point of contact of the circles. If we ſuppoſe the △ to be projected on a plane, the prob. may be reduced to Prob. XII. of *Lawſon on Tangencies*, but the calculation from thence would be exceedingly troubleſome. The ſimpleſt method of conſtruction that has appeared to me is the following:—

Take AB (fig. 334, pl. 20.) = the ſum of the given ſegments, AI = their difference, and AC = the given ſum of the ſides, to the radius OA: draw the indefinite tangent AD to touch the circle at A; biſect AC in Q, AB in R, and AI in S, and draw OQD, ORE, and OSF to meet AD in D, E, and F; take AG a 4th proportional to AD, AE, and AF: draw OG to cut the circle in T, then AT = half the diff. of the ſides; hence AT + AQ = the greater ſide, and AQ − AT = the leſs ſide.

For, by ſpherics and conſtruction, As the tangent AD of half the ſum of the ſides, is to the tangent AE of half the baſe, ſo is the tangent AF of half the diff. of the ſegments of the baſe, to the tangent AG of half the diff. of the ſides; whence the whole is evident.

The ſame may be determined otherwiſe thus.

In fig. 335, pl. 20. take AD, DB = the given ſegments of the baſe to the radius OD, on OD demit the ⊥s AE, BF; take the arch AH = the ſupplement of the given ſum of the ſides, and divide it in G, by prob. 37, Simpſon's Select Exerciſes, ſo the ſines

of

of GH, GA, may obtain the same ratio EO to OF; erect the ⊥ arch DC, and make AC = the complement of GH and BC = the comp. of AG, then is ACB the △ required.

This construction will appear evident by considering that the cosines of the sides are as the cosines of the segments of the base, which is a well-known property of spherical triangles.

If the difference of the sides, instead of their sum, be given, the sides are determined the same way as above.

The same answered by Mr. William Burdon.

By theo. 1. art. 75, pa. 85, Crackell's translation of Maurit's Trigonometry, it is, as the tan. of ½ the base of any sph. △, is to the tan. of ½ the sum of the other two sides, so is the tan. of ½ the diff. or ½ the sum of the sum of the segments of the base formed by the ⊥, according as it falls within or without the △, whence the diff. of the sides, and consequently the sides themselves will become known.

The same, by Mr. Rd. Elliott, *Liverpool.*

Fig. 335, pl. 20. Let AD, DB, be the given segments of the base, and from D erect the ⊥ DC. From A and B, by p. 20. b. 1. cor. in Mr. *Howard's Treatise on Spherical Geometry*, draw AC, BC, to meet DC, and be equal to the given sum of the sides, and it is done.

The same, by Mr. J. T. M'Doneld.

Draw the great circle AB (fig. 336, pl. 20.), making AD, BD, the given segments of the base; at D erect the ⊥ great circle DF; then,

Case 1. When the given sum of the sides is *less* than the semi-circumference of the sphere: about B, as a pole, with the given sum of the sides, describe a circle EG, and by prob. 19, pa. 137, of Mr. Howard's Spherical Geometry, describe the circle EA to touch EG, and have its centre C in DF; join AC, BC, and the thing is done.

Case 2. When the sum of the sides is *equal* to the semi-circumference; let E be the point on the sphere opposite to B; draw the great circle EA, and bisect it by the ⊥ great circle *d*C, which will meet DF in C, the vertex of the △. For EC + CB = AC + CB, the given sum of the sides.

Case 3. When the sum of the sides *exceeds* the semi-circumference. With the given base AB, and sum of the supplements of

AC,

AC, BC, let the oppoſite △ AcB be conſtructed as in *Caſe* 1, and produce the ſides till they meet in the oppoſite ſide of the ſphere in C, the vertex of the △ required. For ∠ C = ∠ *c*, and AC, CB, are the ſupplements of A*c*, *c*B.

Ingenious conſtructions were alſo given by Meſſrs. Boſworth, Gregory, Hill, Howard, *the propoſer*, Johnſon, Merones Minor, *and* Thornoby.

XIII. QUESTION 143, *anſwered by Mr.* John Blackwell.

Let ABCD (fig. 337, pl. 20.) repreſent the given can, the ſolidity of which is eaſily found = 525·337043 cubic inches, EFCD the part immerſed in the water; to find its dimenſions, we have AB = 6·5, CD = 8·7, BH = 11·5, and FD = 2; now BD = $\sqrt{(BH^2 + \overline{\frac{CD-AB}{2}}^2)}$ = 11·55249, alſo, by ſim. △s, as BD: BH :: FD : FG = 1·99094, and BH : ½ (CD — AB) = HD : GF : GD = ·1904348, therefore CD — 2GD = EF = 8·3191304, and the content of the fruſtum EFCD = 113·248 024814. Hence, as 1728 : 1000 :: 113·248024814 : 65·53 705139 the weight of the can; and, as 7300 : 1728 :: 65·53705 139 : 15·5134280 5 the ſolidity of the tin, which, taken from that of the whole can ABCD, leaves 509·82361495 for the content of the inſide. Now put x = the thickneſs of the tin, then will the inſide dimenſions be 6·5—2x, 8·7—2x, and 11·5—x, and by Menſuration, [(6·5—2x)² + (8·7—2x)² + (6·5—2x)·(8·7—2x)] × (11·5—x) × ·2618 = 509·82361495, this equation reduces to x^3—19·1x^2 + 101·94833x = 4·938066, in which x is = ·05, or 1-20th of an inch, nearly = the thickneſs of the tin.

The ſame, by the Rev. L. Evans.

By Menſuration, the content of the outer fruſtum is = 2006·635 × ·2618. And putting x for the thickneſs of the tin, 6·5—2x, 8·7—2x, and 11·5—x, are the dimenſions of the inner fruſtum, and $\overline{(6\cdot5-2x)^2 + 8\cdot7-2x^2 + 6\cdot5-2x \times 8\cdot7-2x}) \times \overline{11\cdot5-x}$ × ·2618, or (2006·635 — 1223·29x + 229·2x^2 — 12x^3) × ·2618 is its content, and 2006·635 × ·2618 — (2006·635 — 1223·29x + 229·2x^2 — 12x^3) × ·2618 = (1223·29x — 229·2x^2 + 12x^3) × ·2618 = content of the tin; and (1223·29x — 229·2x^2

$229 \cdot 2x^2 + 12x^3) \times \cdot 2618 \times 7300 =$ its weight, which, by hydroſtatics, is equal to the weight of the water diſplaced. Now, $\sqrt{(BH^2 + DH^2)} = DB = 11 \cdot 552$, and by ſim. $\triangle$s DB : DH :: DF : DG $= \cdot 1904$, and ſo EF $= 8 \cdot 3192$: alſo DB : BH :: DF : FG $= 1 \cdot 991$, and the content of the fruſtum DFEC $= 432 \cdot 596516 \times \cdot 2618$, and its weight $= 432 \cdot 596516 \times \cdot 2618 \times 1000$; therefore $(1228 \cdot 29x - 229 \cdot 2x^2 + 12x^3) \times \cdot 2618 \times 7300 = 432 \cdot 596516 \times \cdot 2618 \times 1000$, and $x^3 - 19 \cdot 1x^2 + 101 \cdot 94x = 4 \cdot 938$, which gives $x = \cdot 05$, or 1-20th of an inch nearly.

The ſame queſtion was ingeniously anſwered by Meſſrs. Boſworth, Byerley, Gregory, Johnſon, Marrat, *the propoſer, and* Merones Minor.

XIV. QUESTION 144, *anſwered by Mr.* O. G. Gregory, *Cambridge.*

Find a mean proportional between AB and AC (fig. 338, pl. 20.) the length and breadth of the card, and let it be ſet from A to the oppoſite ſide CD, falling on the point *a*. On A*a* let fall the perpendicular, BE, which, with the line A*a*, will divide the card as required. For, let the piece AC*a* be removed to the ſituation BD*b*, and the piece AEB to the ſituation *aeb*; and the ſquare BE*eb* is formed: E*e* a ſide of which is evidently equal to A*a*, which, by the conſtruction, is the ſide of a ſquare equal in area to ABDC.

Remark 1. When CD $=$ 2AC, the point *a* would fall in the middle of CD, and divide the card into 3 triangular parts: and when CD is more than double AC, no line from B could be demitted perpendicularly upon A*a*, without producing it; in which caſe the prob. becomes impoſſible.

Remark 2. In the caſe before us, as the parallelogram is already formed, the ſide of an equal ſquare may be determined in a manner different from the common one of finding a mean proportional: thus, ſet off half the length of the parallelogram from A to G, and make G*a* $=$ AG, then is A*a* the ſide of an equal ſquare; as may thus be demonſtrated.

By theorem 36, Geometry in Dr. *Hutton's Courſe*,
$Aa^2 = AG^2 + Ga^2 + 2AG \cdot GC$, or, as $AG = Ga$, $AG^2 = Ga^2$, and $Aa^2 = 2AG^2 + 2AG \cdot GC$: but $GC = AC - AG$, therefore $Aa^2 = 2AG^2 + 2AG \cdot (AC - AG)$, and conſequently
$Aa^2 = 2AG^2 + 2AG \cdot AC - 2AG^2 = 2AG \cdot AC$.
Whence, as 2AG $=$ AB, it is manifeſt that $Aa^2 = AB \cdot AC$.
Q. E. D.

Merones Minor *anſwered this queſtion.*

XV.

XV. QUESTION 141, *answered by Mr.* O. G. Gregory, *the proposer.*

From a little consideration of the nature of this question, and of similar ones proposed, it will be obvious that the number of chords [illegible] the greatest chord, that is $8\frac{1}{4}$, the greatest chord, is the diameter of the circle. Now, let $8\frac{1}{4}$, the diameter, be denoted by a, $168 \div 8\frac{1}{4}$ [illegible] of the chords, viz. AK and AB (see fig. 339, pl. 20.) [illegible] as manifestly deduced from *prop.* 37, pa. 73, *Emer. Tri.* [illegible] $\div a$, $x = BK$, whence, by Euc. 47. 1. $AB^2 + BK^2 = AK^2$, that is, $x^2 + (2 \div a^2)\, x^2 = a^2$;

from which we get $x = \sqrt{\frac{a^2}{1 \div (2 \div a^2)}} = \sqrt{\frac{-056}{8 + 4\sqrt{3}}} = 4^{\text{R}}$

$\sqrt{2 - \sqrt{3}} = AB$. As 42 is the radius of the circle, and as $\frac{1}{2}$ radius $\times \sqrt{2 - \sqrt{3}}$ is known to be the line of 15°, it is evident that the line of AB just found, is equal to twice the line of 15°, or the chord of 30°: therefore the number of equal parts into which the circumference is divided, is 12, the number of chords 5, and their lengths may be soon determined, are 42, $42\sqrt{3}$, 84, $42\sqrt{3}$, and [illegible].

The same, by Mr. Mervyns Minor.

[illegible] the chord AB, diameter AK, and supplemental chord [illegible] $= AB$, then $AK = 8\frac{1}{4}$, $BK = \frac{1}{2}\sqrt{8\frac{1}{4}^2 - x^2}$, and by [illegible] 11. (AK·BK $\div$ AB $= 8\frac{1}{4} \div$ [illegible] $= 8 + 8\frac{1}{4}\sqrt{3}$, per quest.; hence $= 42\sqrt{2 - \sqrt{3}}$ [illegible] and the number of chords was 5.

The same, by Mr. W. Marrat, *Eyton.*

Since the circumference is divided into an even number of equal parts, the greatest chord will manifestly be the diameter; therefore let AK $= 8\frac{1}{4} = a$, $168 \div 8\frac{1}{4} = a$, and $BK = x$, then $AB = \sqrt{a^2 - x^2}$, [illegible] Cor. 1. *pr.* 9, *pa.* 78, *Eme. Trig.* (AK·BK, AB = sum of all the chords, that is, $ax \div \sqrt{a^2 - x^2} = a$, which gives $x = \frac{1}{2}\sqrt{a^2 \div a^2 + 2^2} = 81132$, and $AB = 21$ [illegible] also (by Cor. El. VI. 8.) $AK = AB \div BK = 5{\cdot}6383$, [illegible] $cf. B - = \frac{1}{2}\sqrt{AB^2 - AK^2} = 21 = \frac{1}{2}$ the least chord. Now,

[illegible]

fig. as $Bn = 42 : Bm = 21 :: \text{rad.} = 1 : \text{fin. } \angle AnB =$ and $180° \div 30° = 6$; conſeq. the number of chords is 5.

The ſame by Mr. W. Burdon, *Acaſter-Malbis.*

The number of equal parts the circumf. is divided into being n, the greater chord will be = the diam. of the circle. Now on the rad. AL = 42 (ſee fig. 340, pl. 20.) deſcribe a ſemicircle, and join LD, LB, &c. then the arc AL will be cut into n parts in the points N, M, &c. alſo BQ = LN, DP = LM, &c. Hence, becauſe the $\triangle$s ANL, AML, &c. are mutually = and ſim. to the $\triangle$s LBQ, LDP, &c. reſpectively, the ſum of LA, LM, LN, &c. is = the ſum of LF, DP, BQ, &c. = $63 + 21\sqrt{3}$. Put x = the leaſt chord LN, then $\sqrt{42^2 - x^2} =$ the greateſt chord, and by the Theorem at pa. 8, in the Introduction to Dr. Hutton's Logarithms we have this proportion, $x : 42 :: x + 42 + \sqrt{42^2 - x^2} : 126 + 42\sqrt{3}$, which gives $42 + \sqrt{42^2 - x^2} = (2 + \sqrt{3}) \cdot x$, and $x = 21 =$ fine of 30°, or $360° \div 30° = 12$, the number of = parts the whole circumf. is divided into. Whence, the number of chords is 5, and their lengths 42, $42\sqrt{3}$, 84, $42\sqrt{3}$, and 42.

Neat ſolutions were given to this queſtion by Meſſrs. Boſworth, Elliott, Evans, Lowry, M'Doneld, *and Miſs* Suſan May.

XVI. QUESTION 146, *anſwered by the Propoſer*, Philo.

Let a = the vel. of the ſtream flowing through a given aperture,
x = the vel. of the wheel after its motion is uniform,
n = the number of particles impinging upon a ſingle wheel board from the time of its entering to the time of its quitting the ſtream,
and t = the time of immerſion. Then it is evident that $(a - x).tn$ will be the force of the ſtream upon a ſingle wheel board in any one revolution, and the number of wheel boards acted upon in a given time, is as the vel. (x), of the wheel; theref. $(a - x).tnx$ is the whole force of the ſtream upon the wheel in a given time, which muſt be a *maximum*. But the time t is always inverſely, and the number of particles n is always directly as the impingent $a - x$, therf. $tn = 1$, and the above expreſſion for the fluent comes $(a - x).x$, which gives $x = \frac{1}{2}a$, agreeable to Theory.

The ſame, anſwered by Mr. Lowry, *Birmingham.*

Put V = the velocity of the water, and v = the velocity of the wheel, then $V - v$ is = the relative velocity, or that with which the water ſtrikes the wheel. Now, let the velocity be what it will, it is evident that, the quantity of water that ſtrikes the wheel will always be as the abſolute velocity of the water, that is, as V, and that the force of any ſingle particle will always be as the veloc. with which it ſtrikes, that is as the relative veloc. $V - v$, theref. the whole force or effect of the water upon the wheel is as $V.(V - v)$; conſeq. $Vv.(V - v) = V^2v - Vv^2$, muſt, by the queſt. be a *maximum*, or, ſince V is ſuppoſed to remain conſtant, $Vv - v^2$ muſt be a *max.*; theref. $Vv = 2v^2$, and $v = \frac{1}{2}V$, agreeing with the Theory of Mr. *Waring*, and the accurate experiments of the ingenious Mr. *Smeaton*.

Other ingenious ſolutions were given by Meſſrs. Boſworth, Byerley, Gregory, Marrat, M'Doneld, *and* Meronas Minor.

XVII. QUESTION 147, *anſwered by Mr.* W. Francis.

Let BF and CGE (fig. 241, pl. 20.) repreſent the two maſts, and ABC the ray. Take the angle it forms with the ſurface of the water, which will be equal to the angle CBG, i.e. which right angled triangle all the angles and the ſide BG will be then known. On the oppoſite ſide of the water, at D, in the direction of the two maſts take the angles ADB, and ADC, from whence the angles CBD, and BCD will be eaſily obtained; the line BC being found by plane Trigonometry, BD and afterwards AD, or the water's breadth, will be obtained in a ſimilar manner.

Mr. *Gregory of Cambridge, after giving out a mode of proceeding, nearly ſimilar to the foregoing, gives an example as follows:*

Suppoſe the diſtance FE between the two maſts of the veſſel to be 6 yards, the angle CAD = 18° 26′ $\frac{1}{2}$, CDA = 36° 52′ $\frac{1}{2}$, and BDA = 24° 26′ $\frac{1}{2}$. Then in the right angled triangle BGC, are given the angle B = CAD, and BG = FE, to find BC = 6.324555, and CG = 2. In the triangle BCD, we have BC = 6.324555, the angle CDB = CDA − BDA = 12° 25′ $\frac{1}{2}$, CBD = CBG + GBD = CBG + BDA = 42° 52′ $\frac{1}{24}$, and BCD = 180 − (CBD + BDC) = 124° 41′ $\frac{1}{24}$, whence CD is found = 2°. And laſtly in the triangle ACD, all the angles and the ſide CD are known; therefore AD is readily determined, viz. breadth of the water required.

The

The ſame, by the Rev. L. Evans.

Let AD be the breadth of the water, CE and BF the maſts. At A let the ∠ of elevation CAD be taken; as alſo the ∠s BDF, CDE. Draw BG ∥ to EF. Then in the rt. ∠d △ BGC are given the ſide BG and all the ∠s to find the ſides BC and GC. In the △ BCD are given the ſide BC and all the ∠s to find the ſide CD. In the rt. ∠d △ DEC are given the ſide CD and all the ∠s to find the ſides DE and CE. And laſtly, as CG : BG :: CE : AE; therefore AE + ED = AD the breadth of the water required.

The ſame, by Mr. T. Barber, *St. John's College, Cambridge.*

Let AD the breadth of the water, EC and FB the maſts; join CB which, by the queſtion, will fall on the verge of the water at A, from B draw BG ⊥ to CE, join, alſo, CD, and DB. Firſt let the obſerver take the comp. of the ∠ CAD, from which the ∠CAD will be known, and theref. it's =, the ∠CBG, conſeq. all the ∠s and the ſide BG in the △ BGC are known, BC may theref. be found. Secondly, let the complements of the ∠s CDA, and BDA be taken, whence the ∠s CDA, and CDB will be had, and alſo the ∠ BCD, theref. all the ∠s and one ſide of the △ BCD are known, from which CB will be had and thence AD.

Ingenious ſolutions were given by Meſſrs. Boſworth, Bardon, Hill, Johnſon, Lowry, Merones Minor, *and* Peacock.

XVIII. QUESTION 148, *anſwered by Mr.* John Lowry.

This prob. is a particular caſe of *Prop.* D, *Art.* 34, *Vol. I. of the Repoſitory*, where it is ſhewn that the *locus* of the required point is a circle given in magnitude and poſition.

For, let AC × m^2 (ſee fig. 342, pl. 20.) be equal to the given ſolid. Then by the queſtion

$PA^2 \times AC + PB^2 \times BC + PC^2 \times AB = AC \times m^2$,

therefore $PA^2 + PB^2 \times \overline{BC \div AC} + PC^2 \times \overline{AB \div AC} = m^2$.

Let AB be divided in F,

ſo that AF : FB :: BC : AC, join CF and divide it in X,

ſo that FX : XC :: AB : AC + CB, join alſo AX, BX and PX.

Then by *Prop.* X. *Art* 34, *Vol.* I. *of the Repoſitory*,

$PA^2 + PB \times \overline{BC \div AC} + PC^2 \times \overline{AB \div AC} = AX^2 + BX^2 \times \overline{BC \div AC} + CX^2 \times \overline{AB \div AC} + PX^2 \times (AC + BC + AB) \div AC = m^2$. But X is a given point, therefore AX, BX, CX are given; conſequently PX^2 is given. Hence

with the centre X, and diſtance PX, deſcribe a circle, ſo ſhall any point in its circumference anſwer the conditions of the queſtion, which of courſe is indeterminate without ſome other *datum*.

The ſum of the ſolids will be a minimum when P coincides with X, but the maximum is infinite.

Conſtructions, ſimilar in purport to the above, were made by Mr Gregory, and Mr. Swale, the propoſer.

XIX. QUESTION 149, *enforced by Mr.* W. Wallace, *of Perth Academy.*

Analyſis. Fig. 243. Pl. 20. Let ABC be the △ required, which, becauſe the baſe, and vertical ∠ are given, will be inſcribed in a given circle. Let the line BD, which biſects the vertical ∠, meet the circle in F; the arch FA will be equal to FC; therefore, if the diameter FH be drawn, it will biſect the baſe AC in E; wherefore DE is half the diff. of the ſegments AD, DC. Join HB meeting AC produced in K; the △s FED, KBD are evidently equiangular, theref. ED : DB = EF : BK; now the ratio of twice ED to DB, and theref. the ratio of ED to DB, is by hypotheſis given, and ſince the circumſcribing circle and baſe are given, the line EF will alſo be given in magnitude, theref. BK is given in magnitude: and it is well known that, if AH be joined, KH · HB = AH², that is, (ſince AH is evidently given) to a given ſpace; theref. BH and KH are both given in magnitude. Hence the following conſtruction:—

Upon the given baſe AC deſcribe a ſegment of a circle AHC that may contain an ∠ equal to the given vertical ∠: biſect AC in E, and draw the diameter HEF. Let the given ratio of the line biſecting the vertical ∠ to the diff. of the ſegments of the baſe made thereby be that of $\delta\beta$ to twice $\delta\varepsilon$: find bk a fourth proportional to $\delta\varepsilon$, $\delta\beta$, and FE; join HA, and in bk produced find h, ſo that $kk \cdot kb$ may be = to AH²: place the chord HB in the circle equal to kb; join AB, CB, and ABC ſhall be the △ required.

Demon. Join FB meeting AC in D. By a well known prop. in Geom. KH · HB = AH² = (by conſtr. ($kk \cdot kb$, alſo, by conſtr. BH = bk, theref. KH = kk, and BK = bk: Now the line BD evidently biſects the vertical ∠ ABC, and the △s KBD, FED are equiangular, theref. DB is to DE as KB, or kb to EF, that is, by conſtr. as $\delta\beta$ to $\delta\varepsilon$: whereſ. DB : DC — DA = $\delta\beta$: $2\delta\varepsilon$, that is in the given ratio as required.

The

The ſame, by Mr. John Lowry,

Conſtr. Upon the given baſe AB (fig. 344. pl. 20.) let a ſegment of a circle be deſcribed capable of containing the given vert. ∠, let the baſe be biſected in F, and through F draw the diameter DFH. Through H let HI be drawn ‖ to AB, from F apply FI, ſuch, that its ratio to 2FH may be = to the given ratio of the line biſecting the vert. ∠ to the diff. of the ſeg. of the baſe made by the ſaid biſecting line. On FI, as a diameter, let a circle be deſcribed, and through its centre O let DEOK be drawn meeting the circumf. in E and K; from D, to the circle ABCD, apply DC = DE, and join AC, CB, ſo ſhall ACB be the △ required.

Demon. Let DC, AB be produced to meet in L.

The △s HDC, FDL are ſimilar, theref. DH : DC :: DL: DF, and by the circle DH:DK::DE:DF; theref. DC·DL = DK·DE: but DC = DE, by conſtr. theref. DL = DK,

conſeq. CL = EK = IF.

Hence, by ſim. △s, As CG, the line biſecting the vert. ∠, is to FG, ½ the diff. of the ſeg. of the baſe made by the ſaid biſecting line, ſo is CL, or IF, to FH, that is, CG is to FG in the given ratio. Alſo, the baſe and vert. ∠s are = the given ones by conſtr.

Q. E. D.

The ſame, by Mr. J. T. M'Donell.

Analyſis. Suppoſe the thing done, viz. that ACB (fig. 345 pl. 20.) is the required △, about which a circle being deſcribed biſect the baſe AB by the diam. DF, and draw CD the line biſecting the vertical ∠. Join CF and let it meet AB produced in H. Now the ratio CE to AE — BE being given, that of CE to EG (½ the diff. of the ſeg.) is likewiſe given, which is the ratio DG to CH, becauſe the △s DGE, HCE are ſim.: but DG, and theref. CH is given. Alſo, FH HC = BH HA, *i. e.* $CH^2 + HCF = 2HAG + AH^2$, and, by Eu. 47. I. $HF^2 = GH^2 + GF^2$, *i. e.* $CH^2 + CF^2 + 2HCF = AG^2 + AH^2 + 2HAG + GF^2$; hence $CF^2 + HC{\cdot}CF = AG^2 + FG^2$, *i. e.* HF FC = BF^2. Whence this conſtruction:—

Upon the given baſe AB, deſcribe a ſeg. of a circle to contain the given vert. ∠; complete the circle, and biſect the baſe with the diam. DF, and join BF; then take GO to GD in the given ratio of the line biſecting the vert. ∠ to the diff. of the ſegments of the baſe made thereby, and in GB produced take GK = BF. With the centre O, and rad. OG deſcribe a circle, and from K draw the ſtraight line KNL, to paſs through the centre O, cut the

circle in N and to meet it in L. Lastly apply the distance KN from F to C which will determine the vertex of the Δ.

The same, by Mr. Syrle, Chesley.

Analysis. Let ACB (see fig. 257, pl. 17.) be the required Δ, the base AB being bisected in L by the diameter of its circumscribing circle EF, which is given in magnitude; draw CE, the line bisecting the vert. ∠, meeting AB. Demit upon AB, AC, EF, EC, the ⊥s CD, FH, CN, LQ. Then CK : KL = CD : LQ, and EF : EL = FC : LQ are given ratios, for, EF and EL are given lines; hence CD : DF is a given ratio, theref. $CD^2 : CF^2$, *i. e.* $LN^2 : (CF^2 =)$ FE·FN is given, hence the prob. is reduced to this, *viz.* To cut the right line LF in N so that the square of LN may be to EF·FN in a given ratio, which is done at Prob. 3, of *Wales' Determinate Section.*

The same, by Mercones Minor.

Analysis. Suppose ABC (fig. 346, pl. 20.) to be the Δ required, BD the line bisecting the vertical ∠, and AE the diff. of the seg. of the base made thereby; join BE, and through any point F, in the bisecting line BD, draw IFGH ‖ to AC. Then, by Em. Geo. 13, II. BD : EA :: BF : GH : but the ratio of BF to GH is given, therefore GH is given with respect to BF. Hence this construction:—

On any assumed line BF, describe the rhombus BLFM, having the ∠ LBM of the given magnitude; then by Simp. Geo. Prob. 21, draw LN, so that NO = GH, (BF being to GH in the given ratio) and through F draw IH ‖ to LN. In IH take IK = the given base, and draw KA ‖ to BI, meeting BH in A, and through A draw AC ‖ to IH, meeting BI in C, and ABC will be the Δ required.

Nearly similar to this was the construction given by Mr. Gregory.

The same, by Mr. Johnson, *Birmingham.*

Constr. Fig. 347, Pl. 20. Draw the lines AC, CB to include and CD to bisect the given ∠, and from any point E in CD draw EF, EI ‖ to AC, BC respectively; join FI and take IT to CE in the given ratio of the diff. of the seg. to the bisecting line; draw TS ‖ to CB, and make IH·HF = IS^2. Now from H, to EI, apply HG = IS, join FG and let it meet AC in K, and in ML, draw ‖ to FK through E, take MV = the given base; through V draw VB ‖ to AC and let it meet BC in B: through B draw AB ‖ to ML, and ACB is the Δ required.

Demon.

Demon. Join HK. By constr. IH·HF = IS² = HG
theref. the △s GHI, HFG are sim. and so are the △s GHI, FKI
hence the ∠ GKI = GHI, and the points G, H, K,
are in a circle,
theref. the ∠ GHK = GIK = TSI, and ∠KGH = KIH = TIS,
wheref. the △s KGH, IST are equiangular, also HG = IS,
theref. KG = IT. Now, by parallels, KF = ME and GF = EL, therefore ME — EL = KG = IT; conseq. CE : ME — EL is in the given ratio, and because the △s MCL, ACB, are sim. it follows that CD : AD — DB is in the same given ratio.
Q. E. D.

And nearly in the same manner was the solution given by Messrs. Hill, Peacock, *and* Thornoby.

XX. QUESTION 150, *answered by Mr.* W. Wallace, *Perth Academy.*

In this question, by the rectangle under the sides and the line bisecting the base, the proposer probably meant the rectangle under the sum of the sides and the line drawn from the vertex to the middle of the base; this being supposed, the *Analysis* may be as follows. Suppose ABC (Fig. 348, Pl. 21.) the △ required, let AC its base, which is given in magnitude, be also given in position, then because the vertical angle ABC is also given, the point B is in the circumference of a given circle: Let EF, the diameter, bisect the base AC in D, the point F will be given, and if FA be joined, the line FA will be given in magnitude. Join FB, DB; By a well known prop. in Geom. AC : AF = AB + BC : BF = (AB + BC) BD : BF : BD, now AC, AF are given in magnitude, and, by hypothesis, (AB + BC)·BD is given, therefore, EB.ED is given. Let G be the centre of the circle; in EF produced take H so that GD·GH = GF², bisect HD in K, and draw BL perpend. to EF; the points H, K will evidently both be given. Now by *Prop* 6, *Stewart's General Theorems* (see Vol. I. page 28, of the Repository.).
2DG : DB = DB : KL, and from the nature of the circle,
EF:FB = FB:FL; theref. 2DG·EF:DB·BF = DB·BF:KL·FL:
Now the rect. 2DG·EF is given, for DG and EF are given, and DB·BF has been shewn to be given, theref. the rect. KL·FL is given, and, since the points K, F are given, the point L is also given. Hence the following construction:—

Let AC be the given base and P² the rectangle contained by the sum of the sides and the line drawn from the vertex to the middle of the base: Upon AC describe a segment of a circle that may contain an ∠ equal to the given vertical ∠; draw the diameter EF bisecting AC at D, join AF. Let G be the centre of the circle

circle, and, GF produced, find H ſo that HG · GD = GF² (or, which is the ſame, draw AH touching the circle) and biſect DH in K. Find a line Q ſo that AC : AF = P² : Q²; and in K, produced, find the point L ſo that 2DG·EF : Q² = Q² : KL·FL; draw LB perpend. to EF, meeting the circle in B; join AB, BC, and ABC ſhall be the △ required.

The conſtruction would have been nearly the ſame, had the difference of the ſides been taken inſtead of the ſum.

The ſame, by Mr. John Lowry.

Analyſis. Suppoſe it done, and that ACB (fig. 349, pl. 20.) is really the △ required. Let O be the centre of the circumſcribing circle, and FODG a diameter ⊥ to, and biſecting the baſe in D. Join OA, GB, and drawn CE ‖ to AB, and EV ‖ to GB, draw alſo ES to make the ∠ ESD = AOD = ACB, and AX to make the ∠ AXD = EVD. Let AB × *m* be = to the given rect. under the *ſum* of the ſides and the line drawn from the vertical ∠ to the middle of the baſe. Then,
AC + CB : AB :: *m* : DC, and by *Pr.* 12 *and* 18, *Simp. Trig.* and ſim. △s, AC + CB : 2AB :: BV : AC + CB :: *m* : 2DC, and IS : CD :: CD : AB, (AD being biſected in I);
therefore, BV × IS : (AC + CB)· CD = AB·*m* :: *m* : 2AB,
hence 2BV × IS = *m*².
Again, becauſe AD, and the ∠s AOD, AXD (= ½ ACB) are given, OD and DX are given, theref. by ſim. △s the ratio of DV to DS is given; let this be as DB : BH, (BH being drawn ⊥ to AB). On BH produced take BK = DI and KL = DS; then it will be as
BD : DV :: BH : KL, and-by compoſition,
BV : BD :: BH + KL : BH; therefore,
2BV × IS (= *m*²) : BD :: 2× (BH + KL)·BL : BH, or
2 × (BH + KL) · BL = (BH ÷ BD). *m*². Hence this conſtruction:—

Upon the given baſe AB deſcribe the ſegment of a circle to contain the given vertical ∠, and draw the diam. FOD ⊥ to, and biſecting the baſe. Join A with the centre O, and draw AX to make the ∠ AXD = ½ the given vertical ∠. Let BL be made ⊥ to AB, in which take BH a 4th proportional to DX, DO, and DB, alſo let BK be taken = ½ AD. By prob. 2nd, Wales's Determinate Section, find the point L ſo that (BH + KL) · BL = (BH ÷ 2BD) · *m*². Let LV be drawn ‖ to DH, draw alſo

The

VE to make the EVD = ∠ the given vertical ∠ and let it meet DF in E, then if EC be drawn ‖ to AB, meeting the circle in C, and CA, CB be joined, ACB will be the △ required.

The ſynthetic demonſtration is evident from the preceding Analyſis.

Note. The given rectangle muſt not exceed 2AF·DE, nor be leſs than AB.BD.

In the ſame manner may the △ be conſtructed when the rect. under the diff. of the ſides and the line drawn from the vertical to biſect the baſe is given.

The ſame, by Mr. W. Peacock, *the propoſer.*

Conſtr. Let a ſegment of a circle be deſcribed upon the given baſe AB, (fig. 350, pl. 21.) capable of the given vert. ∠, let AB be biſected in G, and through G let the diam. DF be drawn. Alſo let AF be joined, and upon DF let a rect. DSVF be conſtituted ſuch, that its ratio to the given rect. may be the ſame as the ratio of DF to 2AF. Now make GQ = DG, and produce FD to H, ſo that QF · DH may be = to the ſquare of DG, and draw DK ‖ to the line joining the points QV. By Simp. Geom. 18. V, make HE · DE = KF · FV, and draw EC ‖ to AB, meeting the circle in C, the vertex of the △ required.

Demon. We have only to prove that (AC + CB) · CG is = to the given rectangle, for AB is = the given baſe, and ACB is = the given vertical ∠ by conſtruction. On AC demit the ⊥ DP, then by conſtruction,

DE : FV :: KF : HE, therefore,

DE·DF : FV·DF :: KF·FQ : HE·FQ; but by conſtr.

HE·FQ = DG² + DE·FQ = CG² (by a known prop.), and

DC² = DF·DE; therefore, by the above proportion,

DC² : FV·DF :: KF·FQ : CG² : but, by parallels,

KF : DF :: VF : FQ, theref. FV·DF = KF·FQ; hence

DC² : FV·DF :: FV·DF : CG², or

DC : FV :: DF : CG, theref. DC·CG = DF·FV; but, by ſim. △s,

AC + CB (= 2CP) : DC :: 2AF : DF, therefore

(AC+CB)·CG : DC·CG (= DF·FV) :: 2AF : DF : but, by conſtr.

DF·FV : the given rect. :: DF : 2AF; therefore by equality

(AC + CB) · CG is = to the given rectangle. *Q. E. D.*

r. I. H. Swale, *alſo anſwered it.*

XXI.

XXI. QUESTION 151, *answered by Mr.* Samuel Thornoby.

By a method similar to some of those given in answer to the Prize Question in Ladies' Diary 1798, let a △ DCE (fig. 366, pl. 21) be determined such, that DC may be = to the line bisecting the base, CE = the line bisecting the vertical ∠, and DE multiplied by the square of the ⊥ CF a *maximum*. Produce CE to meet DG, drawn ⊥ to DB, in G, and draw CI ⊥ to CG meeting GD produced in I; through the points G, C, I, let a circle be described meeting DE, produced both ways, in A and B, and join AC, BC, so shall ACB be the △ required, as is evident, any demonstration being unnecessary in this place.

XXII. QUESTION 152, *answered by Mr.* W. Wallace, *Perth Academy.*

Analysis. **Fig. 351,** Pl. 21. Suppose the lines PD, PE drawn as required, *viz.* so that the ∠ DPE may be of a given magnitude, and the ratio of BD to CE also given. Join PB, and make the angle BPG equal to the given angle DPE, also PG equal to PB, thus G will be a given point. Through the points P, B, G describe a circle meeting AB in H; this circle will evidently be given by position; therefore the point H, and the straight line which joins the points G and H will be given by position; let GH meet PE in K. Because the angle BPG is, by construction, equal to DPE, therefore DPB is equal to EPG, or KPG, now PBH is equal to PGH, that is PBD is equal to PGK; also by constr. BP is equal to PG, therefore the △s PBD, PGK are equal to one another, and BD is equal to GK; now the ratio of BD to CE is given, by hypothesis; therefore the ratio of GK to CE is also given; and since the points G, C, and the lines GK, CE are given by position, also the point P, from which is drawn the line PKE: therefore the problem is reduced to the *Sectio Rationis* of the Ancient Geometricians, the method of constructing which is well known.

Construction. Let the given ratio which BD is to have to CE be the ratio of α to β. Join P and B, either of the given points in the lines given by position. Draw PG so that the angle BPG may be equal to the given angle, and PG equal to PB, through the points P, B, G, describe a circle meeting AB in H, and join GH; and from P draw PKE to cut HG, AC, in K and E so that GK may be to CE as α to β (this may be done as in *Simpson's Geometry*, 2*d Edit. Prob.* 37.), draw PD meeting AB in D so that the angle EPD may be equal to the given angle, that is to GPB,

GPB, and alſo BPD equal to GPK: the lines PD, PE ſhall be drawn as required.

For the angles PGK, PBD are equal, and ſince the angle GPK is equal to BPD, alſo the ſide PG to the ſide PB the △s PGK, PBD are equal to one another, and GK is equal to BD: Now by conſtr. GK is to EC as α to β, therefore alſo BD is to CE as α to β.

The ſame, by Mr. Lowry.

Conſtr. Join P and B, one of the given points (ſee fig 365, pl. 21.), and PF to make the ∠ BPF = the given one, and meet AC in F. On PF (produced if neceſſary) take PI: BP in the given ratio of CE to ED; draw FG to make the ∠ PFG = PBD, and draw alſo PQ parallel to FG. By Prob. V. B. I. *Determinate Section, by Wales*, Cut CQ in E ſo that, PQ·EF: EQ·CE :: PF: PI, join EP, and draw PD to make the ∠ BPD = EPF, and it is done.

By conſtr. the angle DPE = BPF = the given angle, alſo by conſtr.

PQ·EF : EQ·CE :: PF : PI. But by ſim. △s, EQ : PQ :: FF : FG, theref. PQ·EF = EQ·FG;

hence EQ·FG:EC·EQ::FG:EC::PF:PI, or FG : PF :: EC : PI.

Again, by ſim. △s, BD : BP :: FG : PF;

Therefore, BD: CE :: BP : PI, that is in the given ratio by conſtr.

Q. E. D.

XXIII. QUESTION 153, *anſwered by Mr.* John Lowry.

Analyſis. Suppoſe ACB (fig. 352, pl. 21.) the △ required. Let GH be a diam. of the circumſcribing, ⊥ to, and biſecting the baſe at N, join CG biſecting the vertical ∠, and on it demit the ⊥s AF, BE, alſo, from C, upon AB and GH demit the ⊥s CD, CI: draw DK ⊥ to EF, and join ED, FD and HC, then by the queſtion the △ EDF muſt be a *maximum*. Now, becauſe of the right ∠s ADC, AFC, the points A, F, D, C, are in a circle, theref. ∠ ADF = ACF = ½ ∠ ACB, and, becauſe of the right ∠s CDB, CEB, the points B, D, E, C are in a circle, theref. ∠ EDK = ECB = ½ ∠ ACB, and therefore the ∠s ACB, FDE, having the ∠s at R equal, and the vertical ∠s alſo equal, are ſimilar. Therefore △ ABC : △ EDF :: CD^2 : DK^2, but CD^2 : DK^2 :: GH^2 : CH^2 (=GH · HI) :: GH: HI, theref. △ ABC: △ :: GH : HI, and theref. HI × △ ABC = GH × △ EDF; but GH is given and conſtant, theref. when the △ EDF is a max. the △ ACB × HI muſt be a max. *i.e.* AN·NI·IH

must

... max. Now it is evident, from Lu. II. 5, that NH·HI ... when NH is bisected at I, therefore NI²-AN ... quadruple AN·HN² a max. and conse- ... HN²-GN, or its equa. HN²-GN must be a max.

... the same method that *Simpson* deduces his 11th ... Max. et Min. from the 15th, it will appear that ... when HN²-GN is a maximum. *Ergo* ...

... GH at N so that HN : GN :: 5 : 1, bisect ... NI, and IC, both ⊥ to GH, meeting the ... BC, and ACB shall be the Δ ... from the preceding analysis.

... proposed by Mr. J. Fletcher, *Hollinwood, near Manchester.*

... ABCG be the given circle, ADC the ∠ required, and let ... the problem agreeable to the problem; ... that the trapeziums ACBG, FDEN, ... that ... AC, FE, ... follows that the ... trap. ACBG : the trap. FDEN :: GB² : ... HN² × GN : HI² × GN :: HG : HI; but the area of ... = ... N × IN, the area that of FDE will be a 4th ... N × NI, ... as this is to be a ... HI² × NI² × ...N is a maximum, ... or will be a max- ... = ... the expression ... N² ... a maximum. It is evi- ... IN, or HN² × AN² = HN² × HN × NG ... maximum, and this is known to be the ... = HG, ... it has been shewn that NI ... = ½I, ... is manifest.

... QUESTION 114, *proposed by the proposer, Mr. W. Wallace, Perth Academy.*

... BC (fig. ..., pl. 21) be the ∠ formed by lines joining ... the given circles, and let G be the given point ... the ∠ required is to pass. Find a ... P, such that PA, PB, PC being joined, these lines may ... each other as the radii of the circles to the centres of which ... drawn. (This may be done as in Simpson's Geom. 2nd Edit. ...) ... in PG, and upon PG describe a segment of a circle ... contain an angle equal to PBC, let this circle meet the circle

circle whoſe centre is B in E, join EG meeting the circle whoſe centre is C in F. Join PE, PF, and draw PD meeting the circle whoſe centre is A in D ſo that the angle EPD may be equal to BPA, and ſo that EPB may be equal to DPA, and let the points D, E, F be either all upon the convex circumferences of the given circles or elſe all upon the concave circumferences; Join DE, DF, EF and DEF ſhall be the △ required.

Draw the radii BE, AD. Becauſe the angle BPE is, by conſtr. equal to APD, alſo BP : BE = AP : AD, the △s BPE, APD are equiangular, therefore BP : PA = EP : PD, and ſince the angles BPA, EPD are equal, the △s BPA, EPD are equiangular. The △s BPC, EPF are alſo equiangular, for the ∠ PEF is by conſtr. equal to ∠ PBC, and if ∠ EPF be not equal to ∠ BPC, ſuppoſe it to be either greater or leſs, and make the angle EPH equal to BPC, and join CH, then the angle BPE is equal to CPH, and, ſince by conſt. BP : BE = CP : CH, the △s BPE, CPH are equiangular, therefore BP : PC = EP : PH, wherefore the △s BPC, EPH are equiangular and the angle PEH is equal to PBC, that is, by conſtr. to PEF which is impoſſible, therefore EPF is not unequal to BPC, that is, it is equal to BPC, and the △s EPF, BPC are equiangular, therefore the remaining △s FPD, CPA are equiangular, and the whole △s EDF, BAC are equiangular.

XXV. QUESTION 155, *anſwered by the propoſer*, *Mr.* James North.

The fluent of the exponential $\dot{x}e^{\frac{2x}{a}}$ is eaſily found $= \frac{1}{2}ae^{\frac{2x}{a}}$.

Put y the ſine, v the coſine, t the tangent, and s the ſecant of the arc x. Then $y\dot{y} = -v\dot{v} = vt\dot{x}$, and hence the given fluxion $\frac{-\dot{z}}{z^3} = \frac{-\dot{y}}{v^4}$. Again, putting $w = \frac{y}{v^2}$, we have $\dot{w} = \frac{v^2\dot{y} - 2vy\dot{y}}{v^4} = \frac{v^2 + 2y^2}{v^4}\dot{y} = \frac{2v^2 + 2y^2 - v^2}{v^4}\dot{y} = \frac{2 - v^2}{v^4}\dot{y}$ $= \frac{2\dot{y}}{v^4} - \frac{\dot{y}}{v^2}$. Hence $\frac{\dot{y}}{v^4} = \frac{1}{2}\dot{w} + \frac{\dot{y}}{2v^2}$.

Alſo, putting $\tau = t + s = \frac{1+y}{v}$, we have $\dot{\tau} = \dot{t} + \dot{s}$ =

$= \frac{v\dot{y} - \dot{v}\sqrt{1+y}}{v^2} = \frac{v\dot{y} + \frac{y\dot{y}}{v}\sqrt{1+y}}{v^2} = \frac{v^2+y+y^2}{v^3}\dot{y} =$

$\frac{1+y}{v^3}\dot{y}$; and $\frac{\dot{\tau}}{\tau} = \frac{\dot{v}}{v^3}$. Hence, finally, $\frac{\dot{v}}{v^4} = \frac{1}{2}\dot{w} + \frac{\dot{\tau}}{2\tau}$; the fluent of which is $\frac{1}{2}w + \frac{1}{2}$ h. l. τ. But this becomes $\frac{1}{2}$ W $+ \frac{1}{2}$ h. l. T, or $\frac{\text{fin. } b}{2 \text{ cof.}^2 b} + \frac{1}{2}$ h. l. (tan. b + sec. b) $= \frac{\text{fin. } b}{2 \text{ cof.}^2 b} + \frac{1}{2}$ h. l. tang. $\sqrt{45 + \frac{1}{2}b}$ when $x = 0$, or when the exponential fluent $\frac{1}{2}ae^{\frac{2x}{a}}$ becomes $\frac{1}{2}a$. Theref. the correct fluents are $e^{\frac{2x}{a}} = a + \frac{\text{fin. } b}{\text{cof.}^2 b} - \frac{\text{fin. } z}{\text{cof.}^2 z} + \text{h. l.} \frac{\text{tan. } 45 + \frac{1}{2}b}{\text{tan. } 45 + \frac{1}{2}z}$.

Mr. Gregory *alſo favoured us with a ſolution to this queſtion.*

XXVI. Or, PRIZE QUESTION 156, *anſwered by the propoſer, the late ingenious Mr.* John Howard.

Suppoſe the ſphere to deſcend by its own gravity along any curve WTBX (fig. 354, pl. 21.) placed in a plane that is perpendicular to the horizon QO ; it is required to determine that point B where it will quit the curve.

At any ſmall diſtance above the earth's ſurface any body deſcending thereto may be ſuppoſed as acted upon by an uniform gravity acting in parallel lines. This being premiſed, ſuppoſe T that point in the curve from whence the ſphere begins its deſcent, and let BD repreſent a portion of the parabola VBD it will (per gravity) afterwards deſcribe, and whoſe curvature at B will be the ſame as in the curve WIBX. Draw CB the common radius, and

BI

BI the common chord, of curvature at B, and produce IB to meet STE, drawn parallel to QO, in E. Alſo through V draw SVK, perpendicular QO, the axis of the parabola, and from B draw AB its ordinate, and BC the common tangent.

The velocity in the curve at B (in the direction BC) is, by Mechanics, $\sqrt{BE} = \sqrt{AS}$, and the velocity in the parabola at the ſame point (and in the ſame direction) is, by projectiles $\sqrt{(VA + \text{ſublimity of the parabola})}$, and theſe velocities being equal by the problem, VS is the ſublimity of the parabola, hence, by the parabola, $BE = AS = VS + VA = \frac{1}{2}AK = \frac{1}{2}BO = \frac{1}{2}BI$.

Ex. 1. Suppoſe VTBX (fig. 355, pl. 21.) a quadrant of a circle in which the ſphere begins its deſcent at any point T below the vertex.

From T let fall TO $\perp$ upon QX and divide it into three equal parts TI, IL, and LO, and through I draw IB $\parallel$ to QO which will cut the curve TX in B the point required, as is plain from the general ſolution.

Ex. 2. Let the curve VBX (fig. 356, pl. 21.) be the cycloid, whoſe equation is $y = z + \overline{2ax - xx}^{\frac{1}{2}}$, where $VA = x$, $AE = y$ (E being that point where the ordinate AB meets the ſemicircle VEO, whoſe diam. is the abſciſſa VO), $z =$ arch VE, $AB = y + z$, and $VO = 2a$. When the quantity x is ſuppoſed conſtant the half chord of curvature will be generally expreſſed by $\frac{(1 + \dot{y}^2)\dot{y}}{-\ddot{y}}$, which is alſo equal to $2x$, by the preſent general ſolution. Now, in the cycloid $\dot{y} = \dot{z} + (a - x)\cdot\overline{2ax - xx}^{-\frac{1}{2}}$, and $\dot{z} = a \cdot \overline{2ax - xx}^{-\frac{1}{2}}$, hence $\dot{y} = \overline{a - x}^{\frac{1}{2}} \cdot x^{-\frac{1}{2}} = \overline{2ax^{-1} - 1}^{\frac{1}{2}}$, therefore $-\ddot{y} = ax^{-2}\cdot\overline{ax^{-1} - 1}^{-\frac{1}{2}}$, which values being ſubſtituted for $\dot{y}$ and $\ddot{y}$ in the general equation $\frac{(1 + \dot{y}^2)\dot{y}}{-\ddot{y}} = 2x$, we obtain, by proper reduction, $x = a$.

Ex. 3. Suppoſe, in the cycloid, that the ſphere begins its deſcent at the point D (fig. 357, pl. 21.).

Put AV $= x$, draw DC $\|$ to OX, and put VC $= n$, then, ſuppoſing every thing elſe as before, we obtain $\dot{y} = \overline{2ax^{-1}-1}^{\frac{1}{2}}$ and alſo $-\ddot{y} = ax^{-2} \cdot \overline{2ax^{-1}-1}^{-\frac{1}{2}}$, whence $\frac{(1+\dot{y}^2)\dot{y}}{-\ddot{y}} =$

$$\frac{(1+2ax^{-1}-1)\overline{2ax^{-1}-1}^{\frac{1}{2}}}{ax^{-2} \cdot \overline{2ax^{-1}-1}^{\frac{1}{2}}} = \frac{2ax^{-1} \cdot \overline{2ax^{-1}-1}^{\frac{1}{2}}}{ax^{-2} \cdot \overline{2ax^{-1}-1}^{\frac{1}{2}}} =$$

$$\frac{2ax^{-1} \cdot (2ax^{-1}-1)}{ax^{-2}} = 2 \cdot (2a-x) = 2x - 2n, \text{ per}$$

queſtion, and $x = a + \frac{1}{2}n$, which, when $n = 1$, is $x = a + \frac{1}{2}$ the ſame as in the Prize Queſtion, Ladies' Diary 1767.

Ex. 4. There is a building in form of a ſemi-ſpheroid its altitude being 43, and the circumference of its baſe 79 feet. It is required to determine how far a marble, deſcending from its apex, will continue to touch the building, and at what diſtance it will fall from the foot of it?

Let VBXC (fig. 358, pl. 21.) repreſent a ſemi-ſection of the building paſſing through its vertex V, and through B draw the ordinate AB. Put VO $= 43 = a$, VA $= x$, OX $= 12{\cdot}573 = b$, and AB $= y$.

From the general equation we have $\frac{\dot{x}^2 + \dot{y}^2}{-\dot{x}\ddot{y}}\dot{y} = 2x$, and in the ellipſe $\dot{y} = (b \div a) \cdot (ax - x^2) \cdot \overline{2ax - xx}^{-\frac{1}{2}}$, and (making $\dot{x}$ conſtant) $\ddot{y} = (b \div a) \times (-\dot{x}^2 \cdot \overline{2ax - xx}^{-\frac{1}{2}} - \overline{ax - xx} \cdot \overline{2ax - xx}^{-\frac{3}{2}} \cdot \overline{ax - xx})$, and ſubſtituting theſe values of $\dot{y}$ and $\ddot{y}$ we have

$\dot{x}^2 \cdot (b \div a) \cdot (a\dot{x} - x\dot{x}) \cdot \overline{2ax - xx}^{-\frac{1}{2}} + (b \div a) \cdot (a\dot{x} - x\dot{x}) \cdot \overline{2ax - xx}^{-\frac{1}{2}}\Big|^2 =$

$2\dot{x}$, or $(b \div a) \cdot \dot{x} \cdot (-\dot{x}^2 \cdot \overline{2ax - xx}^{-\frac{1}{2}} - (a\dot{x} - x\dot{x})^2 \cdot \overline{2ax - xx}^{-\frac{3}{2}}$

$b^2x^3 - a^2x^3 + 3a^3x^2 - 4ab^2x^2 + 3a^2b^2x + ab^2x = a^3b^2$. In numbers $- 1690 \cdot 92x^3 + 211331 \cdot 184x^2 + 883669x = 12568492 \cdot 7178$, hence x will be found equal to 6 nearly; whence $y = \frac{\sqrt{2ab^2x - b^2x^2}}{a} = 6 \cdot 406$. Now since the path BD which the marble afterwards describes is a parabola, we have, by proper reduction XD $= 4 \cdot 557$ feet the distance required.

The same, by Philo, *of Newcastle-upon-Tyne.*

Let AB (fig. 359. pl. 21.) be a curve placed in a plain $\perp$ to the horizon, and suppose the sphere, by the force of gravity, descends from B down the curve and quits it at A.

Draw AP $\parallel$, and FB $\perp$ to the horizon, continue FB to meet a tangent to the curve, at A, in H; draw BC $\parallel$ to AP, and AO $\perp$ to AH, making AO $=$ the radius of curvature at A, let fall OP $\perp$ to AP. Suppose G to be the vertex of a parabola coinciding with the curve in A, having the same radius of curvature AO, and whose parameter is 4CG (DE being drawn through G $\parallel$ to FH). Then it is evident that a body projected from G, horizontally, with the velocity that would be acquired by falling freely through CG, will have the same velocity and direction at A as the sphere which rolls down the curve from B; therefore both the sphere and the body will move in the parabola GA continued below A. But from the nature of the parabola we have AE $= (\overline{CE - CG} \cdot 4CG)^{\frac{1}{2}}$, AD $= (\overline{CE - CG} \cdot 4CE)^{\frac{1}{2}}$, and AO $= 2CE^{\frac{3}{2}} \div CG^{\frac{1}{2}}$, and by the sim. $\triangle$s AED, APO, $(\overline{CE - CG} \cdot 4CE)^{\frac{1}{2}} : (\overline{CE - CG} \cdot 4CG)^{\frac{1}{2}} :: 2CE^{\frac{3}{2}} \div CG^{\frac{1}{2}} : OP = 2CE$. Hence this general rule.

From the nature of the curve, find the ordinate AF, tangent AH, and radius of curvature AO, all in terms of the abciss

FB and known quantities, then FB will be found from this equation. $AF \times AO = AH \times 2FB$.

(3.) The ſphere will never quit the curve if its nature be ſuch, that the ratio of the tangent to the ordinate be always leſs than that of the radius of curvature to twice the abſciſſa.

The ſame, by Mr. John Lowry, *Birmingham.*

Let APB (fig. 360, pl. 21.) be the given curve placed perpendicular to the plane of the horizon, A the point where the globe begins its deſcent, and P the point where it quits the curve. Draw the tangent PQ and ordinate FP; then it is evident that the globe will quit the curve when its horizontal velocity, or the velocity in the direction of its ordinate FP, is the greateſt poſſible. Now, by mechanics, the flux. of the curve AP, is to the flux. of its ordinate FP, as the velocity of the globe at P in the direction of the tangent PQ, is to its velocity in the direction of the ordinate FP, therefore $(\dot{FP} \div \dot{AP})$ multiplied by the velocity at P in the direction PQ, is a maximum. But the velocity at P in the direction PQ is as $\sqrt{AF}$, the height fallen through, therefore $(\dot{FP} \div \dot{AP}) \times \sqrt{AF}$, or its equal $\dot{FP} \times \sqrt{AF} \div \sqrt{\dot{AF}^2 + \dot{FP}^2}$ muſt be a maximum. Hence this general rule:

Find the value of $\dot{FP}$ from the equation of the curve in terms of AF and known quantities, ſubſtitute the value thus found inſtead of $\dot{FP}$ in the above expreſſion, then the fluxion of this expreſſion being equated or made equal to nothing, we ſhall obtain, by reduction, the value of AF, and from thence the point P is readily determined.

The above is applicable to all caſes where the curve is referred to a fixed axis, but if the curve be a ſpiral whoſe pole is the centre of force, the ſolution will be conſiderably different. In this caſe let SPB (fig. 361 and 362, pl. 21) be the given ſpiral whoſe pole is the centre of force S, B the point where the globe begins its deſcent, and P the point where it quits the curve. Then, ſince the force of gravity is inverſely as the ſquare of the diſtance from the centre of force, it follows, from the known laws of centripetal forces, that the globe would deſcribe a conic ſection whoſe focus is the centre of force S, was it not prevented from ſo doing, by moving in the ſpiral track BPS, in which track it will continue to move till its velocity becomes the ſame as that in a conic ſection at the ſame diſtance SP.

Now let APR be the conic ſection which the globe will deſcribe if it quits the ſpiral, S and I its foci, AR the tranſverſe, and the conjugate axis, draw the tangent PC, which will touch both

both the curves at P, for ſince the velocity is the ſame in each at that point, the curvature of each muſt alſo neceſſarily be the ſame; from the focus S on PC demit the ⊥ SC, join SB, SP, and with the centre S and diſtance SP deſcribe a circle to cut SB in E. Put SB $= a$, the tranſverſe axis $= 2b$, SP $=$ SE $= x$, and SC $= y$; then, by *Emerſon's Centripetal Forces* Prop. xiv. Cor. 1, the velocity in a circle at the point P, is to the velocity in a conic ſection, at the ſame diſtance, as $\sqrt{b}$ to $\sqrt{2b \pm x}$ (the upper ſign being for the hyperbola, and the lower one for the ellipſis): but *ibid.* Prop. v. Cor. 5, the velocity in a circle is reciprocally as the ſquare root of the diſtance, that is, as $x^{-\frac{1}{2}}$; therefore the velocity in the conic ſection will be $= \sqrt{(2b \pm x) \div bx}$.—Again, ſince the velocities are equal at equal diſtances from the centre of force S, it follows that, the velocity in the ſpiral at P, will be the ſame as the velocity at E, acquired by decending along the right line BE; then by *Vince's Fluxions Art.* lxxiii. *Ex.* 3, the velocity at P is $= \sqrt{(2a - 2x) \div ax}$, therefore $(2b \pm x) \div bx = (2a - 2x) \div ax$, and, by reduction $2b = \mp a$; conſequently the tranſverſe axis of the conic ſection which the globe will deſcribe after it quits the ſpiral is allways equal to the diſtance between the point where the globe begins its deſcent and the centre of force. Now to determine the diſtance SP, by prop. xiv. *Emerſon ibid.* the velocity in the ſpiral is to the velocity in a circle at the diſtance x, as $\sqrt{yx} : \sqrt{xy}$, hence, it follows, that, if from the nature of the ſpiral we determine the value of y, we ſhall obtain the ratio of $\sqrt{yx}$ to $\sqrt{xy}$ in terms or x and known quantities, that is, we ſhall obtain the ratio of the velocity of the Globe in the ſpiral, to that in the circle at the ſame diſtance in terms of x and known quantities, let this ratio be expreſſed by that of $n : m$, then by what is done above, it is evident that $\frac{m}{n\sqrt{x}} = \sqrt{\frac{2b \pm x}{bx}}$, or $\frac{m}{n} = \sqrt{\frac{2b \pm x}{b}}$, or $\frac{m^2}{n^2} = \frac{a \pm x}{\frac{1}{2}a}$, from which equation x becomes known, and conſequently the point P.

The ſame, anſwered by Mr. Richard Elliott, *Liverpool.*

Suppoſe BC (fig. 363, pl. 21.) to be the given curve whoſe vertex is B, and let E be the point where the body quits the curve, after rolling from the point G.—Now it is well known

that

that when the body quits the curve, the velocity in the direction of the ordinate DE will be the greateſt poſſible, and it is alſo well known that the velocity of the body in the direction of curve, is to the velocity in the direction of the ordinate, as the fluxion of BE to the fluxion of DE, alſo, by the laws of falling bodies, $\sqrt{FD}$ will expreſs the velocity at E, therefore $BE : DE :: \sqrt{FD} : DE\sqrt{FD} \div BE =$ the velocity of the body in the direction of the ordinate DE, the fluxion of which being made $= o$, and the equation properly reduced, it is evident that BD will be found, and thence the point E.

Let the curve be the quadrant of a circle, the radius of which call a, put $BF = m$, $BD = x$, and $DE = y$; then, by the equation of the circle, $y^2 = x(2a - x)$, moreover by *Simpſon's Fluxions. Art.* 142, the flux. $BE = a\dot{x} \div \sqrt{2ax - xx}$, and the flux. of $DE = \dot{x}(a - x) \div \sqrt{2ax - xx}$, conſequently, from the general proportion above, $(a - x)\sqrt{x - m} \div a$ will be the velocity in the direction DE, which being fluxed and reduced, gives $x^2 + \frac{1}{3}(2a + m)x = \frac{1}{3}(a^2 + 2am)$, from which x may be found.

If $m = o$, the equation becomes $x^2 + \frac{2}{7}ax = \frac{1}{3}a^2$, from which x is found $= \frac{1}{7}a$; hence it appears that when the body rolls from the vertex, in a circle, that BD is exactly $= \frac{1}{7}$ of the radius.

The ſame, by Mr. John Surtees, *Bp. Wearmouth, Sunderland.*

Let AM (fig. 364, pl. 21) be the given curve, VMN a parabola, V its vertex, F its focus, BC = 1-4th of the parameter, and TM a tangent to the parabola and given curve at the point M where the ſphere leaves it.—Put $x = AP$, $y = PM$, and $t = TP$, then, by *Emerſon's Mechanics, Prop.* 38, and the nature of projectiles, $AP = BM = MF$, by conics, alſo by *Em. Flux. pa.* 218, the radius of curvature of the parabola at the point M is $(2x \div y)\sqrt{t^2 + y^2}$, which being put equal to the radius of curvature of the given curve at the ſame point, that which is required may be found.

Sup. AM a circle, rad. $= r$, and vertex A, from whence the ſphere deſcends to M, and there leaves it.

Then $y = \sqrt{2rx - xx}$, $t = (2rx - x^2) \div (r - x)$, and $(2x \div y)\sqrt{t^2 + y^2} = r$, or $\sqrt{3x^2 + 2rx} = r$, hence $x = \frac{1}{3}r$.

The Medal for ſolving the Mathematical Prize Queſtion, belongs to *Philo, of Newcaſtle-upon-Tyne*, who is requeſted to ſend for it to Mr. Glendinning.

ART-

ARTICLE XXXIV.

MATHEMATICAL QUESTIONS.

To be answered in Number XI.

I. QUESTION 183, *by Mr.* John Collins, *Schoolmaſter, Kenſington.*

SUPPOSE a gentleman has 5000l. which he puts out at 5 per cent. per annum, compound intereſt; how much muſt he expend every day ſo that principal and intereſt may be entirely exhauſted in 7 years (365 days being a year)?

II. QUESTION 184, *by Mr.* John White, *Bennington.*

A curious gentleman ordered his gardener to lay out a new garden, in form of a circular ſegment, to contain 2 acres, and the verſed ſine to be 4 chains; and as he intends to have a wall quite round it, 7 feet high and 1 foot and a half thick, he wants to know how many ſolid yards the wall will contain, in order to aſcertain the expence: but his gardener being unſkilled in calculation requeſts that ſome ingenious gentleman will find him the diameter of the circle and the content of the wall?

III. QUESTION 185, *by* Merones Minor.

Required the force of gravity at the earth's ſurface, in lat. 30°, ſuppoſing *that*, at the equator, to be 1, and the ratio of the equatorial diameter to the axis as 231 to 230?

IV. QUESTION 186, *by Mr.* J. Barr, *Student in Mr.* Robert Wallace's *Academy, St. John's Lane, Weſt Smithfield, London.*

If the moon were to revolve about the earth's ſurface in a circular orbit at the diſtance of 100000 miles from the earth's centre. It is required to determine the velocity, and periodic time, allowing the ſemidiameter of the earth to be 21000000 feet?

V.

V. QUESTION 187, *by Mr.* John Blackwell, *Hungerford.*

On April 28, 1799, in lat. 51° 23′, at eight o'clock, I obſerved a right-cone ſtanding on a true horizontal plane, the circumference of whoſe baſe was 15 feet, around which I applied a cord 36 feet long, and taking both ends in my hand I found it would exactly reach the extremity of the cone's ſhadow. Query the ſolidity of the cone?

VI. QUESTION 188, *by Mr.* J. T. M'Doneld, *Holland, near Wigan.*

To find 3 fractions ſuch that the product of any two of them being ſubtracted from unity may leave a perfect ſquare?

VII. QUESTION 189, *by Mr.* Francis, *Hampſtead, Middleſex.*

Two ſhips take their departure from the ſame port at the ſame time, the one ſailing S. by W. ½ W. the other S. by E. ½ E. from the meridian of the place: after being out 2½ hours they found a rock, (which is known to be 9 leagues from the port, and on the ſame meridian,) to be in a direct line between them, and that the wind, being N. E. cauſed a difference of 1½ league per hour in the veſſels' ſailing. Query their diſtance from each other and at what rate they ſailed?

VIII. QUESTION 190, *by Mr.* W. Burdon, *Acaſter Malbis, York.*

Each ſide of the baſe of a triangular pyramid is 5, and its perpendicular altitude 9; which being ſet upon its baſe on a plane that makes an angle with the horiſon of 33 ½, will juſt ſupport itſelf. Query the poſition in which it was placed?

IX. QUESTION 191, *by Mr.* William Marrat, *Boſton.*

A merchant ordered his carpenter to make him a crane for the purpoſe of unloading ſhips, &c. the length of the upright piece to be 8, and the length of the horizontal piece 4½ feet; now the carpenter, being unſkilled in the ſcience of mechanics, thinks, that another piece, reaching from the farther end of the horizontal piece to the lower end of the upright one, will ſupport the horizontal piece with the greateſt force poſſible: but his employer him he is wrong; he, therefore, requeſts the favour of ſome learned

learned gentleman to inform him what length the oblique piece ought to be to have the desired effect?

X. QUESTION 192, *by Mr.* Olinthus Gregory, *Cambridge.*

The generating circle of a cycloid is 16. Now, if half the cycloid revolve about its base and about its axis respectively, it is required to determine the difference of the greatest cylinders which can be cut out of the solids generated by those revolutions?

XI. QUESTION 193, *by the Rev.* L. Evans, *Royal Academy, Woolwich.*

If a conical frustum, the length of which is 20, and the diameters of the greater and less ends 4 and 3 feet respectively, be suspended by the centre of the less end, how many vibrations will it make in a minute, and what will be the distance between its centre of gravity and that of suspension?

XII. QUESTION 194, *by Mr.* Robert Wallace, *Master of the Academy, in St. John's Lane, West Smithfield, London.*

Let two points *n*, *m*, move from two given positions B, C, with given velocities along two given straight lines, CB, CA, making any given angle with each other at the point C. It is required to determine the curve to which the straight line *nm*, joining the points *n*, *m*, shall always be a tangent?

XIII. QUESTION 195, *by Mr.* J. T. M'Donald.

Given the perimeter of a plane $\triangle$ to construct it so, that the line bisecting the vertical $\angle$, and terminating in the base, may have to the segment of the base, intercepted by the said bisecting line and the $\perp$, a given ratio, and moreover that the sum of the squares *plus* the rectangle of the sides may be equal to the square of the base.

XIV. QUESTION 196, *by Mr.* J. H. Swale, *Chester.*

Given the two sides to construct the plane $\triangle$ when the rectangle of the segments of the base made by the $\perp$ has to the square of the $\perp$ a given ratio.

XV.

XV. QUESTION 197, *by Mr.* Harris, *Carmarthen.*

It is required to determine, by a geometrical conſtruction, the neareſt diſtance between two ſtraight lines given by poſition, but which are not in the ſame plane.

XVI. QUESTION 198, *by Mr.* O. G. Gregory, *Cambridge.*

At what depth, below the ſurface of the water, muſt an aperture, 1,5 inches ſquare, be cut in a ſluice door, ſo that 166·2938889 gallons of water may be diſcharged through it in a ſecond?

XVII. QUESTION 199, *by Mr.* J. Johnſon, *Birmingham.*

Given the vertical $\angle$ and the length and breadth of a rectangle inſcribed in a plane $\triangle$ to conſtruct it, when the rectangle contained by the ſegments of the ſides intercepted between their points of contact with the given rectangle and the angular points at the baſe, may be of a given magnitude.

XVIII. QUESTION 200, *by Mr.* W. Peacock, *Birmingham.*

In a given $\triangle$ to inſcribe a rectangle ſuch that its perimeter may be a maximum.

XIX. QUESTION 201, *by Mr.* J. Fletcher, *Hollinwood.*

As it is well known that the $\triangle$ EDF, formed by joining the three points where perpendiculars from the extremities of the baſe and vertical angle meet the line biſecting that angle and the baſe reſpectively, is always ſimilar to the original $\triangle$ ABC (ſee ſolution to queſt. 153), it is evident that another $\triangle$ may, by the ſame operation, be formed from the $\triangle$ EDF which will be ſimilar to both the former, another from the laſt and ſo on *ad infinitum*, now ſuppoſing all theſe poſſible $\triangle$s drawn and circles circumſcribing them. *Query*, where will be the vertex of the $\triangle$ ABC when the ſum of all the diameters of theſe circles is equal to a given line, the diameter HG of the original circumſcribing circle being given.

QUESTION 202, *by Mr.* Newton Boſworth, *Cambridge.*

In the Scholium to Prop. 27, B. I. of the Principia, the following

owing theorem is given without a demonſtration. If a ſtraight ine is drawn through any point given by poſition ſo as to cut a given conic ſection in two points, and the diſtance of the interſections is biſected, the locus of the point of biſection will be another conic ſection of the ſame kind with the former, and having its axes parallel to the axes of the former. Query demonſtration?

XXI. QUESTION 203, *by Mr.* Lowry, *Birmingham.*

From one of the extremes B of the diameter AB of a given ſemicircle, whoſe centre is O, to draw a line BC to terminate in the circumference at C, interſect a chord AD, given by poſition, in E, and the radius DO in F ſo that the ratio CE to EF may be a given one.

XXII. QUESTION 204, *by Mr.* Lowry.

From one of the angular points B of a given plane △ ABC to draw two ſtraight lines BH, BI to terminate in the oppoſite ſide AC at H and I and interſect a line DE given by poſition and parallel to AB in G and F ſo that AH may have to HI a given ratio, and GF be of a given length.

XXIII. QUESTION 205, *by* A. B.

If there be a ſeries of quantities $a + bx + cx^2 + dx^3 + ex^4 +$ &c. and the nth differences of a, b, c, d, &c. be equal among themſelves; it is required to find the ſcale of relation?

XXIV. QUESTION 206, *by*, James Ivory, Eſq.

THEOREM. Let ABC (fig. 367, pl. 21.) be a right angled triangle, right angled at B: draw AD to biſect the angle BAC and biſect BD in E: From the centre A deſcribe the arch BF and draw FG to touch the circle and to meet AE in G: Then will $AB^2 \times FG = (AB + BD)^3 - (AB - BD)^3$. Query a demonſtration.

XXV. QUESTION 207, *by Mr.* Bulmer.

I have a Fahrenheit's thermometer whoſe ſcale is not properly adjuſted to the tube, which is cylindrical, for when the bulb is

 immerged

immerged in water, juſt freezing, the mercury ſinks to 25°, and when in boiling water, it riſes to 230°. Now it is required to determine what intermediate degrees of this ſcale correſponds with the degrees of *temperate*, *ſummer-heat*, and *blood-heat* marked on a Fahrenheit's truly graduated.

XXVI. QUESTION 208, *by* A. B.

If a body move in a curve round a centre of force, and if the equation, expreſſing the relation between the diſtance of the body from that centre and a perpendicular drawn thence to the tangent, be $p = \frac{y^n}{a}$; it is required to find the equation of the curve of aberration. N. B. y = diſtance of the body from the centre of force, and p = perpendicular thence upon the tangent; a and n being any given numbers.

XXVII. QUESTION 209, *by Mr.* Lowry.

Let BC, EF be two circles given in magnitude and poſition, and let A be a given point without them. It is required to draw two ſtraight lines AEF, ABC ſo that the angle FAC may be of a given magnitude, and ſo that the ſum, or difference of the ſquares of the chords EF, BC may be equal to a given ſpace.

XXVIII. PRIZE QUESTION 210, *by Mr.* Wm. Wallace.

Let any number of circles given by poſition interſect each other at a point P; it is required to draw a ſtraight line from P to meet all the circles in A, B, C, &c. ſo that the ſum of the ſquares of PA, PB, PC, &c. may be equal to a given ſpace.

ARTICLE

ARTICLE XXXV.

Method of diſcovering the number of negative and impoſſible Roots, in any equation.

By WILLIAM FREND, *Eſq.*

ALGEBRAISTS, who deal in negative or impoſſible numbers, ſuppoſe, that every equation has as many roots as there are units in the higheſt index of the unknown number in the equation. Conſequently, as the roots of an equation are ſeldom ſo numerous, they complete the number by roots of other equations, which they call the roots of the given equation, and if the ſum of the numbers of the roots, in the two equations, does not amount to the higheſt index in the given equation, the difference between the higheſt index and that ſum gives a number which they call the number of impoſſible roots. To diſcover theſe negative roots, or impoſſible roots, is of no uſe whatever, but they, who are curious after them, may find their number in the eaſieſt way, by the following proceſs.

In any given equation find the number of roots by the proceſs laid down in the ſecond part of my Principles of Algebra. Change the ſigns of the terms, in which the index of x is an odd number. Find the number of roots, in this transformed equation, which is the number of negative roots to the given equation. Add together the two numbers, thus found, and ſubtract the ſum from the higheſt index of x, in the given equation, the remainder is the ſum of the impoſſible roots.

Examples.

Ex. 1. $x^4 - 3x^3 + 75x = 1000$.

$x^4 - 3x^3$ is ſubtractive, as long as x is leſs than 3, but $75x$ cannot evidently equal 1000, if x is leſs than 3, conſequently there can be only one root to this equation.

Change the ſigns of the odd indices, whence the equation becomes

$$x^4 - 3x^3 - 75x = 1000;$$

This equation muſt have one root, and can have but one.

Hence the given equation has one poſſitive and one negative root. Conſequently $4 - 2$ or two is the number of impoſſible roots.

Ex. 2. $x^3 - 17x^2 + 54x = 350.$

$x^3 - 17x^2$ is ſubtractive if x is leſs than 17 and the number to be ſubtracted is the greateſt, when x is equal to $\sqrt{\frac{34}{3}} = \sqrt{11,\ldots}$ $= 3, \ldots$ Conſequently the unknown ſide cannot be equal to 350 if x is leſs than 3, ... and the equation can have only one root.

Change the ſigns of the odd indices

$$-x^3 - 17x^2 - 54x = 350$$

This is a falſe equation, conſequently there are no negative roots and ſubtracting one from three, the number of impoſſible roots is two.

Ex. 3. $x^3 - 39x^2 + 479x = 1881.$

$x^3 - 39x^2$ is ſubtractive if x is leſs than 39 and the number to be ſubtracted is the greateſt when x is equal to $\sqrt{\frac{39x^2}{3}}$ or $\sqrt{\frac{78}{3}}$ or $\sqrt{26} = 5, \ldots$ in which caſe $39x^2 - x^3 = x^2 \times 39 - x = x^2 \times 34 = 5 \times 170 = 850$ nearly and $479x = 2395$ and $2395 - 850$ is leſs than 1881 conſequently this equation can have but one root, and equations in this form cannot have a negative root, conſequently there are two impoſſible roots.

Ex. 4. $14937x - 1998x^2 + 80x^3 - x^4 = 5000$

$14937x - 1998x^2$ is additive, if x is leſs than $\frac{14937}{1998}$ or 7, ... and, it is evident, that one root is leſs than unity, and the greateſt additive number $14937x - 1998x^2$ is when x is equal to $\frac{14937}{2 \times 1998}$ or 3, ... at which time it is far greater than 5000, and the part $80x^3 - x^4$ increaſing till x is equal to 6 it is evident, there cannot be another root till x is greater than 7,.... There muſt be another root at leaſt and as $-1998x + 80x$

$+ 80x^3$ is ſubtractive till x is equal to 249, ... and is the greateſt when $x = 16$, ... if the ſubtractive part is greater than the additive when x is equal to 16, ... there muſt be another root leſs than 16 but $80x^3 - x^4$, in this caſe, $= 16^3 \times \overline{80 - 16} = 16^3 \times 64$ and $1998x^2 - 14937x = 16 \times \overline{1998 \times 16 - 14937} = 16 \times \overline{31968 - 14937} = 16 \times 17031$ but $16 \times 17031 \backsim 16^3 \times 64 = 16 \times \overline{17031} \backsim 16^2 \times 64$ or $16 \times 8 \times \overline{2128,....} \backsim 16^2 \times 8$ or $16 \times 8 \times 8 \times \overline{266,....} \backsim 16^2$. Conſequently, the ſubtractive number being greater than the additive, there muſt be a root leſs than 16. Now $80x^3 - x^4$ is the greateſt when x is $= 60$, in which caſe it is equal to $60^3 \times \overline{80 - 60}$ or $60^3 \times 20$ or 216000×20 which is leſs than $60 \times \overline{1998 \times 60 - 14937}$ or $60 \times \overline{119880 - 14937}$ or 104943×60 conſequently, if the increaſe of $1998x^2 - 14937x$ is greater than that of $80x^3 - x^4$, for every value of x between 16 and 60 this equation could not have another root. To try this, it is to be conſidered, that the rate of the increaſe of $80x^3 - x^4$ is the greateſt when $3 \times 2 \times 80x$ is equal to $4 \times 3 \times x^3 =$ or x is equal to 40 in which caſe $80x^3 - x^4 = x^3 \times \overline{80 - x} = \overline{40}^3 \times 40 = 2560000$ and $1998x^2 - 14937x = 40 \times \overline{1998 \times 40 - 14937} = 40 \times \overline{79920 - 14937} = 40 \times 64983 = 2599320$; conſequently, as by decreaſing x, the part $1998x^2 - 14937x$ decreaſes much faſter than $80x^3 - x^4$, it appears evident almoſt, that there muſt be two more roots. Let $x = 30$ then $80x^3 - x^4 = 30^3 \times \overline{80 - 50} = 27000 \times 50 = 1350000$ and $1998x^2 - 14937x = 30 \times \overline{59940 - 14937} = 30 \times 45003 = 1350090$; Hence $80x^3 - x^4$ increaſing faſter than $1998x^2 - 14937x$ there muſt be two roots between 30 and 40 and conſequently the given equation is a complete equation, having four roots, or the number of its roots is the ſame as the number of the higheſt index. This form of equation is incapable of having negative roots.

Ex. 5. $155x^2 - 20x^3 + 7x^4 - x^5 = 10000$.

$155x^2 - 20x^3$ is additive, if x is leſs than 7·75, and is greateſt when x is $= 5,....$ in which caſe, $7x^4 - x^5 = x^4 \times \overline{7 - 5,...} = x^4$

$= x^4 \times 1,\ldots$ or a number less than 1000, and $155x - 20x^3 = x^3 \times \overline{155 - 100} = x^3 \times 55$; consequently the unknown side is less than 10000; and the equation cannot have a root, if x is less than 5,.... but if x is greater than 5, both additive parts decrease, consequently this equation is a false equation.

Change the signs of the terms, where the index is an odd number, whence the equation becomes

$x^5 + 7x^4 + 20x^3 + 155x^2 = 10000$; but this equation must and can only have one root. There is therefore one negative root, and by subtracting one from 5, it appears, that the equation has four impossible roots. *

ARTICLE XXXVI.

LONDON, *October* 24, 1799.

To the Editor of the Mathematical Repository.

SIR,

AS you have inserted, in your Mathematical Repository, some corrections of the learned Mr. Landen, taken from the London Magazine improved, and the observations there given on the solution of the learned and profound Mr. Emerson, do not appear to me strictly just, I flatter myself, my remarks, thereon, will not be unacceptable to the readers of your Repository,

I am, Sir, your humble servant,

BENJAMIN GOMPERTZ.

After the correction of Mr. Landen, with regard to Simpson's solution of the Tides, he enquires whether Emerson's solution, to the said problem, be right or wrong? and his reasoning thereon tends to prove it erroneous. But had I answered his question, I should have said, though it be not perfectly true, it is an approximation, and as such, should be considered just. Landen begins thus, "Mr. Emerson (to whose characters I refer) makes the gravity at P the same as at A; which, though the difference is very small, occasions a very considerable error in the conclusion." But this does not occasion so very considerable an error as Landen seems to imagine, for it makes the height less than it should be

* See on the mode of finding Roots—*The Principles of Algebra: or the True Theory of Equations established on Mathematical Demonstrations.* By W. FREND. Sold by Robinsons, Paternoster-row; White, Fleet-street; Ridgway, York-street, St James's Square; Jennings, Hatton-garden; and Deighton, Cambridge.

but

art of the whole, (extremely near,) and it is not reaſou-
ppoſe Emerſon was not aware of this. Now the pertur-
e on the particles in the periphery of any circle perpen-
AC, being the ſame throughout, and the boundary of
perpendicular to AC being a circle when the perturbating
not act, will conſequently be a circle when the ſaid force
and the lines parallel to ACB being lengthened and thoſe
PCQ ſhortened, the boundary of any plane parallel to
ll become an ellipſis, therefore the ſea from a ſphere
ne an oblong ſpheroid, whoſe axis is AB. Conſe-
om the corollary at page 373, of *Emerſon's Fluxions*
reaſoning is the ſame whether the ſpheroid be oblate
) we find the gravity at P is nearly: the gravity
$+ (a-b) \div 5 : 1$. Landen proceeds, " his
: perturbating force of S, on a particle at D, is alſo er-
ting $\frac{2}{3}fy$ inſtead of fy) and he has omitted the force of
ticle at E, in the direction EC." Now this remark is
s juſt, for, by the laws of fluids, it is the ſame thing if
in the direction EC, on any particle on the column
t acted on any particle in the column AC, in the di-
, and the perturbating force of a particle in the column
diſtance y from C, in the direction $PC = \frac{1}{3}fy$, which
dered (as above) to act on the particle at D, we have
force at $D = \frac{2}{3}fy + \frac{1}{3}fy = fy$; the ſame as Emer-
it. Indeed this way of proceeding is not perfectly juſt,
are more particles in CA than in CP, and when y be-
ter than CP the force will only be $\frac{2}{3}fy$; but this force
ſo few particles that the effect will be inſignificant, and
e eaſy enough to take all in conſideration if neceſſary.
ng the gravity at A, we have (from above) the gravity

$+\frac{a-b}{b}g$ nearly, therefore the gravity at $E = \frac{gx}{a} +$

therefore Emerſon's quotation becomes by this ſmall

$\frac{gx\dot{x}}{a} - fy\dot{y} = \left(\frac{g}{a} + \frac{a-b}{5a^3}g\right) x\dot{x}$, which expreſſes

n of the equation of the values of the columns that
ance each other, the whole fluent of which gives, $a -$

$+\frac{a-b}{5a^3}b^5 = b + \frac{a-b}{5}$, extremely near, there-

fore

fore, $a - b = \frac{4}{3}\frac{fa^2}{g}$ the ſame as Landen makes it himſe

I do not think it very ſtrange that Emerſon ſhould have c himſelf with his approximation, ſeeing the problem, c the adjuſting the gravity at A and P_1 is much more diffi the propoſed problem itſelf would be without ſuch ad and that the reſult in this manner, differs but by $\frac{1}{3}$ par whole.

ARTICLE XXXVII.

THREE PROPOSITIONS FROM LAWSO

(To be anſwered in No. XI.)

PROP. XXXVII.

LET A and B be two points in the diameter of a circl centre is C, and let the rectangle ACB be equal to th of the ſemidiameter; biſect AB in D, and raiſe the p cular DM; from the point A draw AF to any point circumference, and FE perpendicular to DM; then I ſay ſquare of AF is equal to twice the rectangle contained and FE.

PROP. XXXVIII.

If any regular figure be circumſcribed about a circle, a any point within the figure there be drawn perpendicula the ſides of the figure; the ſum of the perpendiculars will to the multiple of the ſemidiameter of the circle by the nu the ſides of the figure.

PROP. XXXIX.

Let there be any number of right lines interſecting in and making all the angles about the point equal, and let an paſs through the ſame point; I ſay the circumference the be divided by the interſecting lines into as many equal parts are lines.

ARTICLE XXXVIII.

ON PRESSURE.

By Mr. JOHN LOWRY.

PROBLEM I. Fig. 368, Pl. xxii.

GIVEN the direction of two forces, supporting a body in equilibrio, to find the ratio of thoſe forces.

Let the body D be ſupported by the two forces DC, DF, acting in the given directions Dc, D*f*, and let DB, drawn perpendicular to the horizontal plane PQ, repreſent the force and direction of the body D, and compleat the parallelogram BCDF. Then, by the compoſition and reſolution of forces, the force DB is equivalent to the ſum of the forces DC, DF; but DF is = BC; therefore the force DC, acting in the direction Dc, is to the force DF, acting in the direction D*f*, as CD to CB, that is, as the ſine of the angle DBC or BDF to the ſine of the angle BDC.

Note, the lines CD, DF, DB are ſuppoſed to be in the ſame plane.

PROBLEM II. Fig. 369. *Pl.* xxii.

Given the poſition of two planes, ſupporting a given ſphere, to find the preſſure againſt each plane.

Let AE, BE, be the planes given by poſition, with reſpect to the horizontal plane PQ, and OAB the given ſphere, whoſe centre is O. Conceive a plane to paſs through the points of contact A, B, and at right angles to the horizontal plane PQ; this plane, it is evident, will alſo paſs through O the centre of the given ſphere. Draw OAP, OBQ, to meet PQ at P, and Q, and from O demit the perpendicular OC. Then it is evident, that AO is perpendicular to the plane AE, and OB to the plane BE, and ſince O is the centre of gravity of the ſupported body, the preſſures againſt the planes AE, BE, will be exerted in the directions OA, OB, and the force of the ſphere in the direction OC. But, by the laſt problem, the force in the direction OA, is to the force in the direction OB, as the ſine of the angle QEB to the ſine of the angle PEA, therefore the preſſure on the plane AE is to the preſſure on the plane BE, as the ſine of the angle QEB to the ſine of the angle PEA; and ſince the two planes ſuſtain the whole weight of the ſphere, the preſſure on each may be eaſily determined.

PROBLEM. III. Fig. 370. *Pl.* xxii.

If a plane, given in position together with a sphere, given in magnitude and position, support another given sphere it is required to find the pressure on the sustaining sphere and plane.

Let AE be the plane given in position DBI the sphere given in magnitude and position, and OAB the supported sphere. Conceive a plane to pass through D, the centre of the sphere DBI, and at right angles to the plane AE, as well as to the horizontal plane PQ, this plane will evidently pass through O, the centre of the sphere OAB, and the points of contact A, B. Draw DV perpendicular to the plane AE, and on DV demit the perpendicular OS; through B, let a plane be conceived to pass, so as to touch the spheres and meet the plane AE at E. Draw OD, which will evidently pass through B, and on AE demit the perpendicular OA. Then it appears, as before, that AO, OD, are the directions of the forces acting against the plane and sphere respectively; hence, if OC be drawn perpendicular to the horizontal plane PQ, we have by prob. 1, the pressure against the plane AE to the pressure against the sphere, as the sine of the angle DOC to the sine of the angle AOC, that is, by similar triangles, to the sine of the angle AEP.

Now the angle AEP is given, by the problem, and the angle DOC may be easily found by calculation, for VD is given, and VS = AO is also given, therefore DS is given, whence DO and the angle DOS become known, and consequently ($\angle$ DOS + comp. $\angle$ AOV = AEP or) the angle DOC is known.

PROBLEM IV. Fig. 371, *Pl.* xxii.

Let two spheres, given in magnitude and position, support a third given sphere, it is required to find the pressure against each of the sustaining spheres.

Let AR, DB be the two given sustaining spheres, placed at given distance from each other, on the horizontal plane PQ, and AOB the other given sphere. Conceive a plane, at right angles to the horizontal plane PQ, to pass through the centres D and E this plane will, it is evident, pass through O, the centre of the other sphere and likewise through the points of contact A and B. Draw OR, OD, which will pass through A and B respectively, and on PQ drop the perpendicular OC. Then it appears, as in the preceding problems, that the pressure against the sphere AR is to the pressure against the sphere BD, as the sine of the angle DO to the sine of the angle AOC.

To determine theſe angles by calculation, draw RS perpendicular, and DS parallel; to the horizontal plane PQ, and join RD. Then in the right angled triangle RSD, RD is given, conſequently the angle RDS may be found; and in the triangle DOR, the three ſides are given, therefore the angles ROD, RDO may be found, and from thence the angle SDO, and its complement the angle DOC; hence the angle AOC becomes known.

PROBLEM V. Fig. 372, *Pl.* xxii.

Given the directions of three forces, ſupporting a body, in equilibrio, to find the ratio of thoſe forces.

Let the forces AD, AB, AC acting in the directions AD, AI, AH reſpectively, ſuſtain a body at A acting in the direction AE perpendicular to the horizontal plane RQ. Let AE repreſent the force of the body A, and conceive a plane to paſs through AD, AE and interſect the plane which paſſes through AI, AH, in the ſtraight line AF, and compleat the parallelograms EDAF, FBAC. Then, by the compoſition of forces, the ſum of the forces AD, AF is equivalent to the force AE, and the ſum of the forces AB, AC is equivalent to the force AF; hence the ſum of the forces AD, AB, AC acting in the directions AD, AI, AH is equivalent to the force AE.

Conceive a plane to paſs through the point A at right angles to AE, and on it demit the perpendiculars B*b*, C*c*, F*f*, and join A*b*, A*c*, A*f*. Then, by prob. 1,

AD : AF :: ſin. ∠ EAF (or AF*f*) : ſin. ∠EAD; but by Trigonom. AF = A*f* ÷ ſin. ∠ AF*f* (radius being ſuppoſed = 1); therefore, A*f* = AD × ſin. ∠EAD. Again

A*f* : A*b* :: ſin. ∠*b*Ac (ſupp. A*bf*) : ſin. ∠*f*Ac (A*fb*), and A*b* = AB × ſin ∠ EAB (coſin. BA*b*). Therefore, we have

AD × ſin∠EAD : AB × ſin∠EAB :: ſin∠*b*Ac : ſin∠*f*Ac; hence AD : AB :: ſin ∠*b*Ac ÷ ſin ∠EAD : ſin ∠*f*Ac ÷ ſin ∠EAB.

And, in the ſame way, it appears, that

AD:AC::ſin∠*b*Ac÷ſin∠EAD:ſin∠*f*A*b*÷ſin∠EAC. Hence, the three forces AD, AB, AC are as $\frac{\text{ſin}\angle bAc}{\text{ſin}\angle EAD}$, $\frac{\text{ſin}\angle fAc}{\text{ſin}\angle EAB}$, and $\frac{\text{ſin}\angle fAb}{\text{ſin}\angle EAC}$ reſpe.

COROLLARY. Draw DM and EN parallel to AB to meet EM and DN, drawn parallel to AC, at M and N, and join MB, NC; then the figure ACFBDNEM is a parallelopiped whoſe three ſides are AD, AB, and AC and whoſe diagonal is AE. Hence this

THEOREM.

Three forces, acting in the direction and having the prop of the ſides of a parallelopiped, are equivalent to a force, a in the direction and having the proportion of the diagonal.

This theorem is of very extenſive uſe, in the ſcience of chanics.

PROBLEM VI.

Given the poſition of three planes, ſupporting a given ſpl to find the preſſure againſt each plane.

Conceive three planes to be drawn parallel to the given at a diſtance therefrom equal to the radius of the given ſphere, it is evident, that the interſection of theſe planes will be the c of the ſphere, when it reſts in equilibrio, and, if perpendic be demitted from that point, on the given planes, the preſſ againſt the reſpective planes, will be exerted in the direction theſe perpendiculars. Hence the directions of three forces l given, the forces themſelves may be determined by the problem.

The calculation will be extremely eaſy, for if a plane be ceived to paſs through the centre of the ſphere and parallel t horizon, interſecting the given planes, the angles of interſec which are given, will be equal to the reſpective angles of direc

PROBLEM VII. Fig. 378, *Pl.* xxii.

If two planes, given in poſition, together with a ſp given in magnitude and poſition, ſupport another ſphere, it is required to find the preſſure againſt each.

Let O be the centre of the given ſphere, reſting on the zontal plane. Conceive two planes to be drawn parallel given planes, at a diſtance therefrom equal to the radius c ſupported ſphere. Let the ſtraight line EF be the interſecti theſe planes which will be given in poſition, and in whic centre E of the ſupported ſphere will be ſituated. Now the O, and the diſtance OE (equal to the ſum of the radii given ſpheres) being given, the point E is given, hence if I joined and EA, ED be drawn perpendicular to the given p the preſſures will be exerted in the directions EO, EA, ED conſequently their ratios may be determined by prob. VI.

The method of calulation is ſo evident from prob. V anc that it is needleſs to repeat it.

PROBLEM VIII. *Fig.* 374, *Pl.* xxii.

Let a plane, given in position, together with two spheres, given in magnitude and position, support another given sphere, it is required to find the pressure against each.

Let O and Q be the centres of the given spheres, and CL the plane given in position. Conceive the plane FI to be drawn parallel to the given one, at a distance therefrom equal to the given radius of the supported sphere. The centre E of this sphere will evidently be situated in the plane FI, and since the distances QE, OE are given, (equal to the sum of the respective radii), the point E is given; hence, if EC be drawn perpendicular to the plane CL; EC, EA and EB will be the directions of the pressures against the plane CL, the sphere whose centre is Q, and the sphere whose centre is O respectively.

The calculation is easily made out from what is done above.

PROBLEM IX. Fig. 375, *Pl.* xxii.

Let there be three spheres, given in position and magnitude, supporting a fourth given sphere, it is required to find the pressure against each.

Let O, Q, and G be the centres of the spheres given in position, and E the centre of the other sphere. Then the distances OE, QE and GE are given, being equal to the radius of the sphere whose centre is E, together with the radii of the spheres whose centres are O, Q, and G respectively, therefore, the point E may be determined, and so may the directions OE, QE, and GE of the pressures; whence, and from problem VI, the pressures themselves, become known.

Since O, Q, and G are given points and OE, QF and GE given distances, the point E may be determined by calculation, as in the second solution to question 99 of the REPOSITORY, and thence the directions may be found by a single operation in Trigonometry.

When the three spheres, given in position, touch each other, the problem is the same as question 329 LADIES' DIARY, and which is left unanswered in that work. The only solution, that I know of, is that given by the late ingenious Mr. REUBEN BURROW, in his DIARY for 1779, but the method of calculation thence is by no means obvious.

ARTICLE XXXIX.

To the Editor of the Mathematical Repository.

SIR,

AS the following Problem will be found of ſome ſervice, in the ſolution of Problems, which, when done algebraically, riſe to equations of three or four dimenſions; I preſume, that this advantage may apologize for its inſertion in the Repoſitory. The Problem may be conſidered as a ſmall addition to the Lemmas of NEWTON in his *Arithmetica Univerſalis*.

I am, Sir,

Your moſt obedient,

London, March 10, 1800. ROBERT WALLACE.

PROBLEM. Fig. 376, *Pl.* xxii.

Given one of the angles at the baſe ABC, the difference between the baſe and one of the ſides, BC — AB, and one of the ſegments of the other ſide, AO or OC, made by the line BO drawn from the angular point B to make given angles AOB, BOC with AC at the point of interſection O, to conſtruct the plane triangle.

CASE I. When AO is given. Make CD = DF = BC — AB, the given difference, and the ∠ CDF = ∠ ABC = the given angle at the baſe; through F draw FG ∥ to CD, and make the ∠ DFM = the given angle AOB; draw CL ∥ to FM meeting FG produced in L, and CH ∥ to FD meeting MF, GF produced in H and P reſpectively. Take *f* a 4th proportional to AO, HC, and CD, theſe being all given, and to FL, FP and *f* find a 4th proportional *m*; now through C draw CMG, ſo that GM, the part intercepted by GF, FM, may be = to *m* (*vide* Newton's *Arithmetica Univerſalis*, *Archimedes' Lemmas*, *or Pappus' Mathematical Collections*.) and through D draw DA ∥ to CF meeting CG produced in A, alſo draw AB ∥ to FD meeting CD produced in B; ſo ſhall ABC be the triangle required.

Becauſe the △s ABD, FDC are evidently ſimilar, and DF = CD, AB will be = to BD, and BC — AB = CD, and alſo the ∠ ABC = ∠ FDC = the given ∠ at the baſe.

Make the ∠ AOB = ∠ END = ∠ DFM = the given ∠ at the point of interſection. Then by ſim. △s CD : AO :: FE : EN : but FE : EN :: ME : ED; therefore ME =

ME = CD × ED ÷ AO, and theref. HC : MC :: FE : CD × ED ÷ AO. Hence
f(CD × CH ÷ AO) : CM :: FE : ED. Moreover
FE : ED :: GE : CE, and GE : EC :: GF : FP; therefore
f × FP = CM × GF: but GM : MC :: GF : FL. Conſequently
GM × FL = CM × GF = f × FP, and
GM = f × FP ÷ FL = m. *Q. E. D.*

Case II. When CO is given. Make CD = DF = the given difference as before, and draw FG ‖ to CD. Make the ∠ GFM = the ∠ COB = the given ∠ at the point of interſection, and draw CS ‖ to FM meeting FD produced in S. Take a a 4th proportional to CO, TC, and CD, T being the point where FM meets CD produced, and b a 4th proportional to SE, a, and CD. Through C draw CEMG ſo that ME, the part intercepted between FT, FD, may be = to b, and through D draw DA ‖ to FC meeting CG produced in B, and ABC ſhall be the triangle required.

Let lines be drawn as before; then becauſe
the ∠ CDN = ∠ GFM, TC × CD ÷ MC = CN; alſo
CN : FE :: CO : CD, therefore FE × CO ÷ CD = CN. Hence
MC : CD :: TC × CD : FE × CO : but a = CT × CD ÷ CO; theref.
MC × FE = a × CD. Again SE : FE :: MC : ME, theref.
SE × ME = FE × MC = a × CD, or ME = a × CD ÷ SE = b.
Q. E. D.

Corollary. In Caſe I. when CD = 9, AO = 18, ∠ AOB = 28° 30′, and ∠ ABC = 90°, the Prob. becomes the ſame as queſt. 3rd. Ladies' Diary, 1797. In this caſe produce BA till BX = BC, and in CH produced take CY = BC, and join XY; BCXY ſhall be the ſquare required.

The 6th queſt. of the Gent. Diary, for the preſent year, is alſo a particular caſe of this Problem.

ARTICLE XL.

To the Editor of the Mathematical Repoſitory.

SIR,

IN Number IX. of your valuable work, are inſerted ſome inſtances of the mode of ſearch after negatives and impoſſibles, which I recommend to perſons who are curious after thoſe matters. In the third example is an error in the calculation, ariſing probably from the little intereſt I take in ſuch ſubjects, which overthrows the

the refult there ſtated. The inſtance was taken from BAER & MASERES' "Tracts on the reſolution of affected algebraic equations, by various methods of approximation," among which Lagrange's is one, by myſelf, on the number of negative and impoſſible roots, but without any inſtances.

The error in my calculation conſiſts in making $39x^2 - x^3$ the greateſt poſſible when x is equal to $\sqrt{\frac{39 \times 2}{3}}$: but $39x^2 - x^3$ is the greateſt poſſible when $2 \times 39 \times x$ is equal to $3x^2$ or x is equal to 26. Hence the true reaſoning upon the propoſed equation is as follows:—

$$x^3 - 39x^2 + 479x = 1881.$$

The unknown ſide muſt be leſs than $479x$, if x is leſs than 39; if x is equal to 39, the unknown ſide is equal to $479x$, which is much greater than 1881, and, if x is greater than 39, the unknown ſide is increaſed; conſequently each root of this equation muſt be leſs than 39.

Alſo ſince the greateſt ſubtractive value of $39x^2 - x^3$ is 8788, when x is equal to 26, if in this caſe the unknown ſide is greater than 1881, a number greater than 26 cannot make the unknown ſide equal to 1881. But if x is equal to 26, then $39x^2 - x^3$ is equal to $x^2 \times (39 - 26)$ or $13x^2$ and $479x - 13x^2$ is equal to $x \times (479 - 13x)$ or $x \times (479 - 169)$ or 310×13, which is evidently greater than 1881: conſequently each of the roots muſt be leſs than 26.

Now, the unknown ſide of equations, in this form, firſt increaſe from ſome indefinitely ſmall value of x and then diminiſh and afterwards increaſe, or they conſtantly increaſe: in the latter caſe, they have only one root, in the former caſe, they may have one, two, or three roots, and this number is ſeen by trying whether the increaſe of $479x$ is equal to that of $39x^2 - x^3$ or 479 is equal to $78x - 3x^2$, or $159,\ldots. = 26x - x^2$, or $\sqrt{169} - 159,\ldots. = 1, \infty x$, or $3,\ldots = 13 \infty x$. Hence, the increaſe of $479x$, being equal to that of $39x^2 - x^3$, when x is equal to $9,\ldots.$ or $16,\ldots.$, there can be only two roots, if one root is equal to either $9,\ldots.$ or $16,\ldots.$ But the root of the propoſed equation cannot be either $9,\ldots.$ or $16,\ldots.$ ſince it does not admit of a decimal for the root; conſequently, if there is one root leſs than $9,\ldots.$ there muſt be three roots to the equation*: unleſs that root is leſs than or equal to

* See my principles of algebra, on the true theory of equations, eſtabliſhed on Mathematical demonſtration. Part the ſecond. Pages 50, 60, 70.

$= 13 - \frac{2\sqrt{39^2 - 3 \times 479}}{3}$, or $13 - \frac{2\sqrt{1521 - 1437}}{3}$ or $13 - \frac{2 \times 9,\ldots}{3}$, or 6,.....

Hence I try 9, as the neareſt number to 9,.... and I find $x^3 \times (39 - x)$ to be $x^2 \times 30$ and $479x - 30x^2$ or $x \times (479 - 270)$ or 9×209 to be 1881. Hence the equation has three roots.

I have given the method at large, more to exemplify my general rule, than as the one I ſhould have adopted, if this example had been propoſed to me to find the roots themſelves. For I ſhould have ſeen at once, that each root was greater than 1. This would have led me to try 10, as an eaſy number; and finding the reſult to be very nearly equal to, though greater, than 1881, and that the increaſe of $479x$ is leſs than, though very nearly, equal to that of $39x^2 - x^3$, I ſhould conclude at once, that there muſt be two roots nearly equal to 10, the one greater and the other leſs than 10. Of courſe, I try 9, which ſucceeds: then 11 which alſo ſucceeds; conſequently the third root is 19.

In fact, all equations of the third claſs, in the ſimpleſt form, are reſolved with great eaſe, if one of the roots is a whole number, equal to or leſs than 10.

W. FREND.

Inner Temple, London, 20th July, 1800.

ARTICLE XLI.

GEOMETRICAL SECTIONS.

By Mr. JOHN LOWRY.

PROBLEM I.

TO or from two given right lines, to add or cut off two others, having a given ratio, or to add the one and cut off the other, ſo that the ratio, the rectangle, the ſum of the ſquares, or the difference of the ſquares of the two lines thus, compounded, may be given.

CASE I. When the ratio is given. *Fig.* 377, 378, 379, *Pl.* xxii. To or from two right lines AB, CD, to add as in fig. 377, or cut off as in fig. 378, two others BE, CE having a given

...the one and cut off the other as in ... of the ratio of AE to DE may be given.
... with AB and let it be equal to ... given ratio of AE to DE, and BI ... of BE to CE; draw AQ, IQ, and ... FD be drawn to meet the former, pro- ... D: draw FB parallel to DE to meet ... QI. at C. and it is done.
... the triangles, AE : DE :: AB : BQ, and ... CE : BI : the given ratios by construction; also AE, ... = FB ... lines
... Wanted to add the given right lines AB, DC, to ... the sum is cut off as in fig. 381, two others ... E. in a given ratio, or to add the one and cut ... as in fig. 382, so that the rectangle AE × ... to a given square.
Parallel to EP, the side of the given square, draw the ... AHD. GPE. and take PF = AB, and PG = ... GT ... so that IF may be to IG in the given ... CE. O. GF let a semicircle be described, meet- ... in E to Q. and draw GQC, FQB, and parallel ... PA ... from, and it is done.
... B = PF and CD = PG, ... the given ratio by construction. ... ∠ BDC = ∠ BQC = a right angle, ... AD × DB = BF² = the given square.
... the sum of the squares is given ... fig. 383, ... The ... in the given right lines ... cut off as in fig. 384, ... C ... or to add the ... fig. 385. so that the sum of ... to a given square.
... AP ... BF = DC ... and BI : BF in ... interest, from A, ... = ... DE ... IF. at C. and ... AB ...
For ... AP ... BF. as ... is, in the given ... , and AF² — BF² = ... the given sum of the squares, ...
... must ... be sought ... he ... P ... n ... as, ... al ... tw ... or ... ...

C ... E

CASE IV. When the difference of the ſquares is given. *Fig.* 386, 387, 388, *Pl.* xxii. To or from two given right lines AB, AC, to add, as in fig. 386, or cut off, as in fig. 387, two others BE, CF, having a given ratio, or to add the one and cut off the other, as in fig. 388, ſo that the difference of the ſquares of AE, AF may be equal to a given ſquare.

Perpendicular to AC take CI = the ſide of the given ſquare, and make alſo AD to DB in the given ratio of CF to BE; draw AI, and to it, from A, apply AB of the given length, and continue it to meet IE, drawn parallel to AC, at E, and draw EF parallel to IC, and it is done.

For by reaſon of the parallels CF (= EI) : BE :: AD : AB, that is, in the given ratio, by conſtruction, and by Eu. 47. I. $AE^2 - AF^2 = EF^2 = CI^2 =$ the given difference of the ſquares.

PROBLEM II.

To or from two given right lines, to add or cut off two others, or to add the one and cut off the other, ſo that the lines, thus compounded may have a given ratio, and ſo that the rectangle, the ſum of the ſquares, or the difference of the ſquares, of the parts, added or cut off, may be given.

CASE I. When the rectangle is given. *Fig.* 389, 390, 391, *Pl.* xxii. To or from two given right lines AB, CD to add, as in fig. 389, or cut off, as in fig. 390, two others BE, CE, or to add the one and cut off the other, as in fig. 391, ſo that AE may have to ED a given ratio, and ſo that the rectangle BE × EC may be equal to a given ſquare.

Perpendicular to PE, the ſide of the given ſquare, draw the indefinite right line GPF, AED, and take GP = AB, and PF = CD; divide GF at I, ſo that GI may be to FI in the given ratio of AE to ED, and draw EI and continue it to meet a ſemicircle deſcribed on GF at H; draw HGA, HFD, and parallel thereto draw PB PC, and it is done.

For, by reaſon of the parallels, AE : ED :: CI : IF, that is, in the given ratio: alſo AB = GP, and CD = PF = the given lines reſpectively. Moreover, the ∠ BPC = ∠ AHD = a right angle, therefore, Eu. 8. vi. BE × EC = EP^2 = the given ſquare, by conſtruction.

CASE II. When the ſum of the ſquares is given. *Fig.* 392, 393, 394, *Pl.* xxii. To or from two given ſtraight lines AB, CD, to add, as in fig. 392, or cut off, as in fig. 393, two others BE, CE, or to add the one and cut off the

the other, as in fig. 394, ſo that AE may have to DE a given ratio, and ſo that the ſum of the ſquares of BE, CE, may be equal to a given ſquare.

On AB take IA to CD in the given ratio of AE to ED, and draw IQ perpendicular to IA and equal to CD; draw AQ, and parallel thereto draw IC, to which, from B, apply BC = the ſide of the given ſquare; through C let ECD be drawn parallel to IQ to meet AQ, produced if neceſſary, at D, and it is done.

For, by reaſon of the parallels, AE : DE :: AI : IQ (= CD), that is, in the given ratio; alſo ſince the ∠ DEA is a right angle, EB² + EC² = BC² = the given ſum of the ſquares, by conſtruction.

CASE III. When the difference of the ſquares is given. *Fig.* 395, 396, 397, *Pl.* xxii. To or from two given right lines AB, CD, to add, as in fig. 395, or cut off, as in fig. 396, two others BE, CE, (or to add the one and cut off the other as in fig. 397, ſo that AE may have to DE a given ratio, and ſo that the difference of the ſquares of BE, CE may be equal to a given ſquare.

Perpendicular to AB, draw BC = the ſide of the given ſquare, and take AI to CD in the given ratio of AE to DE; draw CI, and parallel thereto AD, to which, from I, apply IQ = CD; through C let DCE be drawn parallel to IQ, to meet AB, produced if neceſſary, at E, and it is done.

For, by parallels, AE : DE :: AI : IQ (= CD), that is, in the given ratio, and becauſe the ∠ CBE is a right angle, CE² − EB² = CB² = the given difference of the ſquares.

PROBLEM III.

To or from two given right lines, to add or cut off two others, or to add the one and cut off the other, ſo that one of the lines, thus compounded, may have to the part added to, or cut off from the other, a given ratio, and ſo that the ratio, rectangle, the ſum of the ſquares, or the difference of the ſquares of the other compounded line, and the part added to, or cut off from the firſt line may be given.

CASE I. When the ratio is given. *Fig.* 398, 399, 400, *Pl.* xxii. To or from two given right lines AB, CD, to add as, in fig. 398, or cut off, as in fig. 399, two others BE, CE, or to add the one and cut off the other, as in fig. 400, ſo that the ratio of AE to CE, and alſo the ratio of DE to BE may be given.

Perpen-

Perpendicular to AB let BF be drawn, and let AB to BQ be taken in the given ratio of AE to CE; draw IQ parallel to AB, and take BQ to QI in the given ratio of DE to BE; draw AQ, BI, and take QF = CD, and from F draw FD parallel to AQ to meet BI, produced if neceſſary, at D; through D draw DCE parallel to FB to meet AQ, and AB, produced if neceſſary, at C and E, and it is done.

For, by reaſon of the parallels, AE : CE :: AB : BQ, and DE : BE :: BQ : QI, that is, in the given ratios, by conſtruction.

Case II. When the rectangle is given. Fig. 401, 402, 403, Pl. 22. To or from two given right lines AB, CD, to add, as in fig. 401, or cut off, as in fig. 402, two others, BE, CE, or to add the one and cut off the other, as in fig. 403, ſo that AE may have to CE a given ratio, and ſo that the rectangle BE × DE may be equal to a given ſquare.

Perpendicular to FE, the ſide of the given ſquare, let the indefinite right lines AED, FG be drawn, and take FG = DC, and FH = AB; divide GH at I, ſo that HI may have IG in the given ratio of AE to CE, and draw EI to meet a ſemicircle deſcribed on HG at Q; draw alſo GQC, HQA, and parallel thereto FD, FB reſpectively, and it is done.

For, by reaſon of the parallels, and conſtruction AE : CE :: HI : IG, that is, in the given ratio, and AB (= FH) and DC (= FG), are the given lines; alſo the ∠ AFD = AQC = a right angle; therefore, Eu. 8, VI. DE × BE = EF² = the given ſquare.

The above ſuggeſts a method different from any I have yet ſeen, of adding a line to a given line, ſo that the rectangle under the whole compounded line and the part added may be equal to a given ſquare.

Perpendicular to AD, (fig. 404, pl. 22.), the ſide of the given ſquare, draw AB = the given line, and alſo the indefinite line CDF; from D draw DI to the middle of AB, and take IC = AI or IB, and draw ACF; then DF is the part required to be added.

For, draw BCE and parallel thereto AG; then the ∠ ACB is evidently a right angle, therefore, by parallels, the ∠ GAF will be a right angle; hence GD × DF = DA², but ſince AI = IB, ED is = DF and AB = GE; therefore (AB + DF) · DF = = GD × DF = DA² = the given ſquare.

Case III. When the ſum of the ſquares is given. *Fig.* 405, 406, 407, *Pl.* 22. To or from two given right lines AB, CD, to add, as in fig. 405, or cut off, as in fig. 406, two others BE, CE. or to add the one and cut off the other, as in fig. 407, ſo that AE may have to CE a given

ratio, and ſo that the ſum of the ſquares of BE, DE may be equal to a given ſquare.

Perpendicular to AB let BI be drawn, and take AB to BI in the given ratio of AE to CE, take alſo IQ = CD, and draw QI parallel to the line joining AI; from B to QD apply BD = to the ſide of the given ſquare, and draw DCE parallel to QB to meet AI, AB, produced if neceſſary, at C and E, and it is done.

For, by reaſon of the parallels, AE : CE :: AB : BI, that is in the given ratio, and AB and CD (= IQ), are the given lines; alſo becauſe of the right angle at E, $BE^2 + ED^2 = BD^2$ = the given ſum of the ſquares.

CASE IV. When the difference of the ſquares is given. *Fig.* 408, 409, 410. *Pl.* 22. To or from two given right lines AB, CD, to add, as in fig. 408, or cut off, as in fig. 409, two others BE, EC, or to add the one and cut off the other, as in fig. 410, ſo that AE may have to CE a given ratio, and ſo that the difference of the ſquares of DE, BE may be equal to a given ſquare.

Perpendicular to AB take BD = to the ſide of the given ſquare, and take alſo AI to CD in the given ratio of AE to CE; join ID, to which, from A, apply AQ = CD, and from D draw DCE parallel to IQ to meet AC, drawn parallel to ID, at C, and AB, produced if neceſſary, at E, and it is done.

For, by parallels AE : CE :: AI : AQ (= CD), that is, in the given ratio, and AB and CD (= AQ) are the given lines, alſo $DE^2 - BE^2 = BC^2$ = the given difference of the ſquares.

(To be continued.)

ARTICLE XLII.

Demonſtrations of Lawſon's Propoſitions propoſed in

ARTICLES XIV. AND XXV.

PROP. XXXI. Fig. 411, 412, 413, *Pl.* 22.

Demonſtrated by Meſſrs. Campbell *and* Lowry. *Fig.* 411, 413.

BY hyp. the rect. $EAF = AB^2 =$ rect. CAD by Eu. 36, III theref AC : AE :: AF : AD, and ∴ the △s ACE, ADF, are equiangular Hence the ∠AEC = ADF = HGE, theref. GH is ‖ to AE.

Q. E. D.

The ſame by Mr. James Cunliffe, *Bolton-le-Moors. Fig.* 411, 412.

Since AF·AE = AB², and AB² = AD·AC, theref. AF·AE = AD·AC;
Whence AF : AD :: AC : AE, Euc. 16, VI, theref. ∠ AFD = ∠ ACE, Euc. 6. VI.
But the figure DCGH is a trapezium inſcribed in the circle, theref. the ∠DCG + DHG = two right angles = ∠ DCG + ACE, by Sim. Geo. 17, III;
whence ∠ DHG=ACE=AFD, and ∴ by Eu. 29. I, GH is ‖ to AE.
Q. E. D.

The ſame by Mr. J. Harris, *Caermarthen, Fig.* 413.

Becauſe, the rect. EAF = AB² = rect. CAD, the points E,F,C,D, are in a circle,
therefore ∠ AFD = ACE = GCD : but GCD = GHD, theref. ∠ AFD = GHD;
wherefore GH is parallel to AF. *Q. E. D.*

PROP. XXXII. Fig. 414, 415, 416, *Pl.* 22.

Demonſtrated by Meſſrs. Campbell, Cunliffe, Harris, *and* Lowry.

By hyp. the rect. FAG=rect. BAE=rect. CAD, by Euc. 35, III; theref. FA:CA::DA:GA, and ∴ the △s FAC, GAD are equiangular.
Hence the ∠CFE=CDK=CHK, theref. HK is ‖ to AB. *Q. E. D.*

PROP. XXXIII. Fig. 417, 418, 419, 420, *Pl.* 22, 23.

Demonſtrated by Mr. Colin Campbell, *Liverpool. Fig.* 417.

Draw AN = AE to meet EB in N, and from A demit the ⊥ AL upon NE.
Then AB² = AG² + BG² + 2BGL (Euc. 12, II.) = AG² + BG² + BG · (GE + GN) = AG² + BGN + GBE = AN² + NGE + BGN + GBE (Sim. Eu. Dat. p. 427, lem.)
But AB² = AC² + BD², by hyp. = EAF + GBE, Euc. 36, III;
hence AE² + NG·BE + GB·BE = EA·AF + GB·BE = AE² + AE·EF + GB BE;
therefore NG·BE = AE·EF; conſeq. NG:AN::EF:BE::GH:HB.
wherefore, ſince ∠ ANG = GEF = GHB, (Euc. 22, III), the △s ANG, GHB are equiangular, and the ∠NGA=HGB (Euc. 6, VI.);
theref. ∠ HGB + EGH = two right angles = NGA + EGH; conſ. A,G,H are in a right line (Eu. 14, I.). *Q. E. D.*

The ſame by Mr. James Cunliffe. *Fig.* 418.

In AB take a point *n*, ſuch that B*n* × BA = BD², and draw the lines *n*F, *n*H, AH, and GH. By hypotheſis
AB² = A*n* × AB + B*n* × AB = AC² + BD², theref. A*n* × AB = AC²; but
AF × AE = AC², and BG × BE = BH × BF = BD², Euc. 36, III; theref. AF × AE = A*n* × AB, and therf. AF : AB :: A*n* : AE, Eu. 16, VI; hence ∠ A*n*F = AEB = FEG, Eu. 6, VI. whence ∠ F*n*B = FHG = ſupplement of ∠ E.
Moreover, by Conſtr. B*n* × BA = BD² = BH × BF, Euc. 36, III. whence BF : BA :: B*n* : BH, Eu. 16, VI; theref. ∠ AHB = F*n*B = FHG, and theref. AGH is a ſtraight line, by Euc. 15, I. *Q. E. D.*

The ſame by Mr. John Harris. *Fig.* 419. 420. *Pl.* 23.

Let the diameter KIL be drawn perpendicular to AB meeting the circle in K and I, and AB in L. Then by hypotheſis,
AC² + BD² = BA²: but, by Prop. XI.
AC² = IL·K + AL², and BD² = IL·X + LB², therefore
2IL·K + AL² + BL² = AB² = AL² + BL² + 2AL·B, i.e. the rect. IL·K = ALB.
Hence, the rect. LAB (= ALB + AL²) = AC² = EA*F*,
and the rect. LBA (= ALB + BL²) = BD² = EBG;
theref. the points L, B, E, F are in a circle, and ſo are alſo the points E, G, A, L;
conſeq. ∠ EFH = ELA or ELB = EGA: but ∠ EGH + EFH = two right angles;
theref. ∠ EGH + EGA = two right angles, and ∴ AGH is a right line. *Q. E. D.*

The ſame by Mr. John Lowry. *Fig.* 418.

In AB take the point *n* ſuch that AB·A*n* = AC², then DB² = AB B*n* = BG·BE, therefore, joining E*n*, (interſecting AG in K,) the △s AGB, B*n*E are equiangular, and, of courſe, ſo are the △s AK*n*, EKG, therefore the ∠ AGE (= A*n*E = AFB) = BGH, as is ſhewn in the demonſtration to Prop. 31; therefore the points A, G, H, are in a right line. *Q. E. D.*

PROP. XXXIV. Fig. 421, 422, 423, *Pl.* 23.

Demonſtrated by Mr. Campbell. *Fig.* 421.

Biſect. CD in I and draw FI. Then by hypotheſis
CAD = AB² + CBD = AB·(AC + CB) + CBD = BAC + CB·AD, conſequently CA·

CA·(AD — AB) = CA·BD, = CB·AD, therefore
AC : CB :: AD : BD, and therefore, by Prop. I. the rect. ABI = CBD. Hence
CAD = EAF = AB² + ABI = AB·(AB + BI) = BAI, conſeq.
AF : AI :: AB : AE, theref. the △s FAI, ABE, having ∠ A common, are equiangular, and therefore the ∠ AEB = AIF. Moreover, becauſe
AB·BI = CB·BD = HB·BF, AB : BH :: BF : BI; theref. the △s ABH, FBI, in which the ∠ ABH = FBI, are alſo equiangular, and AHB = AIF.
Wherefore, ſeeing ∠ GEF = FHG, ∠ GEF + AEB = 2 right angles = FHG + AHB.
Conſeq. the points A, H, G are in the ſame right line. *Q. E. D.*

The ſame, by Mr. James Cunliffe, *Fig.* 422.

Join the point A with O the centre of the circle, and draw BQ, and OI ⊥ to AO and AD reſpectively; produce BQ till it cuts the circle again in T and *t*, and draw AT, as alſo the radius OT.

The line T*t* is biſected in the point Q by Euc. 3, III. and by Euc. 12, II.

AT² = AB² + TB² + 2BQ·TB = AB² + TB·(TB + 2BQ)
= AB² + TB·B*t*, (Euc. 35, III.) = AB² + CB·BD: but, by hypotheſis
AB² + CB·BD = AC × AD, theref. AT² = AC × AD; therefore AT is a tangent to the circle at the point T, by Euc. 36, III.

Now ATO is a right angle, therefore the △s ATO, AQT, are ſimilar; and it is alſo evident that the △s AIO, AQB, are ſimilar, therefore,
AQ:AT::AT:AO, and AQ:AB::AI:AO, and theref. Eu. 16, VI.
AB·AI = AQ·AO = AT² = AB² + CB·BD, wherefore,
AB·BI = AB·(AI — AB) = AB·AI — AB² = CB·BD = BG·GF, whence
AB:BG::GF:BI: but ∠ ABG = FBI, theref. Eu. 6, VI. ∠ BGA = BIF.
Again, ſince AB·AI = AT² = AE·AF, therefore, Euc. 16, VI.
AB:AE :: AF : AI; theref. ∠ AEB = AIF = BIF = BGA.
Moreover ∠ FEH = FGH, theref. ∠ FEH + AEB = FGH + BGA: but ∠ FEH + AEB = 2 right angles, theref. ∠ FGH + BGA = 2 right angles;
Therefore the points A, G, H, are in a right line. *Q. E. D.*

The ſame, by Mr. John Harris, *Fig.* 423.

Through A draw the diam. AK*q*, and ⊥ to it draw BQ cutting

it in Q and the circle T and t; draw alſo EQR, HQI, and join FR, GI, (meeting Aq in P,) AI, AT, and HE.

Then rect. $CAD = AB^2 + CBD = AB^2 + TBt = AB^2 + TQ^2 - BQ^2 = AQ^2 + TQ^2 = AT^2$,

therefore AT touches the circle at the point T; hence, by Prop. XV, $AK : Aq :: KQ : Qq$, and, by Conv. Prop. VII. FR is parallal to Tt; therefore

$\angle HBQ = HFR = HEQ$ (= HER), conſeq. the points H, E, B, Q are in a circle; hence

$\angle BQI = HEG$ (= HEB) = HIG, therefore IG is parallel to Tt, IP = PG: but

$\angle PAI = PAH$, by Prop. IV; theref. AHG is a right line. *Q.E.D.*

The ſame, by Mr. John Lowry. *Fig.* 421.

Draw FI to meet AD at I and make the $\angle FID = FEG$, then the $\angle AIF + GEF = 2$ right angles, therefore the points E, B, I, F, are in the circumf. of the ſame circle; hence the rect. $BAI = EAF = CBD =$, by hypotheſis, $AB^2 + CBD$, therefore taking away the ſquare of AB from the firſt and laſt of theſe equals, the rect. $ABI = CBD$, that is = HBF; therefore the points A, H, I, F are in the circumf. of the ſame circle, conſeq. $\angle AHF = AIF$: but $\angle GHF = GEF$; theref. $\angle AHF + GHF = AIF + GEF =$ two right angles, and theref. the points A, H, G are in a right line. *Q. E. D.*

PROP. XXXV. Fig. 424, 425, 426, *Pl.* 23.

Demonſtrated by Mr. Colin Campbell.

Join GE in Fig. 424, and HE in Fig. 425. Then

$BA \cdot AE = AC^2 = GA \cdot AH$, and $EB \cdot BF = BD^2 = GB \cdot BK = LB \cdot BK$, therefore in

Case I. When the points E and F are taken between the points A and B.

$BA : AG :: AH : AE$, conſeq. the $\triangle$s ABH, AGE are ſim. and $\angle AHB = AEG$:

but $\angle AEG + GEB = 2$ right angles $= AHB + GKL$; hence $\angle GEB = GKL$. And

$BG : BF :: BE : BK$, theref. the $\triangle$s BGE, BKF are ſim. and $\angle GEB = BKF$.

Whence $\angle GKL = BKF$; conſeq. $\angle GKL + LKB = 2$ right angles $= BKF + LKB$.

Case II. When the points E and F are taken without the points A and B.

AE

AE : AG :: AH : AB, and EB : LB :: HB : FB, therefore in the $\triangle$s AEH, EBH, which are reſpectively ſimilar to AGB, BLF, we have $\angle$ AGB = AEH = BLF.

Alſo, BH : BK :: BG : BL, therefore the $\triangle$ BHG is ſim. to BKL, and $\angle$ BGH = BLK. Whence AGB + BGH = 2 right angles = BLF = BLK.

Conſequently the points L, K, F are in a ſtraight line. *Q. E. D.*

The ſame, by Mr. James Cunliffe, *Fig.* 424.

Draw the lines GE, FK, and KL. Then by hypotheſis,

AE·AB = AC^2 = AG·AH, by Euc. 36, III. whence AB : AG :: AH : AE, therefore

$\angle$ AEG = AHB, by Euc. 6, VI; whence $\angle$ GEB = GKL = ſupp. of $\angle$ H. Again

by hyp. BF·BE = BD^2 = BK·KG, whence Euc. 16, VI. BF : BG :: BK : BE, therefore

Eu. 16, VI. $\angle$ FKB = GEB = GKL, from what is done above, therefore the points K, L, F, are in the ſame ſtraight line, Euc. 15, I. *Q. E. D.*

The ſame, by Mr. John Harris. *Fig.* 426.

Join EK, EG, and FG, Becauſe, by hypotheſis,

rect. EAB = AC^2 = GAH, the points E, G, H, B, are in a circle, theref. $\angle$ AEG = GHB = LKB. Again, by the hypotheſis,

rect. EBF = BD^2 = KBG, therefore the points E, F, G, K, are in a circle, conſeq.

$\angle$AEG = FKG; hence $\angle$FKG = LKB, and theref. LKF is a right line. *Q. E. D.*

The ſame, by Mr. John Lowry. *Fig.* 426.

Draw GE to meet the circle at Q, and join QL. Then,

QL is parallel to AB, by Prop. XXXI. and by hyp. EBF = BD^2 = BKG;

therefore the points E, F, G, K are in a circle, hence

$\angle$ FKG = FEG = EQL = (Eu. 22, III.) GHL = LKB, that is $\angle$ FKG = LKB;

therefore the points L, K, F, are in a right line. *Q. E. D.*

PROP. XXXVI. Fig. 427, 428, *Pl.* 23.

Demonſtrated by Mr. Colin Campbell. *Fig.* 427.

About the $\triangle$ circumſcribe the circle ABC, and produce AD to meet it again in *a*; draw the chord *ac* = AC, and in *ac*, *a*A let *af*, *ad* be taken = to AF, AD reſpectively, and join *df*, *cd*, and CB interſecting *a*A in P.

Since

Since arc ac = AC (Eu. 28, III.), arc aC = Ac, and therefore ∠ DAC = dac (27, III.); conſeq. the △s ade, adf, are equal and equiangular to ADC, ADF (Eu. 4, I.). Wherefore ∠ Pca = BAP (27, III) = ADF (29, I.) = adf; therefore the △s /Pa, daf are equiangular, as alſo are BPA, AED, becauſe ∠ AED = AFD (34, I.) = adf (by conſtr.) = aPc = BPA (15, I.) and ∠ BAP common. Whence

aP : af (AF) :: ac (AC) : ad (AD), and PA : EA :: BA : DA (4, VI); theref.

aP· DA = FAC, PAD = EAB (Eu. 16, VI.); hence

aP DA + PAD = DA· (AD· Da) = DA^2 + ADa = AD^2 + BDC = BAE + CAF. *Q. E. D.*

The ſame, by Mr. James Cunliffe. *Fig.* 427.

Deſcribe a circle about the △ by Eu. 5, IV. and produce AD to meet it at a and join Ba, alſo draw the line FG meeting AD and AB in H and G, ſo that the ∠ FGA may be = the ∠ BCA. Then the ∠ BaA = BCA (Eu. 21, III.) = FGA, by conſtr.; whence the △s AHG, ABa are ſimilar, as alſo the △s AFG and ABC. Hence ∠ AHG = DHF (Eu. 15, I.) = ABa by what is done above, alſo ∠ FDH = BAa, (Eu. 27, I.), whence the △s DHF and ABa are ſimilar. Therefore, by the ſim. △s AFG and ABC,

AF : AG :: AB : AC, whence, Euc. 16, VI. AF × AC = AG × AB.

Alſo becauſe of the ſim. △s AHG, ABa, AH : AG :: AB : Aa, whence

AH × Aa = AG × AB = AF × AC. Again, by the ſim. △s DHF and ABa,

DH : DF :: AB : Aa, whence DH × Aa = DF × AB = AE × AB. Therefore

AE × AB + AF × AC = DH × Aa + AH × Aa = AD × Aa = AD × (AD + Da) = AD^2 + AD × Da = AD^2 + BD × DC, Eu. 35, III. *Q. E. D.*

Mr. Harris *refers to* Art. II. Vol. I. Repository, *for a demonſtration.*

The ſame, by Mr. Lowry. *Fig.* 428.

The truth of this propoſition will eaſily appear from Prop. B. Art. 34, Vol. I. Repoſitory. For if FI, EH, be drawn ‖ to BC, and FG, EQ to AD, it is evident that FI is = EH, (or DG = DQ) and AI = DH; hence the rect. DAH + DAI = AD^2. Again, by reaſon of the parallels, EH (QD) : BD :: DC : BC, or QD·BC = rect. BDC: but the rect. BDQ + CDG is = QD·BC; therefore BDQ + CDG = BDC; hence by the Prop. above cited the rect. BAE + CAF = AD^2 + BDC. *Q. E. D.*

ARTICLE

ARTICLE XLIII.

Anſwers to the Mathematical Queſtions propoſed in ARTICLE XXVIII. No. VIII.

1. QUESTION 157, *anſwered by Mr.* John Croudace, *Student under Mr.* Rutherford, *Teacher of the Mathematics at Lancaſter, Durham.*

SINCE the Δ is a right angle, the ſum of the ſquares of the two ſides $=$ to the ſquare of the hypothenuſe, by Eu. 47, 1. that is, $x^{10x} + x^{12x} = x^{14x}$; this equation divided by x^{10x} gives $1 + x^{2x} = x^{4x}$, or $x^{4x} - x^{2x} = 1$; now put $y = x^x$ and the equation will become $y^4 - y^2 = 1$, hence by compleating the ſquare, &c. $y = \sqrt{\frac{1}{2}(\sqrt{5}+1)}$. Conſequently $x^{5x} = y^5 = \overline{\frac{1}{2}(\sqrt{5}+1)}^{\frac{5}{2}} = 3{,}33019 =$ the baſe; $x^{6x} = y^6 = \overline{\frac{1}{2}\sqrt{5}+1}^{3} = 4{,}2366 =$ the perpendicular; and $x^{7x} = y^7 = \overline{\frac{1}{2}(\sqrt{5}+1)}^{\frac{7}{2}} = 5{\cdot}38836 =$ the hypothenuſe; therefore the area is $= 7{\cdot}0534549$.

The ſame anſwered by Mr. John Johnſon, *Birmingham.*

By Eu. I. 47, $x^{10x} + x^{12x} = x^{14x}$, and by diviſion and tranſpoſition, $x^{4x} - x^{2x} = 1$, and by compleating the ſquare, &c. $x^{2x} = \frac{1}{2}(1+\sqrt{5})$. Hence the baſe is $\overline{\frac{1}{2}(1+\sqrt{5})}^{\frac{1}{2}}$

$=$

$= 3\cdot330$; perpendicular $= \overline{\frac{1}{2}(1+\sqrt{5})}^{3} = 4\cdot236$; hy[pothenuſe] $= \overline{\frac{1}{2}(1+\sqrt{5})}^{\frac{7}{2}} = 5\cdot388$, and the area $= \frac{1}{2}\cdot$[illegible] $\overline{\frac{1}{2}(1+\sqrt{5})}^{\frac{11}{2}} = 7\cdot05345$.

The ſame anſwered by Mr. David Davis, *Warminſter School, Wi*[illegible]

By the property of right-angled $\triangle$s, we have $x^{10x} + x^{[illegible]}$ $= x^{14x}$, or $x^{4x} - x^{2x} = 1$; putting $y + \frac{1}{2} = x^{2x}$, the equation will then become $y^2 - \frac{1}{4} = 1$, or $y^2 = 5 \div 4$, or $y = \sqrt{\frac{5}{4}}$; hence $x^{2x} = y + \frac{1}{2} = \frac{1}{2}\sqrt{5}$, or $x^{x} \sqrt{\frac{1}{2} + \sqrt{[illegible]}}$ and conſeq. $x^{5x} = \overline{\frac{1}{2} + \frac{1}{2}\sqrt{5}}^{\frac{5}{2}} = 3\cdot330$, the baſe; $x^{6x} = \overline{\frac{1}{2} + \frac{1}{2}\sqrt{5}}^{3} = 4\cdot236$ the perpendicular; and $x^{7x} = \overline{\frac{1}{2} + \frac{1}{2}\sqrt{5}}^{\frac{7}{2}}$ $= 5\cdot388 =$ hypothenuſe.

Neat ſolutions to this queſtion were alſo received from Meſſrs. Ba[illegible] ron, Blackwell, Buffham, Bulmer, Burdon, Evans, Gregory, Ha[illegible] ris, Lowry, May, Merones Minor, Surtees, Swale, *and* Tyro Phil[illegible] Mathematicus.

II. QUESTION 158, *anſwered by Miſs* Suſan May.

Dr. Hutton hath ſhewn, at pa. 168, vol. i. of his Mathematic[illegible] and Philoſophical Dictionary, that $63551 \times$ log. of $D \div d$ i[illegible] general expreſſion for the altitude, in feet, when d is the denſity [illegible] the air at that height, and D the denſity at the earth's ſurface, t[illegible] air being at a mean temperature, 55°. Hence we have $63551 \times$ log. of $(1 \div \frac{1}{2}) = 63551 \times$ log. of $2 = 19130\cdot757$ feet, t[illegible] height required. On the ſame page the Dr. ſhews how to fi[illegible] the height at any other temperature; thus $1000 \times$ log. of $D \div$ [illegible] is an expreſſion for the height, in fathoms, when the temperature at 31°. And to find it, at any other temperature, increaſe the hei[illegible] found for 31°, by the 435th part of that height for each deg[illegible] above that temperature, and the ſum will be the height required.

ſame, anſwered by Mr. G. Buffham, *Boſton.*

the altitude of the mountain, D the denſity at the
e = 1, and *d* the denſity at the top of mountain = $\frac{1}{2}$.
Courſe of Mathematics, page 235, Vol. II *a* =
g. D ÷ *d* = 63551 × log. 2 = 19130·757 feet the
mountain, when the thermometer ſtands at 55°.

nſwered by Mr. John Barron, *Schoolmaſter, Spilſby, Lincolnſhire.*

36, vol. II. of Dr. Hutton's Courſe of Mathematics
0 × log. (1 ÷ ·5) = 3010·3 fathoms for the height
tain, when the temperature of the air is at 31°.

nſwered by Mr. Wm. Burdon, *Acaſter-Malbis, York.*

e, at pr. 52, cor. 2, of his hydroſtatics, ſays, " from
on the denſity of air at the bottom and top of hills,
Hydroſtatics p. 103.) collected, that, at the altitude
the denſity was 4 times leſs than at the earth's ſurface,
Hence if *x* = the diſt. above the earth's ſurface, in
r = the rarity of the air at that height, it will be as
r :: 7 : *x* = 1·1626 × log. *r* (in miles) = 3$\frac{1}{2}$ miles,
of the mountain. Mr. Vince further remarks, that,
ſuppoſes the denſity to be as the compreſſive force,
t true, unleſs the temperature remains the ſame, but as
ture is found to be very different, at the ſame time, at
itudes, the rule will require a correction, according to
s of the thermometers of the two places. Omitting,
is correction, the denſity of the air, at the altitude of
found to be 7420 times leſs than at the earth's ſurface:
om obſervations on the twilight, the rays of light are
fted by the air at that altitude.

ne, anſwered by Tyro Philo Mathematicus, *of Hull.*

21008460 feet = radius of the earth,

centre, by *Emerſon's Fluxions*, *pa.* 389, D ÷ d = a number whoſe log. is $(r^n - rx) \div mhx$; therefore, (putting the log. of D ÷ d = — ·30103 = — l,) $r^n - rx = -mhlx$, and $x = r \div (r - mhl)$; hence $x - r = mhlr \div (r - mhl) = 20624 \cdot 026$ feet, the height of the mountain as required.

According to one or other of theſe methods the queſtion was anſwered by Meſſrs. Blackwell, Bulmer, Croudace, Gregory, Harris, Johnſon, Lowry, Merones Minor, *and* Thornoby.

III. QUESTION 159, *anſwered by Miſs* May.

Let W = the weight of one ſquare yard of the ſtuff compoſing the balloon,

w = the weight to be raiſed, c = 3·14159,

A = denſity of one ſolid yard of air, at the given height which may eaſily be found, by reverſing the rule uſed in the laſt queſtion,

a = denſity of the air with which the balloon is filled, and

x = the required diameter of the balloon. Then

cx^2 = the ſurface of the ſphere, and $cx^3 \div 6$ = its ſolidity: hence

Wcx^2 = the weight of the balloon, and

$\frac{1}{6}acx^3$ = the weight of the air with which the balloon is filled; therefore

$\frac{1}{6}acx^3 + Wcx^2 + w = \frac{1}{6}Acx^3$, by the nature of the queſtion. Hence

$$x^2 - \frac{6W}{A - a}\,x^2 = \frac{6w}{c(A - a)},$$ the THEOREM required.

The ſame, anſwered by Mr. John Surtees, *Biſhop-Wearmouth, Sunderland.*

Let x = the diameter of the balloon, h = the height to which it will riſe,

W = the weight to be raiſed, w = the weight of one foot of the cover,

d = the weight of one cubic foot of air at the earth's ſurface,

a = the weight of one cubic foot of air with which the balloon is filled,

c = 3·1416, and r = radius of the earth. Then

cwx^2 = the weight of the balloon, and

$\frac{1}{6}acx^3$ = the weight of the contained air. And by *Emerſon's Fluxions*, pa. 389, the weight of one cubic foot of air, at the height h, above

h, above the ſurface of the earth, is $= d \times$ number belonging to the log. of $(r \times - h) \div (68444 \times r + h) = (6cnrx^4 + acx^3 + 6W) \div cx^3$, by the queſtion. Hence x may be found.

The ſame, anſwered by Mr. Croudace, *Lancheſter.*

Let $x =$ the internal diam. of the balloon, $y =$ its thickneſs,
$a =$ the given height it is to aſcend to,
$b =$ the weight to be raiſed,
$c =$ the ſpecific gravity of the air with which the balloon is filled,
$e =$ the weight of a [illegible] f the ſubſtance compoſing it,
$n = .5236$, and d [illegible] e ſpecific gravity of common air. Then
$x + 2y =$ the external diameter,
$n(x + 2y)^3 =$ the whole ſolidity.
$nx^3 =$ the internal capacity,
$cnx^3 =$ the weight of the air its filled with,
$dn(x + 2y)^3 =$ weight of a maſs of air of the ſame ſize as the balloon.
$e((x + 2y)^3 - x^3) =$ ſolidity of the part included between the two diameters, and

$en\overline{(x + 2y)^3 - x^3} =$ its weight; Hence $en \times \overline{(x + 2y)^3 - x^3} + cnx^3 + b =$ the whole weight of the balloon and weight to be raiſed. But as the balloon will aſcend and reſt at ſuch a height in the air where it will be of equal weight with the ſame bulk of it, and theref. where its denſity is to the denſity of the ſurface as

$n\overline{(x + 2y)^3 - x^3} + cnx^3 + b$ to $dn(x + 2y)^3$. Hence by *Emerſon's Fluxions*, 389th *page*, $68444 \times$ log. of $dn \times \overline{x + 2y}^3 \div (en \times \overline{(x + 2y)^3 - x^3} + cnx + b) = a$. And taking y a known quantity, the theorem is manifeſt.

True ſolutions were alſo received from Meſſrs. Blackwell, Gregory, Harris, Johnſon, Lowry, Merones Minor, *and* Thornoby.

IV. QUESTION 169, *anſwered by Mr.* Burdon, *Acaſter-Malbis.*

Fig. 429, Pl. 23. Let ABCD be the oblique cylinder, then if $\perp$ to AB, the pillar will juſt ſupport itſelf by the principles [illegible] manics.—Now, by trigonometry, rad. : AB :: tan. $\angle$ A : BD [illegible] $\sqrt{3}$, and by menſuration the ſolidity is $.7854 \times AB^3$

$= 26{\cdot}548148$ cubic feet, by ſpecific gravity: Whence AB $\sqrt[3]{19{\cdot}51564} = 2{\cdot}6923$ feet the diam. of the baſe of the pil and BD $= 4{\cdot}6632$ feet its perpendicular altitude.

The ſame, anſwered by Miſs May.

Fig. 430, Pl. 23. Let ABCD repreſent the cylinder in inclined poſition; draw the diagonal BC, and from O the mid of BC draw OE $\perp$ to CD. Then, it is evident, from the na of the centre of gravity, that the cylinder will juſt ſupport it from falling when BC is $\perp$ to the horizon CQ. Now, by periments, it is found, that a cubic foot of marble weighs 27 ounces, therefore, the content of the cylinder is $= 26{\cdot}54814$ cub feet, alſo by the queſt. $\angle$ OCE $= 60°$, and $\angle$ COE $= 30°$ hence, by trigonom. OE $=$ CE $\sqrt{3}$, and by menſ. $4\text{CE}^2 \times 2\text{O}$ $\times {\cdot}7854 = (4 \div 3) \times \text{OE}^2 \times 2\text{OE} \times {\cdot}7854 = (8 \div 3) \times \text{OE}^3 \times$ $.7854 = 26{\cdot}54814$. Whence $\text{OE} = \sqrt[3]{3 \times 26{\cdot}54814 \div (8 \times {\cdot}7854}$ $= 2{\cdot}3314$. Conſeq. the height is $4{\cdot}6628$ and the diam. of the baſe is $2{\cdot}6932$ feet.

The ſame, anſwered by Mr. Johnſon, *Birmingham.*

Writers on mechanics have ſhewn, that the diagonal of the pillar, when it juſt ſupports itſelf from falling, will be $\perp$ to the horizon. Hence, as 2700 (the ſp. gr. of marble) : 1 :: 7168 ounces, or two tons: $26{\cdot}54814814$ feet, the ſolid content of the pillar. Again let $x =$ the diam. of its baſe, $a =$ ſine of $60°$ $b =$ its coſine, then as $b : x :: a : ax \div b =$ the len. of the pillar and by mens. $x^3 (ax \div b) \times {\cdot}7854 = 26{\cdot}54814814$; hence $= 2{\cdot}69$, and the length $= 4{\cdot}659$ feet.

The ſame, anſwered by Mr. John Barron, *Spilſby.*

This Gentleman, after premiſing that CB, (fig. 430, pl. 23. muſt be $\perp$ to the horizon, puts $a =$ content of the pillar, $d =$ $.7854$, s and $c =$ ſine and coſine of $\angle$ ABC, $x =$ diam, and $=$ the height of the pillar. By hydroſtatics, as 2700 : 76180 :: 1728 : 45875·2 the content in inches $= a$, and $dx^2y = a$, $y = a \div dx^2$. Again, by trigonom. as $s : y :: c : x$ and $cy = sx$ or $y = sx \div c$. Hence, putting theſe two values of y equal to each other, we have $sx \div c = a \div dx^2$, or $x = \sqrt{ac \div ds} =$ $32{\cdot}3079$ and $y = 55{\cdot}9589573$, the dimenſions of the pillar.

The ſame, anſwered by Mr. Croudace, *Lancaſter*

Fig. 430. Pl. 23. Let ABCD repreſent the [illegible] BC ⊥ to CQ and produce BD to Q. Then [illegible] marble 2 feet : 2 inches :: 1·28 : 43·392 inches = [illegible] content of the pillar. Put CD = x, $a = 1{\cdot}28$, [illegible] = ax, [illegible] DCQ = 30°. Hence $a \div px^2 = BD$, and [illegible] DCQ : DQ = x, alſo DQ : DC :: DC : BD = x — [illegible]

$x \div t = a \div px^2$. This equation reduced gives $x = \sqrt{\frac{a}{pt}}$ = 32·30793 = DC and BD = 55·93896.

This queſtion was alſo ingeniouſly anſwered by Mr. Bulmer, Francis, Gregory, Harris, Lowe, [illegible] Squire, *and* Tyro Philo Mathematicus.

V. QUESTION 161, *anſwered by Mr.* [illegible]

CONSTRUCTION. Fig. 431, Pl. 23. [illegible] the given diff. and produce it to A. [illegible] may be = to the given area. Upon AB [illegible] ſcribed, and at B erect the ⊥ BD making the rect. AB·BD = 2AF·FB, and let DE be drawn ∥ to AB meeting the circumference in C; join AC, CB, and ACB will be the △ required.

DEMONSTRATION. Make AH = AG, BK = BG and at G, H, and K let ⊥s be erected to meet [illegible] of the inſcribed circle. Biſect AB in M and [illegible] ſince BG is = the given diſtance, we have only to prove that the rectangle of the ſegments of the hypothenuſe of any right angled △, made by the point of contact with the inſcribed circle, is = to the area. Now

AG·GB = (AM — MG)·(AM + MG) = AM² — MG², and

4AG·GB = 4AM² — 4MG² = AB² — 4MG²: but

AB² = AC² + BC², and, (becauſe 2GM = AG — BG = AC — BC)

4GM² = BC² + AC² — 2BC·CA. Therefore

2BC·CA = BA² — 4GM² = 4AG·GB, and

BC·CA = 2AG·GB = 2△ ACB: Conſequently the rectangle AG·GB is = to the △ ACB. *Q. E. D.*

COROLLARY. The area of any right angled plane triangle is equal to the difference between the ſquare of half the hypothenuſe and the ſquare of half the difference of the legs.

The ſame, anſwered by Mr. Burdon, *Acaſter-Malbis.*

After conſtructing the problem exactly like Mr. Swale, Mr. Burdon proves that the rect. AG·GB is =, to the △ ACB, by the following

DEMONSTRATION. If L be the centre of the inſcribed circle, and G, H, K the points of contact, then will AG = AH, BG = BK, and HC = CK; conſeq.
AC = AG + HC, and BC = BG + HC, therefore
(AG + HC)·(BG + HC) = AC·CB, and, by Eu. I. 47,
(AG + HC)² + (BG + HC)² = (AG + GB)²; ſubtract twice the former of theſe expreſſions from the latter, ſo ſhall ½ AC·BC = AG·GB, and by Euc. I. 41, the △ ABC = ½ rect. AD = the given area by conſtruction.

Calculation. Dividing the given area of the △ by BG (6) gives AG = 9, theref. AB = 15. Again dividing the given area by AG gives BD = CF = 7⅔: but MC = MB = ½ AB = 7½; hence MF = √(MC² − CF²) = 2 1/20, and AF = 9⅗. Conſeq. AC = √(CF² + AF²) = 12, and BC = 9 chains.

Mr. Andrews, *of Cark*, alſo gives the ſame conſtruction, and thus proves that the rect. AG·GB is = to the area of the △ ACB.

Complete the parallelogram ACBN and through L, the centre of the inſcribed circle, let QK and HT be drawn ‖ to AC and CB reſpectively.

Then, by the lem. on pa. 345, Simpſon's Alg. the area of the △ ACB is = to the rect. QC + the rect. KT: but the rect. NC is = to 2 △ ACB, theref. the rect. LN = the △ ACB. Moreover AG = AH = QL = NT, and BG = BK = LT; conſeq. AG·GB = the given area ACB by conſtruction.

The ſame, inſerted by Mr. Johnſon, *Birmingham.*

ANALYSIS. Suppoſe the problem really ſolved, and lines drawn as in fig. 431, pl. 29, and that BG is = 6 chains the given diſtance. Then, *at page* 369, *Vol. I. Prop. VIII. Art.* 52, *of the Repoſitory*, it is proved, that AG·GB = VM·MW, but AM = MV, therefore AG·GB = AM·MW = the given area of the △ ACB. Now BG being given, AG will alſo be given, and conſeq. the ⊥ CF becomes known: Hence this

CONSTRUCTION. Take BG = the given diſtance, and add thereto GA ſo that the rect. AG·GB = the area of the △; upon AB as a diam. deſcribe a circle, biſect AB in M, and through M at right angles to AB draw VS to cut the circle in V and S, and in MS take MW ſo that the rect. VM·MW may be = the given area; through W, ‖ to AB, draw WC to cut the circle in C and join AC, CB, ſo ſhall ACB be the △ required, as is evident from the Analyſis.

The

The same, enquired by Mr Smith, Mar.

Suppose ACB, (fig. 429, pl. 23) the ∠ required. L the centre of the inscr. circle, and G its point of contact with the hypotenuse; then, by the question, there are given BG and the area. About the △ let a circle be described, and draw CL to meet it at V, which will evidently be the middle of the arc AVB. Now, it is evident, from Cor. 8. Prop. III. Art. III. Book II. of the Repository, that the area of the ∠ ACB is = to the rect. AG·GB, hence AG is given. Again, from the first above quoted, it is evident that VB = VL: Hence this

CONSTRUCTION. Take BG = the given perimeter and make the rectangle BGA = the given area; on AB as a diameter describe a circle and draw GH ⊥ to AB, and from V the middle of the arc AB, to LG apply VL = VB and draw VL to meet the circle at C, and join AC, CB, and it is done, as is evident from the Analysis.

Geometrical solutions were also received from Messrs Carlisle, Harris, Hill, and Lowry.

The same, enquired by Mr Gregson the younger.

Fig. 430, Pl. 23. Let ABC be the right ∠d △, inscribed circle, touching the sides of the △ in the points F, and K. Then BG = BK. AG = AF, CK = CF, LK, L being the centre of the inscribed circle. Now BLF are = △s, the sum of these are = to the rect. BG·GL; for the like reason the rect. AG·GL; and the sq. are = to the sum of these, or (BG + GA = AB) AB·GL = to the area of the △ ABC. But EC = BG − GL; therefore (BG + GL = AG) AG·GL − BG·GL + GL·GL = AG·GL + GL·GL, is = to 2BG·GL + 2GA·GL = 2 times the foregoing. From each of these equals take AG·GL + GL·GL, and there remains AG·GL + GL·GL; but 2 times the area of the ∠ ABC, therefore 2 times the area of the said △ ABC. Now, in the question we have given the chains, and the area of the ∠ = 54 square chains, whence from the above theorem 54 ÷ 6 = 9 = AG. Then 6 + 9 = 15 = AB. But BC = BG + GC = x + 9 and BA = BG + HC = x + 9, and (x + 6)(x + 9) = 2 × 54 = 108, but x² + 15x + 54 = 108, from which equation x is easily found = 3 = HC, and

conſeq. AC $= 3 + 9 = 12$ ch, and BC $= 3 + 6 = 9$ ch, the dimenſions required.

Or, ſince, (AB + GL) · GL = area of ABC, as may be eaſily proved, if x repreſent LG or HC as above, we ſhall have $(15 + x) \cdot x$, or $15x + x^2 = 54$, which completed and reduced gives $x = 3$ chains, as before determined.

The ſame, by Mr. Bulmer, *Sunderland.*

Let ACB be the $\triangle$, and G, H, K the points of contact with the inſcr. circle; then we have given BG $=$ BK $= 6$ and the area $54 = a$.

Put the rad. of the inſcr. circle $= x$, and AG $=$ AH $= y$, then AB $= y + 6$, BC $= x + 6$, and AC $= x + y$. Now, by Eu. I. 47,
$(y+6)^2 = (x + 6)^2 + (x+y)^2$, alſo by the property of the figure, $(x + 6) \cdot (x + y) = 2a$; theſe equations reduced produce
$y^2 + 12y + 36 = x^2 + 12x + 36 + 2xy + y^2$, or
$2x^2 + 2xy + 12x = 12y$, and
$x^2+xy+6x+6y=2a$. Multiply this equation by 2, the product is
$2x^2 + 2xy + 12x + 12y = 4a$; from this equation take
$2x^2 + 2xy + 12x \quad\quad = 12y$, and there remains
$12y = 4a - 12y$, or $24y = 4a$, or $6y = a$, $\therefore y = a \div 6 = 9$.

Again, by the property of the figure, $(6 + x)^2 + (9 + x)^2 = 15^2$, or $2x^2 + 30x = 108$, or $x^2 + 15x = 54$; hence $x = 3$, and the three ſides of the $\triangle$ are 9, 12, and 15 chains.

The ſame, by Mr. T. Squire, *Baldock.*

Let ACB repreſent the triangular field, the area of which is given $= 54$ ſq. chains, which put $= a$, and let L be the cen. of the inſcr. circle, and G, H, K the points where the circle touches the ſides of the $\triangle$. Put BG $= b = 6$ ch. the given diſt. and the rad. of the inſcr. circle $= x$, then BC $= b + x$, and by trigonom. $b : 1 :: x : x \div b =$ tan. $\angle$ GBL, or LBK, for the line BL biſects the $\angle$ B. By *pa.* 34, *Crackell's Tranſlation of Maudit's Trigonometry*, the tan. of the double arc is $= 2bx \div (b^2 - x^2) =$ tan. $\angle$ ABC; again, as $1 : 2bx \div (b^2 - x^2) :: b + x : 2bx \div (b + x) \div (b - x) (b + x) = 2bx \div (b - x) =$ AC, and AC · BC $= 2bx \cdot (b + x) \div (b - x) = 2a$; hence $x = 3$, and the three ſides of the field are 9, 12, and 15 chains reſpectively.

Ingenious algebraical ſolutions were alſo received from Meſſrs. Barron, Blackwell, Buffham, Croudace, Francis, Marrat, Merones Minor, Thornoby, *and* Tyro Philo Mathematicus.

VI.

VI. QUESTION 162, *answered by Mr.* W. Francis, *Teacher of the Mathematics, at Hampstead, Middlesex.*

The propoſer of this queſtion ſeems to have been miſtaken, in the compoſition of his globe, for, it is evident, it could never have produced the effect he deſcribes had it been made of glaſs or any other ſubſtance more heavy than the liquid in which it was immerſed. Let us ſuppoſe he wiſhed only to know the content of that ſegment of the globe which would be forced under the water by the weight of the globe only. Then ſince the ſolidity of a globe one foot in diameter is 904·7808 cubic inches, and that of one 11·9 inches in diameter is 882·3492524, it follows that 22·4315 inches is the content of the glaſ[illegible]nd as 1000 : 2600 :: 22·4315 : 58·3219 inches the conte[illegible] of the water diſplaced by it. Now put x = the depth of the [illegible]ment immerſed, then $(12 \times 3 - 2x) \times x^2 \times \cdot 5236 = 58\cdot3219$ [illegible]uration, that is, $18x^2 - x^3 = 35\cdot69318$, whence $x = 1$[illegible]t now remains to find the content of the ſegment within [illegible]even with the water's ſurface, or 1·8074 in depth, the di[illegible]ng 11·9 inches. Whence by menſuration $(11\cdot9 \times 3$[illegible]$74) \times 1\cdot8074^2 \times \cdot5236$ = 54·8798 ſolid inches th[illegible]ght.

The ſame, by Mr. Olinthus Gregory, *Bookſeller, Cambridge.*

The ingenious propoſer [illegible]f this queſtion has inadvertently fallen into an error in the enun[illegible]iation of it: for ſince glaſs is heavier than water, it is impoſſibl[illegible] according to the principles of hydroſtatics, that the water in a [illegible]ſphere ſhould be on a level with the water in which it is p[illegible]nleſs ſome extraneous power be applied to ſuſtain the ſpher[illegible]n that ſituation. In order, therefore, to render the queſtion cap[illegible]le of a ſolution, conformably to the propoſers views, either the [illegible]iquor in the ſphere muſt be of a leſs ſpecific gravity than water[illegible] the ſhell muſt be compoſed of ſome ſubſtance which is lighter[illegible] water. Firſt, let us ſuppoſe the liquor to be of a leſs ſp[illegible] gravity than water, and let the ſpecific gravities of glaſs and the liquor be r and s reſpectively, that of water being unity. Knowing the diam. and thickneſs of the glaſs ſphere its ſpecific gravity r being given, the relative weight may be readily found, this we will denote by g. Then, if z be put for the altitude of the liquor in the ſphere, a for the axis of the ſphere, t for the thickneſs of the glaſs, and c for ·5236, we ſhall have $c \cdot (3a(z+t)^2 - 2(z+t)^3)$ to denote the relative weight of the water diſplaced, and $cs \cdot (3az^2 - 6tz^2 - 2z^3)$ [illegible] the liquor in the ſphere. Hence we deduce the following equation:

$$c \cdot (3a(z+t)^2 - 2(z+t)^3) = cs \cdot (3az^2 - 6tz^2 - 2z^3) + g.$$

Then

This may be reduced to a final equation of the third degree, from which, when s and t are properly chosen, the value of z may be ascertained. Or, agreeably to the second supposition, let us imagine the shell of the sphere to be composed of elm, the specific gravity of which is 600, that of water being 1000, and let us suppose the thickness of the shell instead of $\frac{1}{10}$ to be $\frac{1}{2}$ an inch. Then, it is evident, that the sphere will rest (as per quest.) when the weight of a portion of water, equal to the spherical segment immersed, is equal to the weight of the water in the sphere, added to the weight of the wood of which the shell is made; and, of course, the solid content of the elm which is immersed, is to that of the whole spherical shell, as the specific gravity of elm to that of water, or as 6 to 10. But the thickness of the shell being small, when compared with the axis of the sphere, we may, for an approximation, suppose the content of the part of the shell immersed in the water, to be *as* the surface immersed, or proportional to the *altitude* of the segment: and then, 11 inches being the internal axis of the hollow sphere, we shall have $11 \times \cdot 6 = 6{\cdot}6$ inches, for the altitude of the water therein. This result will, on a comparison with more accurate methods, be found to vary not more than a-thousandth part from the truth. It must be manifest that in these remarks the effects of air, either within or upon the sphere, have not been taken into consideration.

The same, answered by Tyro Philo Mathematicus.

The enunciation of this question seems to contradict the principles of hydrostatics: but if instead of the words, "till the surface of the water in the sphere was on a level with the surface of the water in the vessel," be inserted, "till the surface of the water in the sphere was n inches below the surface of the water in the vessel," or if instead of water some other liquid, whose specific gravity is given less than that of water, had been contained in the sphere; then the quantity, in either case, may be determined as follows:

Let d = diameter of the sphere, (out side dimensions),
t = thickness of the glass, $\phi = \cdot 5296$,
a, b, τ, = the sp. gr. of glass, water, and the liquid contained in the sphere,
n = the diff. of the altitudes of the liquid in the sphere, and the water in the vessel,
and x = the altitude of the segment, or depth of the liquid in the sphere.

Then we shall have

$$\ldots = \frac{\ldots}{\ldots}(1 + z)^n \ldots$$

$\overline{-x}|^3)\cdot a$ = the whole weight of the glaſs,

$-6tx^2 - 2x^3)\cdot c$ = the whole weight of the liquid contained in the veſſel, alſo

$\overline{c+n+t}|^3 - 2 \times \overline{x+n+t}|^3\cdot b$ = the whole weight ter diſplaced by the ſphere, which muſt be = to the ſum o expreſſions above, by hydroſtatics. Hence we get this ıtion (putting $n + t = r$).

$$\left.\begin{matrix} c \\ b \end{matrix}\right\} x^3 \left.\begin{matrix} +3db \\ -6br \\ +6ct \\ -3cd \end{matrix}\right\} x^2 \left.\begin{matrix} +6dbr \\ -6br^2 \end{matrix}\right\} x \left.\begin{matrix} +3dbr^2 \\ -2br^3 \\ -8at^3 \\ +12adt^2 \\ -6ad^2t \end{matrix}\right\} = 0,$$

ıe valve of r muſt always be taken greater than t, when :o a greater than b.

No notice is here taken of either internal or external

obſervations were alſo received from Meſſrs. Marrat, *the* ınd Merones Minor.

QUESTION 163, *anſwered by Rev.* L. Evans.

urn to Prop. XXII. Book II, of the Principia, we ſhall the property of the hyperbola (vid. the figure to prop. oned) SB : SA : : Aa : Bb = SA × Aa ÷ SB = SA² ecauſe SA and Aa are equal. In like manner Ff = SA² Therefore Aa — Bb = (SA × SB — SA²) ÷ SB, - Ff = (SA × SF — SA²) ÷ SF. Conſequently, Aa ıa — Bb = (SA × SF — SA²) ÷ SF : (SA × SB — SB = (SF — SA) ÷ SF : (SB — SA) ÷ SB = AF AB ÷ SB = AF × SB : AB × SF, as it ought to have ted in the queſtion.

: ſame, anſwered by Mr. Geo. Saunderſon, *London.*

ıing, in the queſtion, AF × SB to AF × SA, the ſolube thus:

: property of the hyperbola SA : SF : : Ff : Aa; and : : Bb : Aa. Whence (by inver. and conver.) SF : ıa : Aa — Ff; and SB : AB : : Aa : Aa — Bb; or : : Aa — Bb : Aa; this compounded with the third,

SF ×

SF × AB : AF × SB :: Aa — Bb : Aa — Ff ; or as 1 : $\frac{AF \times SB}{SF}$ = 1005825, becauſe AB = 1, which is a more ſimple expreſſion than 1 : $\frac{Aa - Ff}{Aa - Bb}$ its equal.

VIII. QUESTION 164, *anſwered by* Tyro Philo Mathematicus

As 81 : 70 :: rad. : tang. of 40° 50′, the apparent altitude of the ſun's upper limb, and applying the ſemidiam.—16′ 4″, refraction—1′ 3″, and parallax + 7″, the ſun's true central altitude will be = 40° 33″; now, in the ſpherical △ ZPS (fig. 432, pl. 23.), let P be the pole, Z the zenith and S the ſun's place. Then there are given ZS the co-alt. = 49° 27′, ∠ PZS the azimuth = 87°, and ∠ ZPS the hour angle = 54°, to find PS and ZP thus. As ſine ∠ ZPS : ſine ZS :: ſine ∠ PZS : ſine PS = 69° 42′ 20″ = co-declination, and as ſine ½ (PZS ∽ ZPS) : ſine ½ (PZS + ZPS) :: tan. ½ (PS ∽ ZS) : tan. ½ ZP = 30° 39′ 43″, and ZP = 61° 19′ 26″ = co-altitude; hence the day was the 21ſt of May, and the latitude 28° 40′ 34″ N.

The ſame, anſwered by Mr. T. Squire, *Baldock.*

By trigonom. as 70 : 81 :: rad. : 1·1571428 = the cotang. of 40° 50′, the ſun's apparent altitude, but allowing for ſemidiameter and refraction, we have the ſun's true alt. = 40° 32′ 51″; then in the ſpherical △ ZPS (fig. 432. pl. 23.) we have given ZPS = ∠ of time, ZS = co-alt. and ∠ PZS = azimuth angle, to find PS = 69°,, 42′⅓ = co-dec. and ZP = 61° 19′ 26″ the co-latitude.

The ſame, anſwered by Mr. J. Blackwell, *Hungerford,*

Firſt, as $\sqrt{70^2 + 81^2}$: rad. :: 70 : ſine of 40° 50′ the alt. of the ſun's upper limb, and by allowing for ſemidiam. and refrac. we have the true alt. of the ſun's centre = 40° 30′ 57″ ½. Hence in the oblique angled ſph. △ SZP (fig. 432, pl. 23.) there are given the co-alt. ZS = 49° 27′ 02″½, the ∠ SZP = 87°, and the ∠ SPZ = 54° the meridian diſtance, to find ZP = 61° 19′ 36″ the co-lat. and SP = 69°, 42′, 26″. Whence the latitude was 28° 40′ 24″ North, and the ſun's declination anſwers to the 21ſt of May.

Solutions

Solutions to this queſtion were alſo received from Meſſrs. Parr, Burdon, Croudace, Gregory, Johnſon, Lowry, Mariat, *and* Merones Minor.

IX. QUESTION 165, *anſwered by the propoſer, Mr.* George Brown, *Teacher of the Mathematics, at Newcaſtle-upon-Tyne.*

Let x and y = the ſine and coſine of the required latitude,
z and v = the ſine and coſine of the required declination;
then per ſpherics
$1 : x \div y :: z \div v : a$, that is, $xz = avy$, a being = fine of 30° the aſcenſional diff. and
$1 : x :: z : b$, that is, $ax = b$ = fine of 15° 35′ the altitude at at ſix, therefore,
$vy = b \div a$, by equating the two values of xz. Hence

$vy - xz = \overline{(1 \div a) - 1} \times b = .2686396$ = cos. of 74° 25′ the ſum of the lat. and dec. and

$vy + xz = \overline{(1 \div a) + 1} \times b = .8059188$ = cos. of 36° 18′ 4″ their difference; therefore
$\frac{1}{2}(74° 25′ + 36° 18′ 4″) = 55° 21′ 32″$, the latitude, North, and
$\frac{1}{2}(74° 25′ - 36° 18′ 4″) = 19° 3′ 28$, the declination anſwering to May 16.

The ſame, anſwered, by Merones Minor.

Put a = nat. coſine of 15° 35′, x = nat. coſine of (co-declination — co-latitude), and y = the nat. coſine of their ſum, to radius 1. Then, *by Emerſon's Trigonom. Cor. I. Prop.* 46, *B. III.* we have
$x - y : x - a :: 2 : 1$, the verſed ſine of 90° = 6 hours, and
$x - y : x :: 2 : 1.5$ the verſed ſine of 120° = 8 hours :: 4 : 3.
Hence $x = 3a = .8059188$ the coſine of 36° 18, and $y = -a$ the coſine of 105° 35′. Wherefore the latitude was 55° 21½ north, and the declination 19° 3′½ N, anſwering to the 16th of May.

The ſame otherwiſe, by Mr. Burdon, *Acaſter Malliſ, York.*

By ſpherics,
rad. : tan lat. :: tan. dec : fine of ∠ of time from ſun riſe to ſix, and
rad. : ſin lat. :: ſin. dec : fine of the altitude at fix, therefore

$$1 : \frac{\text{tan. lat.}}{\text{fin. lat.}} :: \frac{\text{tan. dec.}}{\text{fine dec.}} : \frac{\text{ſin. of ∠ of time}}{\text{ſin. of alt. at ſix}}, \text{ or}$$

1 : ſe-

1 : sec. lat. : : sec. dec. : $\frac{\text{sin. of } \angle \text{ of time}}{\text{sin. alt. at fix}}$. Hence

are given the rectangle of the tangents of two arcs and the rect: of their secants to the same given radius to find those arcs, of w this may be the

CONSTRUCTION. Fig. 433, Pl. 23. Take CD = given radius and produce it to E so that CD·DE may be = t rect. of the tangents; bisect CE in F with the perpend. FG, from E, to FG, apply EG = a 3rd proportional to 2CD an side of the square denoting the rectangle of the secants. W with the centre G and radius GE or GC describe a circle cut AB (drawn ⊥ to CD) in A and B, so shall AD, DB, be tangents of the required arcs.

For AD·DB = CD·DE = the given rect. of the tangents, and AC·CB = 2EG·CD = the given rect. of the secants, by co

The *Calculation* easily follows from the construction, gi BD = tangent of 19° 3′½ the sun's declination, answering to 16th of May, and AD = tangent of 55° 21′½ the latitude of place of observation.

The same again, by Tyro Philo Mathematicus, *of Hull.*

This quest. may be solved by the orthographic projection of sphere, thus, let the circle HZR (fig. 434, pl. 23.) be the m dium, the arc H*m* the sun's meridian altitude, its line *bm*; th R*n* the midnight depression, its line *cn*; EQ the equator, parallel of declination; P*p*, at right angles to EQ, the fix o'c line; *g* the sun's place at rising; and *sr* the sink of his altitu fix. Then by the principles of this projection, it will be as

O*f* : *f*Q (: : *rg* : *gn*) :: *sr* : *cn*, or, as

sin 30° : radius — sin 30° : : sin 15° 35′ : sin 15° 35′ = R*n*, a

O*f* : *f*E (: : *rg* : *gm*) : : *sr* : *bm*, or as

sin 30° : radius + sin 30° : : sin 15° 35′ : sin 53° 42′ = H*m*, th

HE = ½ (H*m* + R*n*) = 34° 38′½ = the colatitude, and

E*m* = ½ (H*m* — R*n*) = 19° 3′½ = the declination. Hence latitude 55° 21′½ N, and the day the 16th of May.

Mr. Croudace *likewise answered this Question.*

X. QUESTION 166, *answered by* Merones Minor.

Let ABRS be a section of the frustum, *f*HFII that of sphere, and produce AR, BS to meet et D and draw the lines as in figure 435, plate 23.

Then in [illegible]

AC — [illegible]

DS = [illegible]
RS: [illegible]
CB: [illegible]
DB: [illegible]
HD = [illegible]
GH = [illegible]
IH = [illegible]

Now, [illegible] tricle [illegible] twee[illegible] [illegible] before [illegible] **DMN** = [illegible] of the [illegible] [illegible] [illegible] the [illegible] Therefore [illegible] join [illegible] the [illegible] [illegible] [illegible] cone [illegible] [illegible] inches, [illegible]

The same [illegible]

[illegible] common [illegible]

BC: **CD** = [illegible]

by [illegible] [illegible] above [illegible]

[illegible]

[illegible] neares[illegible] Now, [illegible]

is equal to the fruſtum MS — the ſegment rHe, that is, the fruſtum LS = fruſt. MS + the whole ſphere — the ſegment rh, that is, $(LO^2 + RS^2 + LO \cdot RS) \times TP \times .2618 = (MN^2 + RS^2 + MN.RS) \times Sp$ or $TQ \times .2618 + (2EF)^3 \times .5236 - 3(Eh^2 - \overline{Eh - hP}^2 + hP^2) \times hP \times .5236$, that is, $1802 \cdot 656546 46 - 204 \cdot 301424655x + 7 \cdot 600329x^2 - .94248x^3 = 408 \cdot 408$ (fruſt. MS) $+ 904 \cdot 7808 - 18 \cdot 8196x^2 + 1 \cdot 0472x^3$, and, by tranſpoſition and diviſion, we have $x^3 - 23 \cdot 17228x^2 + 178 \cdot 9846x = 428 \cdot 8134$, in which equation the value of x is $4 \cdot 548728$; conſeq. $DF + TQ + hP = 21 \cdot 215394$, and $QP = 26 \cdot 8806 - 21 \cdot 215394 = 5 \cdot 665206$ inches the height to which the water will riſe by the immerſion of the globe.

The ſame, anſwered by Mr. John Harris, *Caermarthen.*

Fig. 435, Pl. 23. Let ABRS repreſent the veſſel, MN the ſurface of the water, before the immerſion of the globe, LO the ſame after the immerſion, and F, f, the two points where the globe touches the ſides of the veſſel. Let lines be drawn as in the figure. Then in the right $\angle$d $\triangle$ AtR are given the ſides $At = \frac{1}{2}(AB - RS) = 6$, and $tR = 20$, to find the $\angle$ $ARt = ADC = 16^\circ\ 41'\ 57''$, theref. $ED = 20 \cdot 88061$&c. = coſecant of the $\angle$ EDf to rad. $Ef (= 6)$, and $HD = 14 \cdot 88061$; again, by ſim. $\triangle$s $At\ (6) : tR\ (20) :: RT\ (2) : TD = 6 \cdot 66666$, theref. $HD - TD = HT = 8 \cdot 213944$ the height of the globe above the height of the veſſel. Put $a = HD = 14 \cdot 88061$, $b = HQ = 1 \cdot 786056$, $c = MN = 8$, t = tan. $\angle$ ADC to rad. 1, $n = .26179$ &c, $d = 12$ the diam. of the globe, and $x = HP$ the height of the ſegment immerſed; then, by menſuration, the ſolidity of the fruſtum LIKH will be $= (LO^2 + IK^2 + LO \times IK) \times nx$: but $IK = 2at$ and $LO = 2t(a + x) = 2at + 2tx$; theref. $\overline{2ta}^2 + 8t^2ax + 4t^2x^2 + \overline{2ta}^2 + 4t^2ax + \overline{2ta}^2) \times nx =$ the ſolidity of the fruſtum LIKO, and the content of the ſegment of the globe immerſed will be $= (3d - 2x) \cdot 2nx^2 = (6dx^2 - 4x^3) \cdot n$. Now, it is evident, that their difference, or the ſpace included between the fruſt. LIKO and the ſurface of the globe muſt be equal to the ſolidity of the fruſtum MIKN (= the quantity of water above IK). Hence this equation, $x^3 + \frac{12ta - 6d}{4(t^2 + 1)} \times x^2 + \frac{12t^2a^2}{4(t^2 + 1)} \times x = \frac{(4t^2a^2 + 2tac + c^2) \times b}{4(t^2 + 1)}$, in which the value

value of x is $= 7{\cdot}451259$, conſeq. HP + HT $=$ PT $=$ $5{\cdot}663206$ inches the height required.

XI. QUESTION 167, *anſwered by the Rev.* I. Evans.

Fig. 436, Pl. 23. Put AB $=72$ inch. $=a$, DC $=9=b$, EF $=$ $6=c$, $b-c=3=d$, and BI $=x$; then GH $=\frac{dx+ac}{a}$; put alſo $\cdot7854=p$, and ſp. gr. of fir $=\frac{550}{1728}=s$, and we have

$$\frac{3b^2+2b\times\frac{dx+ac}{a}+\left(\frac{dx+ac}{a}\right)^2}{b^2+b\times\frac{dx+ac}{a}+\left(\frac{dx+ac}{a}\right)^2}\times\frac{a-x}{4}=\text{MI the diſ-}$$

tance of the centre of gravity of the fruſtum DG from the point I

$$\left(b^2+b\times\frac{dx+ac}{a}+\left(\frac{dx+ac}{a}\right)^2\right)\times\frac{a-x}{3}\times p\times s=$$

weight of DG. Again,

$$\left(3b^2+2bx\times\frac{dx+ac}{a}+\overline{\frac{dx+ac}{a}}^2\right)\times\frac{a-x}{4}\times\frac{a-x}{3}\times$$

$ps=$ MI $\times$ weight of DG. Again,

$$\frac{3\times\overline{\frac{dx+ac}{a}}^2+2c\times\frac{dx+ae}{a}+c^2}{\overline{\frac{dx+ac}{a}}^2+c\times\frac{dx+ae}{a}+c^2}\times\frac{x}{4}=\text{BN the diſt. of}$$

the centre of grav. of the fruſtum GE from the point B.

$$\left(\overline{\frac{dx+ac}{a}}^2+c\times\frac{dx+ac}{a}+c^2\right)\times\frac{x}{3}\times ps=\text{wt. of the fruſt. GE.}$$

$$\left(3 \times \overline{\frac{dx \times ac}{a}}^{\frac{2}{3}} + 2c \times \frac{dx + ac}{a} + c^2\right) \times \frac{x}{4} \times \frac{x}{3} \times ps$$

BN × weight of GE. By the nature of the queſtion, MI weight of DG + AI × W = BN × weight of GE + × w, that is,

$$\left(3b^2 + 2bx \times \frac{dx + ac}{a} + \overline{\frac{dx + ac}{a}}^{2}\right) \times \frac{a - x}{4} \times \frac{a - x}{3} \times$$

$+ (a - x)$. W is equal to $\left(3 + \overline{\frac{dx + ac}{a}}^{2} + 2c \times \frac{dx + ac}{a} + c^2\right.$

$\times \frac{x}{4} \times \frac{x}{3} \times ps + xw.$

Hence $3a^2b^2 \times (a^2 - 2ax + x^2) + 2ab \times (a^2 - 2ax + x^2)$ $\times (dx + ac) + (a^2 - 2ax + x^2) \times (d^2x^2 - 2acdx + a^2c^2) -$ $3x^2 \times (d^2x^2 - 2acdx + a^2c^2) - 2acx^2 \times (dx + ac) - a^2c^2x^2 +$ $\frac{12a^2}{ps} \times (a - x)$. W $- \frac{12a^2xw}{ps} = 0$. Whence

$$\begin{aligned}
&+ 3a^4b^2 - 6a^3b^2x + 3a^2b^2x^2 + 2abdx^3 - 2a^2x^4 = 0.\\
&+ 2a^4bc + 2a^3bdx + 4a^2bdx^2 - 2ad^2x^3\\
&+ a^4c^2 - 4a^3bcx + 2a^2bcx^2 - 7acdx^3\\
&\frac{12a^3W}{ps} + 2acdx + a^2d^2x^2\\
&\qquad - 2ac^2x - 2a^2edx^2\\
&\qquad - \frac{12a^2W}{ps}x + 5a^2c^2x^2\\
&\qquad - \frac{12a^2w}{ps}x
\end{aligned}$$

from which the value of x may be found.

XII.

XII. QUESTION 168. Several of the Editor's correspondents inform him, that they have attempted solutions to this question, but with hopeless success. The major part of them are induced to conclude, that some error has crept into the statement, which renders a satisfactory answer impossible; or that the proposer himself has not succeeded so well as might be wished. The Editor takes the liberty of requesting, that the gentleman who proposed it will favour him with an accurate statement of the question, [illegible] for appearance in a future number.

XIII. QUESTION 169, *answered by* [illegible]

The latitude, day, and hour being given, the sun's [illegible] = 41° 31′ 2″·7, this + the sun's [illegible] — [illegible] = [illegible] 16″·7 the sun's true alt. the sine and cosine of which [illegible] = c, to rad. 1, also put a = [illegible] = [illegible] and $2x$ denote the transverse and conjugate axes of the [illegible] frame. Then by trigonom. [illegible] ÷ [illegible] = [illegible] axis of the included space on the plane of the [illegible] $(2cx \div s) \cdot 31 \cdot b = c$, by the question = [illegible] $\sqrt{ac \div 6bc} = 5{\cdot}690615$, hence the sum of the sines are 17·098845 and 11·39923. Whence [illegible] inches.

The same, answered by Mr. Gregory, *Late Fellow and Tutor of the Mathematical School at Cambridge.*

This question is not expressed so clearly as might be wished; but I suppose the proposer means that the two circular parts, on which the shadow was received, to obtain the lower part of the conjugate axis of the ellipsis—for, on any other supposition there are not sufficient *data*. Now, the latitude, declination, and hour angle being given, the alt. of the sun's upper limb is found by spherics, after proper correction for refraction, &c. to be 41° 47′ 50″; and as the shadow of the ellipsis will be the greatest, under the circumstances allowed by the question, when its transverse axis is perpendicular to the solar rays, we shall have one axis of the shadow (which it is well known will be either an ellipsis or a circle) equal to the transverse axis of the elliptical frame, and the other axis of the shadow in proportion to the conjugate of the frame, as the cotangent of the sun's apparent alt. to radius. Therefore, putting $3z$ and $2z$ for the transverse and conjugate of the frame, c for ·785398 &c. and t for the cotang. of the alt. to rad. 1, we

have $c \times 3z \times 2zt = 6ctz^2 = 172{\cdot}8$ the area of the ſhadow, whence we get $z = \sqrt{(172{\cdot}8) \div 6ct} = 5{\cdot}725639$, and the axes of the frame are 17·176917 and 11·451278 reſpectively.

But in the queſtion the area alone is required, and this will be repreſented by $3z \times 2z \times c$, or $6cz^2$: theref. as $6ctz^2 : 6cz^2 :: 172{\cdot}8 : 172{\cdot}8 \div t = 172{\cdot}8 \div 1{\cdot}1185482 = 154{\cdot}48599$ ſquare inches, area of the frame.

The ſame, anſwered by Mr. Wm. Francis.

Suppoſing the lower end of the conjugate diameter to touch the horizontal plane, the ſolution will be thus:

Having found, from the data, the apparent alt. of the ſun's upper limb $= 41^\circ\ 50'\ 39''$, and ſince the conjugate and tranſverſe axes are as 2 to 3; then as $41^\circ\ 50'\ 39'' : 2 :: 48^\circ\ 9'\ 19'' : 2{\cdot}235$. Therefore the axes of the ſhadow are in proportion to each other as 3 to 2·234. Let $3x$ = the tranſverſe, then $2{\cdot}234x$ = the conjugate. And $3x \times 2{\cdot}234x \times {\cdot}7854 = 172{\cdot}8$ by the queſtion, that is, $5{\cdot}2637508x^2 = 172{\cdot}8$, or $x = 5{\cdot}7296$. Hence as $3 : 3 \times 5{\cdot}2796 :: 2{\cdot}234 : 12{\cdot}8$ the conjugate axe of the ſhadow. Again as $2{\cdot}234 : 2 :: 12{\cdot}8 : 11{\cdot}4592$ the conjugate axis of the frame alſo $2 : 11{\cdot}4592 :: 3 : 17{\cdot}188797$ the tranſverſe axis. Hence the area of the frame $= 154{\cdot}[illegible]0128$ ſquare inches.

Ingenious ſolutions were alſo received from Meſſrs. Croudace, Evans, Lowry, May, Merones Minor, *and* Thornoby.

XIV. QUESTION 170, *anſwered by Mr.* John Lowry.

Case I. For the greateſt ellipſis that can be inſcribed in the parabola. Fig. 437, Pl. 23.

Suppoſe the thing done, and that DNCKEL is the greateſt ellipſis that can be inſcribed in the given parabola HCPEG, having one of its axes (NL) parallel, and the other (DK) perpendicular to the baſe of the parabola. Through the points of contact C and E, draw the tangents BCA, and BEF to interſect in B, and meet HG produced both ways at A and F; join AE, FC, and CE, and draw KR to touch the ellipſis at K, the extremity of the axis DK. Then, by Simpſon on the *Max. et Min.* the ellipſis DCE is the greateſt that can be inſcribed in the $\triangle$ ABF, when the ſides AB, BF, AF, are biſected at the points of contact C, E, D; much more, then, will it be the greateſt that can be inſcribed in the parabola HCPEG, which is wholly included within the $\triangle$, therefore we have only to draw the tangent AB, or BF, ſo as to be biſected at the point of contact C, or E.

Now, by the property of the parabola IP = BP, and ſince BE = EF, by hypotheſis, BI will be = ID, theref. 3PB = PD,

and

and therefore BD = [illegible] bisected by the [illegible] DB, as [illegible] O, will be the [illegible] of the ellipsis. [illegible] Δ, $2OD = BD$ [illegible] $= \frac{1}{2}$ $= 2OD = a \div$ [illegible] $= a$.

Again, by the [illegible] PD : PG $= 5^2 \div \frac{1}{2}$ [illegible] $= \frac{1}{2} \sqrt{3}$: but by the [illegible] DF·KF $\div 3^2$, therefore NL = [illegible] other axis. Wherefore the area [illegible] $\times$ .7854.

Note. When the [illegible] integral, a circle, and BL [illegible] $= 2GL$; DP [illegible] or DP : HG : [illegible] $\div$ [illegible]. Hence if parabola, the [illegible] HG to the than $8 \div$ [illegible] the greater axis of the but if in a [illegible] the leſs axis will be [illegible]

Case II. To [illegible] greateſt parabola [illegible] given ellipſis. Fig. 438, Pl. 23.

Suppoſe that BAC is the greateſt parabola in the given ellipſis BACE. Through B, C, draw the tangents GAF, GBH, [illegible] the extremity of one of the axes of the [illegible] contact of the ellipſis and parabola. [illegible] parabola BAC will be a *max*, which has a conſtant ratio, is a *max*; and the *min*. that rectangle will be the [illegible] ſected at the points A, B. C.

Now, if BF be drawn [illegible] of the ellipſis, thereſ AI = [illegible] $+ DE = 3DE = \frac{3}{4} AE$. [illegible] if C be put to repreſent the [illegible] the tranſverſe AE : C :: [illegible] BC $= \frac{1}{2} C\sqrt{3}$, and therefore [illegible] $\times \frac{1}{2} C\sqrt{3} = \frac{1}{4} AE \times C\sqrt{3}$.

It is evident, that we ſha [illegible] area, if the point of contact A [illegible] of the ellipſis, ſo that the baſ [illegible] either of the axes of the ellip [illegible] ſame.

The ſame, anſwe[illegible]

Fig. 439, Pl. 23. [illegible] the ſemiellipſis inſcribed [illegible] draw the tangent TM*t*, [illegible] $\perp$ to T*t*. Put $a =$ [illegible]

bola and $x = BP$. Then, by *Emerſon's Conics*, III. 3, $OP = \frac{1}{3}p$, alſo *ibid* 17, I. *Cor.* 2, EP : ET :: PD : DT, and, by compoſition, EP + ET : PD + DT :: EP : PD, that is, $2a : 2x :: a - x : x(a - x) \div a = PD$; then $CD = \frac{1}{2}(EP + PD) = (a^2 - x^2) \div 2a$, $PC = CD - PD = (a - x)^2 \div 2a$; alſo, *ibid.* 15, 1. *Cor.* 3, PC:PO::CD²:CA², that is, $\sqrt{PC} : \sqrt{PO} :: CD : CA = 5(a + x) \div 8\sqrt{a}$ the ſemitranſverſe. Hence the area of the ellipſis being as $CD \times CA$, we have $((a^2 - x^2) \div 2a) \times (5(a + x) \div 8\sqrt{a})$, or $(a^2 - x^2)(a + x) = a\ max.$ which, being fluxed and reduced, gives $x = \frac{1}{3}a$, and the area $= \cdot 314159 \times 10a\sqrt{a} \div 27 = (80 \div 27) \times \cdot 314159 = 9\cdot 308422677$.

Again, let EFMP be the inſcribed ſemiparabola, and MP an ordinate to the conjugate axis DE of the ellipſis. Put $a = CA$, $b = CD$, and $x = CP$. Then, by *Emerſon's Conics*, 7, 1. CD² : CA² :: DPE : PM², *that is*, $(a \div b)\sqrt{(b^2 - x^2)} = PM$. Hence, the area of the parabola being as EP × PM, we have $(b + x)(a \div b)\sqrt{(b^2 - x^2)}$, or $(b + x)\sqrt{(b^2 - x^2)} = a\ max.$ In fluxions &c. $x = \frac{1}{2}b$, and the area $= ab\sqrt{3} = 5\sqrt{3} = 8\cdot 660254038$.

N. B. The area of the parabola will be the ſame if the vertex be at A inſtead of E.

XV. QUESTION 171, *anſwered by Mr.* Gregory, *the propoſer.*

It has been ſhewn, by writers on optics, that when the apparent altitude of the ſun's upper limb is equal to, or exceeds, 54° 7′ there can be no ſecondary rainbow, and that when it equals or exceeds 42° 2′ there can be no primary rainbow. We have, therefore, given, in the firſt caſe, 54° 7′ apparent altitude of the ſun's upper limb,—41″ refraction + 5″ parallax,—15′ 47″ ſun's ſemidiameter, or 53° 50′ 37″, true altitude of the ſun's centre, latitude 52°N, declination on July 1ſt, 23° 8′ 29″N, the complements of which form the three ſides of a ſpherical triangle, to find one of the angles, namely the hour angle from noon, 28° 27′ 12″ equivalent to 1h. 53m. 59s, which doubled gives 3h. 47m. 38s. for the time during which no ſecondary bow can be ſeen. In the ſecond caſe, we have 42° 2′ for the apparent altitude of the ſun's upper limb, which corrected for the ſemidiameter (—15′ 47″), refraction (—1′ 3″), parallax (+ 6″) gives 41° 45′ 16″ true altitude of the ſun's centre; whence, taking the latitude and declination as before, we get 51° for the hour angle, equivalent to 3h. 24m. and twice this or 6h. 48m. is the time during which there can be primary bow.

In the above ſolution, no notice has been taken of the change of declination during the reſpective intervals: and it has been ſuppoſed

posed that the meridian of the place does not [illegible] from that of Greenwich.

The same, answered by Mr Wm. [illegible]

By *pa.* 383, *Vol. II. of [illegible]* when the alt. of the sun's [illegible] bow will be depressed below the horizon [illegible] is = 54° 10 the summit of the [illegible] horizon. Therefore we have given the [illegible] = 23° 08', and the sun's alt. = [illegible] ∠s or times from noon. Now, when the [illegible] time from noon is found = [illegible] 6h. 44m. 16s. = the [illegible] interior bow, and when the [illegible] is = 1h. 51m. the double of which [illegible] time during which there can be no [illegible]

The same, answered by [illegible]

By Emerson's Optics, p. [illegible] radius of the primary bow = [illegible] = 54° 22', and the depression of the [illegible] alt. of the sun. Therefore, having the [illegible] and lat. 52°, I find, by spherics, the [illegible] in like manner with the [illegible] afternoon; and their sum = [illegible] required for the primary bow. [illegible] 23° 8' and 25° 8', the alt. 54° 22' and [illegible] ∠s 27° 18' and 29° [illegible] 38m. 24s. the time required for the [illegible]

The same, answered by [illegible]

From the principles of [illegible] apparent altitude be greater than 41° [illegible] mary rainbow, and if greater than [illegible] secondary rainbow. Call [illegible] dec. 23° 8, and alt. of the [illegible] refrac. 1' 3' and parallax [illegible] = 41° [illegible] (allowing for refrac. 41' and [illegible] hours, which come out 5h. 28m. [illegible] noon, these doubled give 6h. 44m. [illegible] intervals for the primary and secondary [illegible]

This question was also answered by Messrs. [illegible] May, *and* Thornoby.

XVI. QUESTION 172, *anſwered by Mr.* W. Peacock *Birmingham.*

Fig. 440, Pl. 23. Suppoſe ACB the △ required, HE a
of the circumſcribing circle drawn ⊥ to the baſe, O the c
and OD the rad of the given inſcr. circle. Put $a =$ EH, $b =$
and $x =$ EF, then FH $= a - x$, and by *Prop. II. Cor.* 2,
Prop. III. Cor. 7, *Art. II. Vol. II. of the Repoſitory*, $AF^2 :$
$(2EF + OD) :: FH \cdot EH : AC \cdot CB =$, by the queſt. to
$= 4FE \cdot FH$, hence $AF^2 : OD \cdot (2EF + OD) :: EH : 4FE$,
is, in ſpecies, $(a - x)\ x : b\ (2x + b) :: a : 4x$, therefore
$- 4x^3 = 2abx + ab^2$, and conſequently $x^3 - ax^2 + \frac{1}{2}abx$
$\frac{1}{4}ab^2 = 0$. From this equation EF becomes known, and
thence the ſides and baſe of the △ are readily determined.

Mr. Blackwell *ſent a true ſolution to this queſtion.*

XVII. QUESTION 173, *anſwered by Mr.* James Cunlif

Analysis. Fig. 441, Pl. 23. Suppoſe the thing done,
the lines drawn, as directed by the queſtion. BgC and BhC
right ∠s, a ſemicircle whoſe diam. is BC will paſs through g a
Euc. 31, III, and $\angle ghB + ghi = ghB + gCB = 2$ righ
Euc. 22. III, theref. $\angle ghi = gCB =$ DCB. For the
reaſon a ſemicircle whoſe diam. is DB will paſs through i a
and $\angle giB\ (gih) =$ gDB $=$ DCB, Euc. 21. III, therefo
△s igh and DBC are ſimilar. And in the ſame way it m
ſhewn, that the △s ieh and DAC are ſimilar, and conſeq
the trapeziums $eighe$ and ACBDA; therefore $ie = ig$. By
of the ſim. △s igh, DBC, DB : BC : : ig : gh, alſo by th
△s ieh, DAC, DA (DB) : AC : : ie (ig) : he, theref.
$AC \cdot BC :: ig^2 : gh \cdot he$, and theref. by equating the produ
extremes and means, $DB^2 \cdot gh \cdot he = AC \cdot BC \cdot ig^2$; multipl
ſides of the equation or expreſſion by $AC \cdot BC \cdot ig^2$ and it be
$DB^2 \cdot gh \cdot he \cdot AC \cdot BC \cdot ig^2 = AC \cdot CB \cdot BD \cdot DA \cdot ie \cdot ig \cdot gh$
$AC^2 \cdot BC^2 \cdot ig^4$ which is to be a *max.* by the queſtion, th
$AC \cdot BC \cdot ig^2$ muſt alſo be a *max.* From what is deduced
preceding part of the *Analyſis*, it will eaſily be perceived, t
△s iEg and igh are ſim. therefore $iE : ig :: ig : ih$, th
$iE \cdot ih = ig^2$. Now, produce the ⊥ Di, till it meets the
again in H, and draw CG ⊥ to it.—DH is a diam. of the
and it is well known that $Di \cdot GH = iE \cdot ih = ig^2$. Alſo, by
25, B. III. Simp. Geo. $AC \cdot BC =$ DH·iG. Th
$AC \cdot BC \cdot ig^2 =$ DH·iG·$Di \cdot GH$ muſt be a *max.* from what
above, that is, $iG \cdot Di \cdot GH$ muſt be a *max.* becauſe the dia
is given. Therefore the problem is evidently reduced to t

To divide the given diam. DH into three such parts that the solid contained under them, may be the greatest possible. It is well known, will be the case, when D... equal to each other, that is, when each is ... Hence the trap. ABCD may be eas... constructed.

Remark. Because AI and GC are equal to each other, it is evident that AC muſt paſs through the centre of the circle, and be a diam. thereof. And as A... in DL the points L and ... will coincide. Alſo, because *ei* is generally parallel to LB, the points B and *k* muſt alſo coincide.

The ſame, anſwered by Mr. Lowe.

ANALYSIS. Fig. 442. Prop. Suppoſe the thing required to be ſolved, and let the ſeveral lines be drawn as in the preceding problem. The diam. DL being drawn through D, it will evidently paſs through *i* the middle of the chord... and LE and ... ⊥ to DE, and DC meeting ... in ... Then ... known, that the trapezium ... are ... EC is = to the diam. of a circle circumſcribing the ... by Euc. C. VI.

AC·CB = DE·FL AD·DB = DE·D... = LI ... *circ.* = EC ... *circ.*

But, by ſim. △s, EC : EF :: C... = F... *am.*

Hence EF·FL = EC *bis* = ...

In the ſame way it is ſhewn, that LF·D = ... Hence AC·CB·BD·DA *orig.* ... = ... LE·F·D·EF ... the queſtion, it is to be ... that the ... LE ... tity, therefore LE·D·EF ... the ... and this, it is well ... ... and EF are equal. The method of ... ...

COR. Since LI = LF, the ... ... AC will be a diam. of the ..., and the ... with B, and the ... with D.

The ſame, ſolved by Mr. J. H. Swale.

ANALYSIS. Fig. 442. Prop. ... The ... ſolved, and the lines ... Di to meet the circle at E, ... upon DE and DC reſpectively. ... ⊥ to AB, we have *Ai* = iB, therefore ... centre O, and is equal to the given diameter.

Now AC·CB·AD·DB *orig.* ... is the ſame, AD·... AC·CB *gk* ...

DB, and *ie* is = to *ig*, by *Harriſon's Propoſitions in the Repoſitory*.

Again AC·CB = DE·*Ch*, AD² = DE·*Di*, *ie*² = EC·*ik*, and *gh*·*he* = EC·*hm*; for *by Prop.* 15. *Pa.* 235, *Vol. II. Repoſitory*, EC is = the diam. of the circle circumſcr. the trap. *ighe*, therefore, when the original ſolid is the greateſt, the product DE²·EC²·*ik*·*hm*·*Ch*·*Di* will be the ſame, that is EC⁴·*Di*²·*Ei*² muſt be a *max.* ſince, by ſim. △s DE·*ik* = *Di*·EC, and DE·*hm* = *Ch*·EC.

Moreover, becauſe EC⁴·*Di*²·*iE*² is a *max.* the product EC²·*Di*·*Ei* is likewiſe a *max.* or, becauſe ED·EF = EC², and ED conſtant, the ſolid EF·*Fi*·*i*D muſt be a *max.* which will evidently obtain, when EF = *Fi* = *i*D, each being = to one third of the diam. DE. Hence the conſtr. is evident.

Cor. The angles ABC, ADC are right angles.

Mr. Fletcher, *the propoſer, alſo anſwered this queſtion.*

XVIII. QUESTION 174, *anſwered by* Mr. John Lowry, *Birmingham.*

Put r = the radius, and x = the tangent of the mean arc; then, by the propoſition, $(a \div b)x^2$ will repreſent the tangent of the greater arc. And by *Prop. IX. of Emerſon's Trigonometry*, $r^2 + (a \div b)x^2 \div r^2 :: (a \div b)x^2 - x : r^2(ax^2 - bx) \div (rb^2 + ax^3)$, the tangent of the difference of the ſecond and third arcs, therefore, when the difference of thoſe arcs is the greateſt poſſible, $(ax^2 - bx) \div (rb^2 + ax^3)$ muſt be a *max.* This expreſſion put into fluxions and reduced gives $b^2r^2 + a^2x^4 = 2abr^2x + 2abx^3$, or $r^2 + \overline{(a \div b)x^2}^2 = (2a \div b)x(r^2 + x^2)$. Multiply each ſide of this equation by r^2x and divide by $r^2 + \overline{(a \div b)x^2}^2$ and $r^2 + x^2$, and it becomes $r^2x \div (r^2 + x^2) = 2r^2(a \div b)x^2 \div (r^2 + \overline{(a \div b)x^2}^2)$. Now, by *Prop. II. Schol. I. ibid.* $2r^2x \div (r^2 + x^2)$ and $2r^2(a \div b)x^2 \div (r^2 + \overline{(a \div b)x^2}^2)$ are the ſines of twice the arcs, whoſe tangents are x and $(a \div b)x^2$ reſpectively. Therefore the ſine of twice the greater arc is equal to half the ſine of *twice* the mean arc, as it ſhould have been printed in the queſtion.

The ſame, anſwered by Mr. John Surtees.

Let rad. = 1, $b \div a = n$, and the tangents of the other two arcs nx, and nx^2. Then the tangent of the difference of thoſe arcs will be $(nx^2 - nx) \div (1 + n^2x^3) = a\ max.$ by the queſtion, which put into fluxions and reduced gives $2nx^2 \div (1 + n^2x^4) = nx \div (1 + n^2x^2)$.

$+ x^2x^2$.) Hence it appears, that the difference of thoſe arches will be a maximum when the ſine of twice the greater arc is equal to half the ſine of *twice* (not *thrice* as per queſt.) the mean or the ſum of the two unknown arches.

The ſame, anſwered by Mr. Geo. Sanderſon.

Let A, B, and C repreſent the three arches,

$d = \frac{a}{b}$ the tang. of A, one extreme

T = tang. of B the mean, and
t = tang. of C the other extreme, radius 1.

Then $\frac{T - t}{1 + tT}$ = tang. of B — C, (Simpſon's Trigon. pa. 55.)

which will be a maximum, when B — C is a maximum.

Or, becauſe $\frac{T^2}{d} = t$, $\frac{T - \frac{T^2}{d}}{1 + \frac{T^2}{d}} = \frac{dT - T^2}{d + T^2}$ = a max.

whoſe fluxion made = o, and the equation reduced, we have $d^2 - 2dT - 2dT^2 + T^4 = o$; and by ſubſt. $\frac{T^2}{t}$ for its equal t, $\frac{T^4}{t^2} - \frac{2T^2}{t} - \frac{2T^2}{t} + T^4 = o$; whence, by proper reduction, $(T^2 + 1) \times 2t = (t^2 + 1) \times T$. That is, the ſquare of the ſecant of B, multiplied by twice the ſine of C and divided by the coſine of C, is equal to the ſquare of the ſecant of C, multiplied by the ſine of B divided by the coſine of B. But the ſquare of the ſecant is $= \frac{\text{radius}}{\text{ſquare coſine}}$; whence, by putting m and n for the ſine and coſine of B, and s and c for thoſe of C, we have $\frac{1}{n^2} \times$

$\frac{2s}{c} = \frac{1}{c^2} \times \frac{m}{n}$; which reduced, $2sc = mn$; but by Simpſon's Trig. page 56, the ſine of the double of any arch is equal to twice the rectangle of the ſine and coſine of the ſingle arch. Therefore ($2sc$) the ſine of twice C, the extreme arch, is equal to (mn) half the ſine of twice B, the mean, as it ought to have been printed in the queſtion.

XIX. QUESTION 175, *anſwered by Mr.* Lowry, *Birmingham.*

CASE I. When the ſum of the ſquares of the ſides, together with the ſquare of the perpendicular, is given. Fig. 443, Pl. 23.

CONSTRUCTION. Biſect the given baſe AB at D, and draw BC to make the ∠ ABC = to the given one: draw DI ⊥ to BC, and DH ⊥ to AB, alſo divide BD at E ſo that EB : ED :: 2BH : IH, and let the rect. 2BD·DG be taken = to the diff. between the given ſpace and twice the ſquare of AD or BD, and from E, to BC, apply EC = to a mean proportional between EG and EB; join AC, ſo ſhall ACB be the △ required.

DEMONSTRATION. Produce DC to meet a circle deſcribed through the three points B, C, G, at Q, and join QG, QB; draw DK ‖ to EC to meet BC produced at K, and on AB demit the ⊥ CP. Then ſince EC^2 = BE EG, by conſtr. EC is a tangent to the circle at C, theref. Euc. 32. III the ∠s ECB, BGC are =: but DK is ‖ to EC, therefore the ∠s DKB, ECB, are =, and ſo the ∠s DKB, BGC or its = BQD, are =, and, therefore the points D, K, Q, B are in the circumference of the ſame circle; hence

BC·CQ = BC·CK, wherefore

DC·CQ (BC·CK) : BC^2 :: CK : BC: but by ſim. △s and conſtr.

CK : BC :: ED : EB :: IH : 2BH, therefore

DC·CQ : BC^2 :: IH : 2BH

:: IH·BH (= DH^2) : $2BH^2$ :: PC^2 : $2BC^2$, hence

$2DC \cdot CQ = PC^2$. To each of theſe equals add twice the ſquare of DC, then

2DC·DQ or, its =, 2BD·DG is = $PC^2 + 2DC^2$; but

$AC^2 + BC^2 = 2AD^2 + 2DC^2$, therefore

$AC^2 + BC^2 + PC^2 = 2AD^2 + 3BD \cdot DG$: but by conſtr.

2BD DG = to half the diff. between the given ſpace and $2AD^2$; therefore $AC^2 + BC^2 + PC^2$ is = to the given ſpace. *Q. E. D.*

CASE II. When the difference of the ſquares of the ſides together with the ſquare of the perpendicular is given. Fig. 444. Pl. 23.

CON-

CONSTRUCTION. Biſect the given baſe AB at D and draw BC to make the $\angle$ ABC $=$ to the given one; draw DI $\perp$ to BC, and DH $\perp$ to AB, and let the rect. HI·IQ be taken $=$ to the given ſpace, and take DV : DB : : IB : IH, and SD $= 4$VD $-$ 2DB. By Euc. 29. VI. cut BD at P ſo that the rect. SP·PD may be $=$ to the exceſs of the rect. BI·IQ above the ſquare of DB, and draw PC $\perp$ to AB and let it meet BC at C; join AC, ſo ſhall ACB be the $\triangle$ required.

DEMONSTRATION. Since by conſtruction, SD $= 4$VD $-$ 2DB, the rect. SP·PD is $= (4\text{VD} - 2\text{DB})\cdot\text{PD} + \text{PD}^2$, to each of theſe equals add the ſquare of DB, then Euc. 4. II.

$\text{SP}\cdot\text{PD} + \text{DB}^2 = 4\text{VD}\cdot\text{PD} + \text{PB}^2$: but by conſtruction,

$\text{SP}\cdot\text{PD} + \text{DB}^2 = \text{BI}\cdot\text{IQ}$, therefore

$4\text{VD}\cdot\text{PD} + \text{PB}^2 = \text{BI}\cdot\text{IQ}$. Again by ſimilar triangles,

$\text{PC}^2 : \text{PB}^2 : : \text{DH}^2 (\text{BH}\cdot\text{HI}) : \text{DB}^2 (\text{BH}\cdot\text{BI})$

$: : \text{HI} : \text{BI} : : \text{DB} : \text{DV}$, by conſ.; hence

$\text{PC}^2 : 4\text{DB}\,\text{PD} : : \text{PB}^2 : 4\text{DV}\cdot\text{PD}$, therefore by compoſition,

$\text{PC}^2 + 4\text{PDB} : \text{PC}^2 : : \text{PB}^2 + 4\text{PDV} (= \text{BIQ}) : \text{PB}^2$; conſeq.

$\text{PC}^2 + 4\text{DB}\cdot\text{PD} : \text{BI}\cdot\text{IQ} : : \text{PC}^2 : \text{PB}^2 : : \text{HI} : \text{BI}$

$: : \text{HI}\cdot\text{IQ} : \text{BI}\cdot\text{IQ}$, therefore

$\text{PC}^2 + 4\text{PDB} = \text{HIQ}$: but $4\text{PDB} = \text{AC}^2 - \text{BC}^2$, therefore

$\text{AC}^2 - \text{BC}^2 + \text{PC}^2 = \text{HIQ}$, i.e. $=$ the given ſpace, by conſtr.

Q. E. D.

The ſame, anſwered by Mr. J. H. Swale.

ANALYSIS. Fig. 443, Pl. 23. Suppoſe ACB the $\triangle$ required, let D be the middle of the baſe, and join DC, and drop the $\perp$ CP. Then, in the firſt caſe, when $\text{AC}^2 + \text{CB}^2 + \text{CP}^2$ $(= 2\text{DC}^2 + \text{CP}^2 + 2\text{DB}^2 = 3\text{CP}^2 + 2\text{DP}^2 + 2\text{DB}^2)$ and the $\angle$ BAC are given, $3\text{CP}^2 + 2\text{DP}^2$, and the ratio of $\text{AP}^2 : \text{CP}^2$ will be given, let the latter be equal to $m : n$, then $3n \times \text{AP}^2 + 2m \times \text{DP}^2$, or $(2m + 3n) \times \text{DP}^2 + 6n \times \text{AD}\cdot\text{DP}$ will be given, and the conſtruction is evident from 29. VI. of *Playfair's Edition of Euclid's Elements*. And, in the ſecond caſe, when $\text{AC}^2 - \text{CB}^2 + \text{CP}^2 (= \text{AP}^2 - \text{BP}^2 + \text{CP}^2 = \text{CP}^2 + 2\text{AB}\cdot\text{DP})$ and $\text{CP}^2 : \text{AP}^2 (= n : m)$ are given, we ſhall have $n \times \text{AP}^2 + 2m \times \text{AB}\,\text{DP}$, or $n \times \text{DP}^2 + (4m + n) \times \text{AD}\cdot\text{DP} =$ to a given ſpace. Hence the conſtruction is eaſily effected by means of the Prop. above cited.

Mr. Louis Hill *alſo conſtructed this Problem.*

XX. QUESTION 176, *anſwered by Mr.* W. Burdon, *Acaſ Malbis, York.*

ANALYSIS. Fig. 445, Pl. 23. Suppoſe ACB to be △ required, and CF the diameter of its circumſcribing circ join AF and let it be produced to meet the ⊥ CP, produced, D; draw FK ⊥ to AB and join FB, FE. Now, the ∠ AFC be = ∠ ABC, and the ∠ FAC = ∠ FBC, = a right angle, the △s AF FBC, are ſim. Again, the ∠s ACD. FCB, ſtanding upon = arch are equal, and the ∠ DAC = ∠ FBC = a right angle, therefo the ∠ FDC = ∠ CFE, and the △s FCE, DCF are ſimil Therefore AC : CB : : CF : CE : : CD : CF, but CF and the ra of AC : CB are given, therefore CD and CE are given. Her BP (AK) : BK (AP) : : PG : KF (PE) : : KF (PE) : PD, therefo PG·PD = PE² = a given ſpace: but DG is given; therefo PD will be found by Prop. 29, B. VI. *Profeſſor Playfair's Ele of the Elements.*

The ſame, anſwered by Mr. John Lowry.

CONSTRUCTION. Fig. 445, Pl. 23. Take CD to given diameter, as the greater ſide to the leſs, and in CD take C a 3rd proportional to CD and the given diameter; make the r GPD = to the ſquare of the given prolongation of the ⊥, t through P draw APB ⊥ to CD to meet ſemicircles deſcribed CD and CG, as diameters, at A and B; join AC, BC, and A is the △ required.

DEMONSTRATION. About the △ ABC let a circle deſcribed, and draw BG to meet it at F, and join CF, DA, A and EF, E being the point where the circle cuts CD. Then bec the ∠ FBC, in the ſemicircle GBC, is a right angle, CF is diameter of the circle ACB. therefore the ∠ CAF is a r angle, and the ∠ CAD is in the ſemicircle, by conſtr. there the ſtraight line joining the points A and D paſſes through the p F. Again, FE is evidently parallel to AB, therefore the arcs EB are equal, and therefore the ∠s ACF, BCE are alſo eq therefore the right angled △s ACF, BCG are equiangular, ſo are alſo the △s FCD, FCG: hence CG : CF : : CF : C but by conſtr. CG : diam. : : diam. : CD, conſequently CD = the given diam. Moreover AC : CB : : CF : CG : : CD: that is in the given ratio by conſtr. Again, by the circle, A PC : : PE : PB, or AP² = CP·PD : CP² : : PE² : PB² = CP·D therefore PE² = PD·DG = the ſquare of the given prolonga by conſtr.; therefore PE is = to the given prolongation of perpendicular.

This queſtion was likewiſe anſwered by Meſſrs. Cunliffe, Hill, Johnſon, Squire, *and* Swale.

XXI. QUESTION 177, *anſwered by Mr.* Lowry, *the propoſer.*

Moſt of our ingenious correſpondents, who attempted to anſwer this queſtion, founded their ſolutions on a principle, which, to us, ſeems not true, viz. that, "*The baſe of the greateſt triangle, which can be inſcribed in a given circle, divides the diameter, drawn from its verter, into ſegments which have the given ratio of* 3 *to* 1. This is undoubtedly true, in ſome particular caſes, but not generally ſo, nor is it ſo, we preſume, in the inſtance before us. The problem does not appear to us to admit of a GEOMETRICAL CONSTRUCTION, and our opinion is corroborated, by the following *fluxionary* ſolution, which we have received from the propoſer.—Suppoſe ACB (fig. 446, pl. 23,) the required $\triangle$, BD the $\perp$, and O the centre of the circumſcribing circle; draw the diam. EG $\perp$ to the baſe, and draw BS $\perp$ to EG, draw alſo the radii OB, OC. Put $4a =$ the given diff. of the ſquares of the ſides, $r =$ OC the rad. of the given circle, and $x =$ half the baſe $=$ FC. Then by a well known theorem $AB^2 - BC^2 = AC \cdot 2FC$, hence $FD = BS = a \div x$, and by Euc. 47. I, $\sqrt{r^2 - x^2} =$ OF and $\sqrt{r^2 - a^2 \div x^2} =$ OS, therefore $BD = FS = \sqrt{r^2 - x^2} + \sqrt{(x^2r^2 - a^2) \div x^2}$, and the area of the $\triangle$ ABC is $= x\sqrt{r^2 - x^2} + \sqrt{x^2r^2 - a^2}$, which, by the queſtion muſt be a *maximum*. This expreſſion, put into fluxions, gives $(r^2\dot{x} - 2x\dot{x}) \cdot (r^2 - x^2)^{-\frac{1}{2}} + r^2x\dot{x}(x^2r^2 - a^2)^{-\frac{1}{2}} = 0$*, and, by reduction, we have $r^4x^2(r^2 - x^2) = (2x^2 - r^2)^2 \cdot (x^2r^2 - a^2)$, which being further reduced produces $x^6 - \overline{(3r^4 + 4a^2) \div 4r^2} \times x^4 + (r^4 \div 4 + a^2) \times x^2 = r^2a \div 4$, whence the baſe AC becomes known.

COROLLARY. It is evident, from what is done above*, that BD : OS :: 2FC^2 : OC^2, when the triangle is a maximum.

XXII. QUESTION 178, *anſwered by Mr.* John Andrew, *Cark.*

ANALYSIS. Fig. 447, Pl. 24. Let ACB be the $\triangle$ required, AB the given baſe and D, P, the two given points therein; let ACB be the circumſcribing circle and O its centre. Let fall the $\perp$ OG. draw the diam COF, and join OB. Then AG $=$ GB (Euc. 3. III.), and it is evident that DG, GP are given by the queſtion, therefore, by ſim. $\triangle$s DG : GP :: DO : OC $=$ OB: Hence this

CONSTRUCTION. With the two given ſegments DG, GP, and DO, OB in the ſame ratio of DG to GP, make the △ DOB by *Prop. XXII. Appendix to Simpſon's Algebra*, and produce DO till OC = OB, alſo produce BD till AG = GB; join AC, BC, and ACB will be the △ required.

The ſame, anſwered by Mr. Burdon.

ANALYSIS. Fig. 447, Pl. 24. Suppoſe the thing done, ACB the required △, CF the diam. of the circumſcribing circle, and D, P, the given points. Biſect the baſe AB with the ⊥ GO, and draw Of ∥ to AB. Then by ſim. △s,
OC:DC::Of:DP, or FC = 2OC:DC::2Of:DP, therefore dividendo,
DF (= FC — DC) : DC :: PG — DG (2Of — DP) : DP: but the points D, G, P, are given, therefore the ratio DF : FC is given. Hence this

CONSTRUCTION. Having biſected the given baſe AB in G, take DL : DP :: GP — DG : DP, apply DM ſuch that LD · DM = AD · DB, and upon DM deſcribe a ſemicircle cutting the ⊥ PC in C; join AC, CB and ABC will be the △ required.

DEMON. By ſim. △s DL : DP :: DF : DC, in the given ratio; and DL : DF :: DC : DM, therefore DF·DC = DL × DM = AD·DB, by conſtruction. *Q. E. D.*

The ſame, anſwered by Mr. James Cunliffe.

ANALYSIS. Fig. 447. Pl. 24.) Let ACB, the required △, be circumſcribed by a circle, the diam. N*n* being ⊥ to the baſe AB, which it therefore biſects in G (Euc. 3. III.); make the chord CI ∥ to N*n*, interſecting AB in P, *and draw the diam.* CF *to cut* AB in D: alſo make FL ∥ to N*n*, and join FI; draw CM ⊥ to CF, meeting AB produced in M. The ∠ CIF is a right angle (Euc. 31. III.), whence FI is ∥ to AB and is biſected by N*n* in *g* (Euc. III. 3.) therefore *g*I = GP = GL.—Now the △s DLF, DCR, are manifeſtly ſimilar, therefore DL : DF : : DC : DM, whence DL × DM = DF × DC (Euc. 16 VI.), alſo AD × DB = DF × DC (Euc. 35. III.); therefore DL × DM = AD × DB: but AD, BD and DL are all given by the queſtion, therefore DM becomes known, and the following Conſtruction is ſuggeſted.

Biſect the baſe AB in G, and therein take GL = GP, on DB take DM a 4th proportional to DL, DA, and DB; then upon DM as a diam. deſcribe a ſemicircle, cutting a ⊥ to AB, drawn through P, in C; join AC, BC, and ACB is the △ required.

The

The ſame, anſwered by Mr. Louis Hill.

CONSTRUCTION. Fig. 448, Pl. 24. Let AB be the given baſe, and D, P, the given points therein. On DP deſcribe a ſemicircle, and from A and B draw tangents thereto, as AQ, BO. On AB form the △ ACB, ſo as to have its vertex in the line PC, drawn ⊥ to AB, and its ſides AC, CB in the given ratio of AQ to BO, then is the △ ACB the required one.

DEMONSTRATION. Through the points D, P, C, deſcribe a circle to interſect AC at G and BC at F, join GF, and produce CP, CD, to meet the circumſcribing circle at I and E; then by conſtr.

AC : CB :: AQ : BO, or

$AC^2 : CB^2 :: AQ^2 (= DAP) : BO^2 (= PBD)$: but

DAP = GAC, and PBD = FBC; hence

$AC^2 : CB^2 ::$ GAC : FBC; theref.

AC : CB :: AG : FB, theref GF is ‖ to AB, and therefore the arcs GD, PF are equal, and, ſo are alſo the arcs AE, BI; therefore EI is ‖ to AB, and the ∠ EIC a right ∠; conſequently CE is the diam. of the circle circumſcribed about the △ ABC, and paſſes through the given point G, as required.

The ſame, anſwered by Mr. J. Johnſon.

ANALYSIS. Fig. 447, Pl. 24. Suppoſe it done, and that ACB is the △ required, D and P the given points in the baſe AB, which is alſo given. Make FL ⊥ to AB, then AL = BP, therefore the point L is given: but LD and DP are given lines, hence by ſim. △s LD : DF : : DP : DC, therefore the ratio of FD : DC is given : but the rect FD·DC = AD·DB is a given rectangle; therefore there are given the ratio and the rect. of two lines FD, DC to determine them. The conſtruction is eaſy from the Analyſis.

The ſame, anſwered by Mr. Lowry.

CONSTRUCTION. Fig. 447, Pl. 24. Let AB be the given baſe, and D and P the given points therein. On AB deſcribe a ſemicircle, and let the ⊥s PK, DH, let fall from P and D, meet it at K and H; draw KE ‖ to AB, and from D, to KE, apply DE a third proportional to PK and DH, produce ED to meet KP continued in C; the vertex of the required △ ACB.

DEMONSTRATION. Through the points A, C, B let a circle be deſcribed, interſecting CK at I and CE at F, then we have only

only to prove that EDC is the diam. of the circle ACBF; for the $\perp$ CP passes through the given point by constr. Join FI, then by sim. $\triangle$s,

DC : CP :: DE : PK :: DE · PK : PK², but by construction

DE · PK = DH² (= AD · DB = DC · DF), and

PK² = AP · PB = CP · PI; hence

DC : CP :: DC · DF : CP · PI :: DF : PI,

consequently FI is $\parallel$ to AB, and the $\angle$ FIC is a right angle; hence FC is the diam of the circumscribing circle and passes through the given point G.

The same again, by Mr. J. H. Swale.

Fig. 447, Pl. 24. Imagine ACB the $\triangle$ to be found, and that the diam. of its circumscribing circle, drawn from the vertex, meets the base at D. Then it is very evident that the ratio of DO to OC, = the ratio of DG to GP, is given; consequently the ratio OC − OD (= DF) to OC + OD (= DC) is given: but the rect. FD · DC = AD · DB is likewise given. Hence, we have given the ratio and the rect. of the two lines FD, DC to determine them, the method of doing which is well known. These lines then being determined, draw PC $\perp$ to AB and to it apply DC of the given length; join AC, CB, and it is done.

Mr. Nicholson *sent a neat solution to this question.*

XXIII. QUESTION 179, *answered by Mr.* Lowry.

CONSTRUCTION. Fig. 449, Pl. 24. On the indefinite straight line IC, take AE such, that its square may be equal to the rectangle of the given differences, and let the solid AQ · AE² be taken equal to the given difference of the cubes of the sides. Draw KE $\perp$ to AE and equal to a mean proportional between AE and a third part of EQ; bisect AE at L, and, with the centre L, and distance LK, let a circle be described to meet IC at I and C. With the lines AC, AI and the given difference (CH) of the segments, made by the $\perp$, form the $\triangle$ ACH, and draw AB $\parallel$ to HC, and to it apply CB = AH or AI: so shall ACB be the $\triangle$ required.

DEMONSTRATION. Through the points A, B, C describe the circumference of a circle, draw the diam. FG $\perp$ to the base AB and join FC. Then AE is = to the diff. of the sides AC, CB, and 2GP = HC = the given diff. of the segments made by the $\perp$. But, by a well known property, the square of the diff. of the sides is = to four times the rect. GP · GD, that is, AE² = 4GP · GD = the rect of the given differences, by constr.; therefore 2GD,

that

that is, the diff. of the segments of the base [illegible] the line biſecting the vertical angle, is equal to the given diff. [illegible] by Prop. E. Art. XXII. Vol. II. of the Synopſis [illegible] of the cubes on the ſides AC, CB, [illegible] cube on AE together with three [illegible] AC = IE, therefore AC·EC = [illegible] thrice the ſolid AE·AC·EC is = [illegible] conſtr.) AE³·EQ. Conſeq. the diff. of the cubes [illegible] = the cube on AE together with [illegible] the ſolid AE²·AQ = the given ſolid of the [illegible]

The ſame, anſwered by Mr. James [illegible]

ANALYSIS. Fig. 449, Pl. 24. Suppoſe ABC [illegible] △, G the middle point of the baſe AB, [illegible] and biſect the ∠ ACB by the line CF meeting AB [illegible] GP are each given by the queſtion, and [illegible] ½ (AC² — BC²) = GD·GP, from which AC — BC [illegible] known. But AC² — BC² = (AC — BC)·(AC² — AC·BC + BC²) is alſo given by the queſtion, and AC — BC [illegible] found above, therefore AC² + AC·BC + BC² becomes known. Again take (AC — BC)² from the preceding, and there will remain 3·AC·BC, which alſo becomes known, and therefore AC·BC becomes known. Now the diff. and rect. of the ſides being found, the ſides themſelves may eaſily be found, and the problem eaſily readily conſtructed.

The ſame, anſwered by Mr. Richard Nicholſon, Private Teacher of the Mathematics, at Liverpool.

ANALYSIS. Fig. 449, Pl. 24. Suppoſe the thing done ABC the required △, and G the middle of the baſe, and join GC. Then GP = half the given diff. of the ſegments of the baſe, made by the ⊥ CP, and GD = half the given diff. of the ſegments of the baſe, made by the line CG, biſecting the vertical angle. Now,

AC³ — BC³ = (AC² + AC·BC + BC²) × (AC — BC), but AC — BC is given, its ſquare being = 4GDP, by P. IV. p. 205, V. I; therefore AC² + ACB + BC² is given: but, Sim. Geo. II. 11. And Eu. I. 47,

AC² + BC² = 2AF² + 2DC² + 2FD²; and by Sim. Geo. III. 26, and Eu. I. 5.

AC·BC = AF² + DC² + ED² — EF², therefore

AC² + AC·BC + BC² = 3AF² + 3DC² — EF² + 2FD² + ED²; hence

AF² + CD² is given, conſeq. AC·BC is given, therefore

AC² + 2AC·BC + BC² is given, conſeq. AC + BC is give

the ſides [illegible]n : but $AC^2 - BC^2 = AB \times 2GP$ is given; therefore [illegible] given and the conſtruction is obvious.

Solutio[illegible]e alſo received from Meſſrs. Andrews, Burdon, Hill, *and* S[illegible]t.

XXIV. [illegible]ESTION 180, *anſwered by Mr.* James Cunliffe.

GEOMETRICAL ANALYSIS. Fig. 450, Pl. 24. Let CSD and EOF, be the two given circles, O and S their centres, and P the given point in the line OS, and ſuppoſe PF the line required to be drawn, forming the chords CD and EF. Deſcribe a circle concentric to CSD and equal to EOF, in OP produced take S*p* = OP, and thro[illegible] F cutting the laſt deſcribed circle in *e* and F[illegible] *m'* be drawn ⊥ to *p*F, the latter cut[illegible]ing PD[illegible] it *e*F' = EF, and, by Euc. 3. III, the chord[illegible]cted by the ⊥ S*m'*; in the points *m'* and *n*. D[illegible] which produce till it cuts *p*F in T; then b[illegible]ls PD, *p*F, SP : P*p* :: SC : CT, and becauſ[illegible]all given, CT from thence becomes known.[illegible]F = C D + *e*F is a given length, by d[illegible]r halves C*n* + *m*F = R*m* + *m*F = [illegible] given length. Now TRC being a right angl[illegible]int R will be the periphery of a ſemicircle whoſe [illegible] given line CT, as appears from Euc. 31. III, from which concluſions the following CONSTRUCTION is derived.

In the circle CSD draw any radius SC, which produce till CT is a 4th proportional to SP, P*p* = OS, and SC; upon CT, as a diameter, deſcribe a ſemicircle:—then, by *Prop. 7th of Burrow's reſtitution of Apollonius on Inclinations*, draw the line TRF', cutting the ſemicircle in R, and meeting the circle *e*SF' in F, ſo that RF may be equal to half the given ſum of the chords, and draw S*m'* ⊥ to TF. Again, upon PO as a diameter deſcribe another ſemicircle, in which apply the chord O*m* = S*m'*, and through *m* draw the line P*m*F, cutting the given circles in the points C, D, and E, F, and the thing is done, as appears from the Analyſis.

If the diff. of the chords is given, the conſtruction will be the ſame, only the line TR muſt be ſo drawn, by the prob. before referred to, that *e*R may be equal to half the ſaid given difference.

The ſame, anſwered by Mr. Lowry.

GEOMETRICAL ANALYSIS, Fig. 451, Pl. 24. Let FQE be the given circles and P the given point, in the l[illegible]ing their centres O and Q, and ſuppoſe the line CPE to

s required, viz. ſo that the ſum or diff. of the chords
may be equal to a given line On CE demit the ⊥s
n, and deſcribe the circle ILQ, concentric to EQF and
COB; join OB, OC, and QE, and draw QK ‖ to OB
PE at K, and the circle at I; draw IL ‖ to PE meeting
, and on PE demit the ⊥s IN, Lm. Then, it is evident,
is equal and parallel to OB) that the chords IL and ⌣B
and that FE and IL are biſected, by the ⊥ Qm, at m
herefore when the ſum or difference of the chords FE,
en, the ſum or diff. of Em, IH (Nm) is alſo given, that
Es is given. Now, by reaſon of the parallels, OP :
Q : QK, therefore QK is a given line, and QE is alſo
erefore the diff. of their ſquares, or the diff. of the
Es, mK is given. Let a ſemicircle be deſcribed upon
therein apply mG = to mK, and let ESs be drawn ‖ to
hich is the ſame ⊥ to GE) to meet the line joining the
I, Gm, at S and s. Then, by Euc. 47. I, the ſquare of
to the diff. of the ſquares of Em, mG (mK), that is,
a given ſpace, therefore GE is a given line. Again by
Q (OB) : QK :: mN (mn) : Km (Gm) :: NE : ES : :
therefore ES or Es becomes known, according as the
liff. of the chords is given: Hence we derive the

TRUCTION. Take the line GE ſuch that its ſquare
: to the diff. of the ſquares of QE and QK, (QK being
h proportional to OP, PQ, and OB). Draw SEs and
⊥ to GE, and take ES or Es a 4th proportional to OP,
half the given ſum or diff. reſpectively, and draw GS, or
pply thereto EN, or Es, equal to the ſaid ſum or diff.
ly, as the caſe may be, then, the interſections of theſe
the ⊥ Gm, will determine the chords Em, mN (or mn),
eing done, the method of drawing CPE is evident.
ad of the ſum or difference, the rectangle of the chords
given, the problem might have been conſtructed with
e ſame facility; for, by reaſoning as above, it would ap-
the rectangle and diff. of the ſquares of Em, mK would
and therefore the chords might be determined by the 87th
Euclid's Data.
e ſum or diff. of the ſquares had been given, then the
the ſum or diff. of the ſquares of mK, mN, would ap-
given, and the chords might be eaſily determined by the

ien the ſum or difference of the ſquares of the chords is
e problem may be done, in a more general manner, for
, through which the required line is to paſs, may be given

out

out of the line joining the centres, as will be evident from the following

ANALYSIS, Fig. 452. Pl. 24. Let OB, QF, OP, QP, and OQ, be drawn, and on OQ as a diameter, deſcribe a circle to interſect OG, drawn ‖ to CE, at G, and join GQ; on OG demit the ⊥ PK, and on CE the ⊥ OD; produce KP till KL = KG, and draw GL to meet the circle at I; join OL, IP, and PG. Then, ſince the ſum or diff. of the ſquares of FE, CB, is given, the ſum or diff. of the ſquares of their halves, FH, BD, is alſo given. Let us, in the firſt place, ſuppoſe that the ſum of their ſquares is given; then, by a well known property of plane △s, the ſum of the ſquares of OP, BD, is equal to the ſum of the ſquares of OB, PB, and the ſum of the ſquares of PQ, FH, is equal to the ſum of the ſquares of FQ, PH; therefore it is evident, that when the ſum of the ſquares of FH, BD, is given, the ſum of the ſquares of PH, PD, is given: but PH is = KG = KL, and PD = KO, hence, the ſum of the ſquares of PH, PD, is = to the ſquare of OL, hence OL is a given line. Moreover, ſince KG = KL, and the angle at K a right angle, the arc OI will be a quadrant of the circle OIQG, and I will be a given point; and ſince the angles KGL, GLK are equal, each being half a right angle, the angle PLI will be three fourths of two right angles, and conſequently the locus of the point L will be a circle deſcribed through the points P, I, to contain the given angle PLI, and ſince O is a given point, and OL a given line, the point L will be determined, and the Conſtruction manifeſt from the Analyſis.

Again, when the diff. of the ſquares is given; it will appear, as before, that the diff. of the ſquares of OD (HG) and HQ, will be given: but the diff. of the ſquares of HG, HQ is = to the diff. of the ſquares of PG, PQ, and PQ is a given line, therefore PG is given, and the method of Conſtruction evident.

Meſſrs. Hill, Nicholſon, *and* Swale, *ſent ſolutions to this queſtion.*

XXV. QUESTION 181, *anſwered by Mr.* Lowry.

CONSTRUCTION. Fig. 453, Pl. 24. Join PB, and draw PF to make the ∠ BPF = to the given ∠, and meet AC at F; make the rectangle PB·PI = to the given one, and draw FG to make the ∠ PFG = to the ∠ PBD, and, parallel thereto, draw PQ meeting AC in Q. Then, by Prob V. B. I. of the determinate ſection, by Wales, cut CQ at E ſo that CE·EF : PF·EQ :: PI : PQ; join EP, and draw PD to make the ∠ DPE = to the given ∠ BPF, and the thing is done.

DEMONSTRATION. By conſtr. the ∠ DPE is = to the ∠ BPF, that is, equal to the given angle. Alſo by conſtr.

CE·EF

CE · EF : PF · EQ :: PI : PQ, and by ſim. △s
EF : FQ :: FG : PQ, therefore
CE. EF : PF . EQ :: FG · CE : PQ · PF :: PI : PQ
:: PI · PF : PQ · PF, hence,
the rectangles FG · CE, PI · PF are equal, conſequently
CE : PI :: PF : FG, and by ſim. △s, BD : BP :: FG : PF.
Wherefore it is evident, that the rectangle BD · CE is equal to the given rectangle PB · PI. *Q. E. D.*

The ſame, anſwered by Mr. Richard Nicholſon, *Liverpool.*

GEOMETRICAL ANALYSIS. Fig. 434. Pl. 24. Join the points P, C, and P, B; draw BF ∥ to CP, and PG to AC, the former to meet EP in F, and the latter BF in G. Then (Cor. 4. VI.) EC : CP :: GP : GF, therefore EC · GF = CP · GP a given rectangle, and EC · BD is given, by hyp.; therefore (Cor. 65.) the ratio of GF to BD is given. Make the ∠ GPI = FPD the ſup. of the given ∠ EPD, and PH = FP, let H be in AB, and I in PD; then (Cor. 4. VI.) PG : PH :: GF : HI, but PG and PH are given, therefore the ratio PG : PH is given, and, it has been ſhewn, that the ratio of GF to BD is given; therefore (Cor. 65.) the ratio of BD to HI is given. Draw BK and HL ∥ to PI, BM and DN ⊥ to AB, and DN meeting HL in O, and let QO meet BF in S. Then (Simp. Geo. 21. IV.) RM : OS :: BK : OQ = HL, therefore OS · RK = RM · BH. And, by reaſon of parallel lines, HI = DO = BS : BD = OS :: BM : MR, but the ratio of HI to BD has been ſhewn to be given, therefore the ratio of BM to MR is given, and BM is a given line, therefore MR is given, and BH being given, the rectangle MR · BH will be given; therefore, its equal, the rectangle OS · RK is given, but the rectangle KP · KR is given (becauſe OS = BD = KP). Hence the rectangle KP · KR, and PR being given, the point K may be found by Euc. 27, 28. VI.; conſequently the poſition of PD becomes known.

CONSTRUCTION. Draw PC, PB, draw alſo BG, CG, parallel to CP and AC reſpectively, and take any two lines *m* and *n* in the given ratio of the rect. CP · PG to EC · BD; make the ∠ GPH = the ſup. of EPD, and PH = the ſup. of BGC; take the line *p* a 4th proportional to PG, *m* and *n*, draw HI, ∥ to AB and BM ∥ to HI; and take MR a 4th proportional to *p*, *n*, and BM. Divide PR in K ſo that the rect. KR · KP may be = MR · BH, join KB and parallel thereto draw PD, and draw PE to make an ∠ with PD equal to the given ∠, and the thing is done.

Meſſrs. *Cunliffe* and *Swale* obſerve, from Mr. *Wallace's* Analyſis to Queſt. 152, that this prob. is reducible to what follows.

Fig. 351. Pl. XXI. The lines GK and CE are given by poſition, and C and G are given points therein; it is required to draw a line FKE through another given point P, to cut the former lines in K and E, ſo that the rectangle GK · CE may be equal to a given ſpace.

This prob. may be ſeen done in all its various caſes in a ſmall work of Dr. HALLEY's, printed at *Oxford* in the year 1706, entitled, *Apollonii Pergæi de ſectione ſpatii libri duo reſtituti.*

XXVI. *Or*, PRIZE QUESTION 182, *anſwered by* Arithmeticus, *of Newcaſtle upon Tyne.*

Let ABCD (fig. 455. pl. 24) repreſent a ſection of the veſſel, E the place of the eye, E*g* an incident ray refracted at *g* into *gt*, and let E*g* be continued to *b*. Suppoſe *c*s*V*d to repreſent the bottom of the veſſel as it appears to the eye at E; then the point *t* will be ſeen at *s*, F at V, &c.—Draw s*R* ‖ to C D and put E*G* $= a$, G*F* $= b$, E*R* $= x$ and s*R* $= y$; then will E*s* $= \sqrt{(x^2 + y^2)}$, and the ſine of the ∠ of incidence *s*E*R* $= y \div \sqrt{(x^2+y^2)}$: but by ſim. $\triangle$s $x : y :: a : ay \div x = Gg =$ F*H*, and $y - ay \div x =$ H*t*, whence the ſine of the ∠ of refraction *tg*H $= (xy - ay) \div \sqrt{(b^2x^2 + \overline{xy - ay}^2)}$, which is known to be in the conſtant ratio of 3 to 4 to the ſine of the ∠ of incidence, theref. $3y \div \sqrt{(x^2+y^2)} = (4xy - 4ay) \div \sqrt{(b^2x^2 + \overline{xy - ay}^2)}$, which reduced gives $y^2 = 9b^2x^2 \div 7(x-a)^2 - 16x^2 \div 7$. When $y = 0$, then $x = a + \frac{3}{4}b$, or F*V* $= \frac{1}{4}$F*G*, and when $y =$ C*F* or 1, then $x = 1.1501574$ nearly.

The fluxion of the capacity A*Bcd* (putting $p = 3.14159$, &c.) is $(16x^2\dot{x} - 9b^2x^2 \cdot \overline{x-a}^{-\frac{1}{2}}) \times \frac{1}{7}p$, the correct fluent of which is $\frac{16p}{21}\left(x^2 - \overline{\frac{4a+3b}{4}}^2\right) + \frac{9pb^2}{7}\left(\frac{9b^2 - 16a^2}{12b} - \frac{x^2-2ax}{x-a}\right) + \frac{18ab^2p}{7} \times$ hyp. log. of $\frac{3b}{4x-4a}$, which, when $a = \frac{1}{2}$, $b = 1$, and

$x =$

$= 1{\cdot}1501574$, is $= {\cdot}0490014\ p$, and therefore the capacity of the veſſel ABcd, as it appears from E, is ·6991588 p, which taken from the whole capacity leaves ·3008412 p = ·9451197 cubic inches, the difference required.

Mr. *Gregory*, *of Cambridge*, gave a ſolution, nearly ſimilar to the above, by Arithmeticus, (making the required difference 6·24 inches, or 5·8022 ale gallons,) at the end of which he added the following obſervations:

1. From the inveſtigation of the nature of the curve given above, it will appear, that it bears a near affinity to the *ſuperior conchoid* of *Nicomedes*, the ſurface of the water being an aſymptote to the apparent bottom, and correſponding to the *directrix* of the conchoid.

2. As the eye approaches the ſurface of the water, the extremities of the apparent bottom will have a greater curvature; approaching more and more to the nature of an ellipſis as the eye approaches the water.

3. In the above ſolution I have ſuppoſed the ſines of incidence and refraction to be as 4 to 3, for the ſake of ſimplifying the calculation; this, though not quite accurate, is ſo near the truth, as to produce any material error. I have alſo ſuppoſed that the point C, in the circumference of the baſe, is refracted ſo as to appear at c exactly above it: this (though nearly accurate in the preſent inſtance) will not be the caſe when AG and GF are very great in proportion to EG. A rule for finding the poſition of the refracted point c, in moſt caſes, may be found in Dr. BARROW's *Optics*. Lect. 16.

The ſame, anſwered by Mr. James Cunliffe, *Bolton.*

Fig. 455, Pl. 24. Let ABCD repreſent a ſection of the cylinder, by a plane paſſing through the axis FG, E the place of the eye therein: alſo let the curve cVd repreſent the image of the line CD, as it appears in the water to the eye at E. Draw $tn \perp$ to CD, interſecting the curve in s, draw alſo Es meeting AB in g and nt in t; then will s be the apparent place of the point t, the $\angle$ $Esn = Esn$ will be the $\angle$ of incidence, and the $\angle$ ntg that of refraction of the ray Es: *ſee Emerſon's Optics*, *Cor.* 6, *Prop.* 9, *III and Prop.* 29 *of the ſame Book*.—Now in order to find the nature of the curve, which is the *locus* of all the points s, upon nt demit the $\perp$ sR: put $EG = b$, $GV = d$, $FV = c$, $VR = x$, and $Rs = y$, then $ER = b + d - x$, and by Euc. 47,

$Es = \sqrt{ER^2 + Rs^2} = \sqrt{(b + d - x)^2 + y^2}$. By reaſon of parallels G_o, Rs, $ER : Es :: GR : gs = \overline{(d - x} \div \overline{b + d - x)} \sqrt{}$

$\sqrt{b+d-x)^2+y^2}$, and becauſe of the ſim. $\triangle$s ERs and *sng*, ER : Rs :: *ns* = GR : *ng* = $y(d-x) \div (b+d-x)$, whence, and by Euc. 47. I. *tg*

$= \sqrt{(tn)^2+(ng)^2} = \sqrt{(d+e)^2+y^2(d-x)^2 \div (b+d-x)^2}$

$= \sqrt{(d+e)^2(b+d-x)^2+y^2(d-x)^2} \div (b+d-x)$.

Moreover, by the principles of trigonometry and optics, *tg* and *gs* are in the conſtant ratio of the ſine of the angle of incidence to that of refraction, that is, as $d+e$ to d, or

$\sqrt{(d+e)^2(b+d-x)^2+y^2(d-x)^2} \div (b+d-x) : (d-x)$

$\sqrt{(b+d-x)^2+y^2} \div (b+d-x) :: d+e : d$; by equating the product of the means and extremes, and proper reduction, we have $y^2 = (d+e)^2(2dx-x^2)(b+d-x)^2 \div (2de+e^2)(d-x)^2$, being an equation expreſſing the nature of the curve cV*d*, when the abſciſſa and its correſponding perpendicular ordinate begin together. Now $y^2px = (d+e)^2(2dx-x^2)(b+d-x)^2(2de+e^2)^{-1}(d-x)^{-2} \times px$ will expreſs the fluxion of the ſolid generated by the revolution of the ſpace VsR about the axis FE. Put $v = d-x$, and the preceding expreſſion becomes $(d+e)^2(d^2-v^2)(b+v)^2(2de+e^2)^{-1} \times pv^{-2} \times -\dot{v}$. But $(a^2-v^2)(b+v)^2 \times v^{-2} \times -\dot{v}$ is $= -a^2b^2v^{-2}\dot{v} - 2ba^2v^{-1}\dot{v} - a^2\dot{v} + (b+v)^2\dot{v}$, the fluents of which are $a^2b^2v^{-1} + \frac{1}{3}(b+v)^3 - a^2v - 2ba^2 \times$ hy. log. of v; therefore, the expreſſion for the content of the ſolid generated by the revolution of the ſpace VsR about the axis FE will be $p(d+e)^2(2de+e^2)^{-1} \times (a^2b^2v^{-1} + \frac{1}{3}(b+v)^3 - a^2v - 2ba^2 \times$ h. l. of $v) = p(d+e)^2(2de+e^2)^{-1} \times (a^2b^2(d-x)^{-1} + \frac{1}{3}(b+d-x)^3 - a^2(d-x) - 2ba^2 \times$ h. l. of $(d-x)$, by reſtoring the value of v, (p being $= 3{\cdot}14159265$.). But the whole expreſſion ought to vaniſh when $x = 0$, therefore the correct expreſſion for the content of the ſaid ſolid will be $p(d+e)(2de+e^2)^{-1} \times (2bd^2 \times$ h. l. of $d(d-x)^{-1} + b^2e^2(d-x)^{-1} + \frac{1}{3}(b+d-x)^3 - b^2a - \frac{1}{3}(b+e)^3)$. The preceding part of the ſolution is general, and will anſwer *mutatis mutandis* for any other medium beſides water.

When

When s coincides with c, then $y = GA = GF = f$ — the queſtion, therefore $y^2 = f^2 + x^2 =$ — $(b + d - x)^2 (2dc + c^2 -$ — $(c - x)^2 = (2dx - x^2 (b + d - x)^2$ taking the ratio of the ſine of the angle of incidence to that of refraction, in water, as 4 to 3, we have $a =$ — $=$ — $= \frac{4}{3}$, wherefore, the preceding equation becomes $\frac{16}{9}$ — $= \frac{16}{9}$ $— x^2) (15 — x)^2$, or $x^4 — 2^9 x +$ — $=$ — from whence $x = 1{\cdot}1981314$ B and the foregoing general expreſſion for the of the part generated by the the axis FE will be found to be the content of the cylindrical 1899·1899 cubic inches, whereas their cubic inches is the content of the tween the apparent and real bottoms of the veſſel, before, the quantity of water required.

OBSERVATIONS. 1. The line AB to the continuation of the curve cV branches or legs. For, in the equation y becomes infinite or an aſymptote. to have inadvertently been of his Optics, for he there would meet the ſurface of the the line CD is to the ſurface.

2. From the equation of the curve $y =$

$$\times \sqrt{2dx - x^2} = \frac{d + e}{\sqrt{2d + e}} \times \frac{d + e}{e} \times \sqrt{2dx - x^2}.$$

Now, if a ſemi-ellipſis be deſcribed, having GV for its ſemi-conjugate, and $\frac{d + e}{\sqrt{2d + e}}$ for its ſemi-tranſverſe axis, and an ordinate to GV, and the point where the centre G be joined; then if a parallel to the latter line, it will meet the ſaid ordinate in a point which will be in the curve cV*d*.

3. By comparing the equation of the curve, $y = \frac{d+e}{\sqrt{2de+e^2}} \times \frac{b+v}{v} \times \sqrt{d^2 - v^2}$, with that in *Cor. to Prop. IV. Emerſon on the conchoid*, it will appear that the curve is really a ſpecies of the conchoid. Now, if inſtead of the quadrant of a circle, as in the *III. Prop. of Emerſon*, we imagine the quadrant of the ellipſis, mentioned in the laſt obſervation, to be deſcribed, the property of the conchoid, ſtated in that propoſition, will equally appertain to the curve cVd.

4. The curve may thus be deſcribed by continued motion. If an ellipſis whoſe principal ſemi-axis is $\frac{(d+e)d}{\sqrt{2de+e^2}}$ and its ſemi-conjugate d, be moved along its plane always coinciding with the plane paſſing through the line AB and the point E; and its tranſverſe axis at the ſame time coinciding with the line AB: then if a right line be drawn through the point E and the centre of the ellipſis, the interſection of this line with its periphery, on the ſide of AB, which is fartheſt from E, will generate the curve cVd.

For let *psvq* (fig. 456, pl. 24.) repreſent this ellipſis as its tranſverſe axis moves along the line AB: through the centre g draw the line Egs to meet the periphery of the ellipſis in s, draw alſo sn and sR perpendicular to pq and VE reſpectively, the latter interſecting the ſemi-conjugate axis vg in r. Put the ſemi-tranſverſe $gp = \frac{(d+e)d}{\sqrt{2de+e^2}}$, the ſemi-conjugate $gv = \text{GV} = d$, $\text{EG} = b$, $\text{VR} = vr = x$, and $s\text{R} = y$; then $\text{ER} = b + d - x$, and $\text{GR} = gr = d - x$. By the well known property of the ellipſis, $rs = \frac{(d+e)\sqrt{2dx - x^2}}{\sqrt{2de+e^2}}$, and by reaſon of the ſimilar $\triangle$s srg and sRE, $sr : rg :: s\text{R} : \text{RE}$, whence $s\text{R} = \frac{sr \times \text{RE}}{rg}$, this, being expreſſed algebraically, is $y = \frac{d+e}{\sqrt{2de+e^2}} \times \frac{b+d-x}{d-x} \times \sqrt{2d}$.

$\sqrt{2dc - x^2}$ which is the [illegible]
inſtead of an ellipſis, we [illegible]
ſame manner, the [illegible]
Nicomedes, as is ver[illegible]
the *generating circle*, [illegible]
oned ellipſis may not [illegible]

The ſame, another way.

Fig. 455, Pl. 24. Let [illegible]
ſindrical veſſel, [illegible]
place of the eye, and [illegible]
it appears to the eye at [illegible]
the ſurface of the [illegible]
and draw *nst* ⊥ to CD [illegible]
CD, to meet the axis [illegible]
principles of Optics, [illegible]
ſtraight direction [illegible]
direction *gt*, [illegible]
s, and *gs* will [illegible]
ſame manner, the [illegible]
and EV will [illegible]
25. Now, [illegible]
$VF = c = [illegible]$
Euc. I. 47, $\overline{EK}^2 = [illegible]$

$+ y^2 (b - x)^{-\frac{2}{3}} [illegible] = [illegible]$

$= y^2 (b - x)^{-\frac{2}{3}} [illegible] = [illegible]$

$+ y^2 (b - x)^{-\frac{2}{3}} [illegible]$

$(m - x)^{-\frac{2}{3}} [illegible]$

GV^2 or $b^2 : GF^2$ or $[illegible]$ and [illegible]
tremes, we obtain, after reduction, $y^2 = [illegible]$,

$(m - x)^2 (2dx + c^2)^{-\frac{2}{3}} [illegible] = n^2 [illegible]$

$(m - x)^2 (b - x)^{-\frac{2}{3}}$ an equation expreſſing the nature of the curve cVd, n being $= (b + c)^2 (2dx + c^2)^{-\frac{1}{3}}$.

Now, when s coincides with c, y is $=$ to CF $= 1$, by the queſtion, and, in that caſe, the above equation, by reduction, be-

comes

comes $x^4 - (6m + 2b)x^3 + (4m + m^2 + \frac{1}{2}n)x^2 - (2bm^2 + 2b \div n)x + b^2 \div n = 0$, in which the value of x is ·0998443.

Again, to find the content of the ſolid, generated by the revolution of the ſpace VsR about the axis VR, put $p = 3{\cdot}1416$, then $py^2\dot{x} = pn(b^2 - \overline{b-x}^2)(m-x)^2(b-x)^{-2}\dot{x}$ is the fluxion of the ſaid ſolid. And to find the fluent, it is evident that $(b^2 - \overline{b-x}^2)(m-x)^2(b-x)^{-2}\dot{x}$, may be reſolved into $(b-x)^{-2} \times a^2b^2\dot{x}$, $(b-x)^{-1} \times 2ab^2\dot{x}$, $b^2\dot{x}$, and $-(m-x)^2\dot{x}$, the fluents of which (found by the common rules) are $a^2b^2(b-x)^{-1}$, $-2ab^2 \times$ hy. log. of $(b-x)$, b^2x, and $\frac{1}{3}(m-x)^3$ reſpectively; therefore the fluent of $py^2\dot{x}$, or the expreſſion for the content of the ſolid generated by the revolution of the ſpace VsR about VR is $= pn \times (a^2b^2(b-x)^{-1} - 2ab^2 \times$ hy. log. of $(b-x) + b^2x + \frac{1}{3}(m-x)^3)$. But when x is $= 0$, this expreſſion becomes $pn \times (a^2b^2(b-x)^{-1} - a^2b + 2ab^2 \times$ hy. log. of $(b \div \overline{b-x}) + b^2x + \frac{1}{3}(m-x)^3 - \frac{1}{3}m^3)$, which, when x is $=$ VR′ $=$ ·0998443 (as found above), becomes ·153948 cubic feet, the content of the ſolid, generated by the revolution of the ſpace VcR′ about VR′. And the content of the cylinder CcdD is eaſily found $=$ 1·099068; conſequently their difference, or 94514 cubic feet is the quantity of water required.

When x is $= b$, the value of y in the above equation of the curve is infinite, which ſhews, that AB produced is an aſymptote to the continuation of the curve. This is alſo evident from another conſideration, for if the water be extended on all ſides indefinitely, it is plain that, every part of the curve will fall below the ſurface of the water, when that ſurface is conſidered as a plane parallel to the horizontal plane CD. The curve will therefore have two infinite legs and in form will much reſemble the conchoid of Nicomedes.

The ſame, anſwered by Mr. John Surtees, *Biſhop Wearmouth, Sunderland.*

Let Fig. 455, Pl. 24, repreſent a vertical ſection of the veſſel, E the ſituation of the eye, EG $= \frac{1}{2}$, AG $=$ GF $= 1$ foot, $sR = nG$,

$= nG = y$, GR $= x$, $c = 3{\cdot}1416$, and $m = 2{\cdot}302585\,09$ hyp. log. of 10. By fim. $\triangle$s ER $(\frac{1}{2} + x) : sR\,(y) :: sn\,(x) : ng = 2xy \div (1 + 2x)$, and by Euc. I. 47, $\sqrt{nl^2 + ng^2} = gl = \sqrt{1 + x^2y^2} \div (\frac{1}{4} + x + x^2)$ and $\sqrt{ns^2 + ng^2} = sg = \sqrt{x^2 + x^2y^2} \div (\frac{1}{4} + x + x^2)$, which, by the nature of refraction, are to one another as 4 to 3; hence $y^2 = (1 \div 28x^2) \times (9 \times 36x + 20x^2 - 64x^3 - 64x^4)$, and when $y = 0$, $x = \frac{3}{4}$, and when $y = 1$, $x = {\cdot}65016$.—Now, the fluxion of the folid, generated by the revolution of the area R'sV about R'V, $= cy^2 \times -\dot{x}$, and the fluent $= (c \div 28) \times (9x^{-1} - 36m \times \log. x - 20x + 32x^2 + (64 \div 3)x^3$, and, when duly corrected, the fluent is $= (c \div 28) \times (9x^{-1} - 24 + 36m \times \log.(3 \div 4x) - 20x + 32x^2 + (64 \div 3)\,x^3$, which, when $x = {\cdot}65016$, becomes $= {\cdot}15416$; therefore $(R'c)^2 \times R'F \times c - 15416 = 1 \times {\cdot}34984 \times 3{\cdot}1416 - {\cdot}15416 = {\cdot}945$ folid feet $= 1632{\cdot}96$ cubic inches $=$ the folid generated by the area CcVF about VF. W. W. R.

True folutions were alfo received from Meffrs. Hill, Nicholfon, Thornoby, *the propofer, and* Robert Wallace. *Other gentlemen attempted folutions but without fuccefs.*

The Prize Medal is decided in favour of Mr. James Cunliffe, *who will pleafe to fend for it to Mr.* Glendinning's.

ARTICLE XLIV.

MATHEMATICAL QUESTIONS.

(To be anfwered in Number XII.)

I. QUESTION 211, *by Mr.* John Croudace, *Lanchefter School, Durham.*

ON the firft day of May, 1800, the fum of the fun's declination, altitude at fix, and amplitude was found to be equal to the latitude of the place. Query where?

II.

II. QUESTION 212, *by Mr.* Francis, *Hampſtead.*

There is a certain hill from whoſe ſummit may be ſeen four ſeparate objects, A, B, C, D, diſtant from each other 23, 20, 30, and 18·909 miles reſpectively. Now admitting all thoſe objects to be in the horizon, or utmoſt extremity that the eye can reach, from that elevation, what is the perpendicular height of the hill, conſidering the earth as a perfect ſphere 25000 miles in circumference.

III. QUESTION 213, *by Mr.* John Blackwell, *Hungerford, Berks.*

There are two circular ſegments, of equal altitude, touching each others vertex, viz. the convex vertex of the leſs touching the concave vertex of the greater ſegment, the length of whoſe arcs are 50 and 60 yards reſpectively. Now, it is obſerved, that the area of the leſs ſegment is to the area of its greateſt inſcribed circle, in the ratio of 5 to 2. Whence it is required to find the area included between the arcs of thoſe circles from which the ſegments were cut.

IV. QUESTION 214, *by Mr.* Wm. Marrat, *Boſton.*

Three ſailors, having found an ingot of gold, in form of a cone, the length of which is 4 and the diameter of its baſe 6 inches, are determined to have it divided equally among them, by ſections parallel to one ſide: but finding the operation to be rather difficult, requeſt ſome of the learned contributors to the Repoſitory will favour them with a method of ſolution.

V. QUESTION 215, *by Mr.* Thomas Squire, *Baldock.*

One night I made obſervation on an eclipſe of the moon, but clouds interpoſing I could not ſee the beginning; however at 11*h.* 49*m.* 48*s.* it cleared up, when ſhe was exactly 6 digits eclipſed on her lower limb, at the ſame time I meaſured the *diſtance* between the extremities of the illuminated cuſps, and found it to be 30′ 15″: the air continuing bright, at the time of the greateſt obſcuration, I found the digits eclipſed to be 7d. 27m. 36s—ſoon after clouds appearing put a ſtop to all farther obſervation. By calculation I found the moon's ſemidiameter 15′ 24″, and her horary motion, from the ſun, on the relative orbit, 29′ 45″. From theſe data, the beginning, middle, and end are required.

V.

QUESTION 216, *by Mr.* **W. Burdon,** *Acaſter-Malbis.*

ven the baſe, the diameter of the circumſcribing circle, and lifference between the perpendicular and the radius of the in-ed circle, to conſtruct the plane △.

I. QUESTION 217, *by Mr.* **Newton Boſworth,** *Cambridge.*

here is a lever, equally thick throughout, the length of which 5 feet and weight 40lbs averdupoſe, from one end of which is ended a globe of lead of 6 inches diameter: it is required to rmine where the fulcrum muſt be placed ſo that the whole may ain in equilibrio.

II. QUESTION 218, *by Mr.* **Jonathan Barr,** *Student in Mr. Wallace's Academy, St. John's-Lane, near Smithfield, London.*

Given the ratio of the baſe to the diameter of the circumſcribing cle, the ratio of the difference of the ſegments of the baſe to perpendicular, and the radius of the inſcribed circle, to conſtruct the plane △.

IX. QUESTION 219, *by Mr.* **Thomas Simpſon Evans.**

Required a ſimple and accurate expreſſion to reduce the apparent itude to that at the centre?

X. QUESTION 220, *by Mr.* **Wm. Peacock,** *Birmingham.*

In a given △ ABC it is required to inſcribe another EDF, hoſe angular points E, D, F ſhall fall on the ſides AB, BC, AC reſpectively, ſo that the two ſides FE, DE ſhall paſs through wo given points I, H, and moreover, that the third ſide FD ſhall ut off from the given △ another FCD of a given magnitude, or hat the ſum of CF, CD ſhall be equal to a given line; and ſhew he limits in both caſes.

XI. QUESTION 221, *by Mr.* **John Byerley,** *Stockton-upon-Tees.*

At what point in a plane paſſing through the centres of the ſun, earth, and moon, muſt a particle of matter be placed ſo as to be equally attracted by theſe bodies, their diſtances and magnitudes being given?

XII.

XII. QUESTION 222, *by* Tyro Philo Mathematicus, *of Hull.*

A certain water pool, the area of which is 10 acres, has two ſmall outlets through two ſimilar and equal rectangular apertures, each 3 feet wide parallel to the horizon and 4 feet high perpendicular thereto:—Now admitting the ſurface of the water to be level with the top of the apertures when it begins to iſſue out through them; I would know in what time the ſaid ſurface will be on a level with the bottoms of the ſaid apertures?

XIII. QUESTION 223, *by* E. R.

Suppoſing a ſolid of a given baſe and altitude, to be generated by the rotation of a given equilateral hyperbola, round one of its aſymptotes, as an axis;—It is required to determine the length of a ſpiral, making a given number of turns round the ſame, from the baſe to the top, in ſuch a manner, that if ordinates were drawn from the beginning of each turn to the aſymptote, they would divide the axis into the ſame number of equal parts?

XIV. QUESTION 224, *by Mr.* Olinthus Gregory, *Bookſeller, Cambridge.*

The ſides of a △ are 50, 26 and 24. Now if this △ be placed on fulcrums at the angular points, and a weight of 100 lbs be laid on the centre of the inſcribed circle, what part of this weight will be ſupported by each of the three fulcrums; and how far from the angular points muſt the weight be laid ſo that each fulcrum may ſupport an equal part of it?

XV. QUESTION 225, *by Mr.* James Whalley.

Required the ſum of n terms of the ſeries $\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} + \&c.$?

XVI. QUESTION 226, *by Mr.* Gregory.

There are two wheels (the weights of which are ſuppoſed to be condenſed into their circumferences;) one of them whoſe radius is 2 feet, weighs 50lbs, and from a cord, coiled round an axle of $\frac{1}{4}$ of a foot radius, a weight of 5lbs is ſuſpended: the other wheel,

in

in radius 1½ feet, weighs 40 lbs and has a weight of 5 lbs. suspended in like manner, from an equal axle [illegible] wheels to move freely on their respective axles; it is required to shew, how long they have been in motion when the weight affixed to one axle, has descended as far as that suspended at the other?

XVII. QUESTION 229. *by Mr.* James Cunliffe. *Leigh-Moor, Lancashire.*

Required the least dimensions of a triangular piece of land, such that if perpendiculars be drawn to the line from the other angles, the lengths of the line between them and the extremities of the sides, made thereby, may be expressed in whole chains?

XVIII. QUESTION 228. *by* E. R.

To describe a circle through two given points to cut an equal right line, given by position, and which is limited between the two given points, so that the two parts between the ends of the given points to the points of intersection of the right line and circle, may include a given angle.

XIX. QUESTION 230, *by Mr.* Robert Wallace, *Lawton.*

Suppose a corpuscle of matter to be situated in the axis produced of a given paraboloid. It is required to determine the whole force of attraction of the paraboloid upon the corpuscle, supposing each particle to exert a force as the *n*th power of its distance.

XX. QUESTION 230, *by Mr.* James Cunliffe, *Bolton.*

To find two rational fractions such that their sum, sum of their squares, and sum of their cubes shall all be rational square numbers; and moreover, either of them being added to the square of the other shall make a rational square number.

XXI. QUESTION 231, *by Mr.* Smith, *of Liverpool.*

Given one side, an adjacent angle, and the area to construct the trapezium, when the side, opposite to the given side, produced passes through a given point in the given side produced, and the angle, opposite to the given angle, lies in a straight line given by position.

XXII. QUESTION 232, *by Mr.* Louis Hill.

Required a new, concife and elementary demonftration of the rule, in common ufe, for finding the area of a trapezium inferibed in a circle, when the four fides are given.

XXIII. QUESTION 233, *by Mr.* John Johnfon, *Birmingham.*

In any plane triangle, if a diameter of the inferibed circle be drawn perpendicular to the bafe, to meet the line bifecting the bafe at P, and Q and R be the points of contact of the circle with the other two fides of the $\triangle$; the points Q, P, R lie in a ftraight line. Required the demonftration?

XXIV. QUESTION 234, *by* Geometricus.

In the figure to Prop. 47, Book I. of all the editions of Euclid's Elements which I have feen, the ftraight lines joining the points F, C; B, K, appear to interfect the perpendicular AL in the fame point. It is required to demonftrate whether they really do meet in the fame point or not. See the figure in Dr. Simfon's or Profeffor Playfair's Editions.

XXV. QUESTION 235, *by Mr.* Louis Hill.

Though a given point within or without a given circle, to draw a right line to interfect the circle fo that if perpendiculars be drawn from the points of interfection to that diameter of the circle which paffes through the given point, the fum, difference or rectangle of the faid perpendiculars may be of a given magnitude.

XXVI. QUESTION 236, *by Mr.* Lowry, *Birmingham.*

To determine a point, fuch, that if three ftraight lines be drawn from it to meet three other ftraight lines, given by pofition, in given angles; the fum of the lines fo drawn may be equal to a given line; and moreover, that the rectangle contained by two of them may be equal to the rectangle contained by the third and given line.

XXVII. QUESTION 237, *by the Rev.* L. Evans.

Mr. Worldly-wife-man lends a tradesman, newly fet up in b

fine

ſineſs, 1000l. to bear compound intereſt at 5 per cent. per annum, till the ſame be liquidated, by quarterly payments, thus, 1l. at the end of the 1ſt quarter, 2l. at the end of the 2nd quarter, 3l. at the end of the 3d quarter, &c. It is required to find, by a clear and general inveſtigation, how long the tradeſman will be in acquitting himſelf of this obligation?

Note. A ſimilar queſtion, in effect, has been propoſed before, but the ſolution given to it is not ſatisfactory.

XXVIII. QUESTION 238, *by Mr.* Geo. Sanderſon.

If there be three arches or angles δ, η, φ, where δ is given, the expreſſion $\frac{\sin(\delta-\eta-\varphi)}{\sin\varphi} \times \sqrt{\cos\eta}$ will be the greateſt poſſible when the $\tan(\delta-\eta) = \frac{1}{2}\tan\eta \times \frac{2-\tan\eta\times\tan\varphi}{\frac{1}{2}-\tan\eta\times\tan\varphi}$; the relation between the angle η and φ being expreſſed by the equation, $\cos\eta = (a^2 \div 2c^2) \times \tan\varphi^2$.—See page 222, Watſon's Tranſlation of Euler's conſtruction and properties of veſſels. Required a demonſtration?

XXIX. QUESTION 239, *by Mr.* Lowry.

Let AB, AC be two ſtraight lines given by poſition, B and C given points in theſe lines, and P a given point without them. It is required to draw two ſtraight lines PD, PE, meeting the given lines in D and E, ſo that the angle DPE may be of a given magnitude, and the ſum or difference of the rectangles PD × M, PE × N may be equal to a given ſpace; M and N being given lines.

XXX PRIZE QUESTION, by Mr. Lowry.

In a circle given in [illegible] deſcribe a trapezium [illegible] they may intercept [illegible] its diagonals may also pass [illegible]

ARTICLE XLV.

SEVEN PROPOSITIONS FROM LAWSON.

(To be anſwered in No. XII.)

PROP. XL.

IF there be two t… …EF, which have one angle A in one equal to … … other, and another angle B in the firſt equal to t… …les D and E in the ſecond; then ſhall the ſides AC… … proportional.

PR… XLI.

The ſquare of the line … …rtical angle of any triangle is a mean proportional … …ifference of the ſquares of each ſide including that angle, and the ſquare of the adjacent ſegment of the baſe made thereby.

PROP. XLII.

If from the ſame point two tangents be drawn to a circle, and a line be drawn joining the points of contact, and another line to be intercepted between the tangents cut the foregoing which joins the points of contact, ſo as to be biſected in the point of interſection; then I ſay that the part of that line which is a chord of the circle will alſo be biſected by the ſame point.

And, converſely, if the chord cutting the line joining the points of contact be biſected by the point of interſection, then the continuation of the ſame to meet the tangents will alſo be biſected by the ſame point.

PROP. XLIII.

If from one of the equal angles of an iſoſceles triangle a perpendicular be drawn to the oppoſite ſide; then I ſay that the rect-

angle

ngle contained under that ſide and its ſegment intercepted by the perpendicular and the baſe is equal to half the ſquare of the baſe.

PROP. XLIV.

If in a line AB two points C and D be taken, then I ſay that $(AB + AD) \times BC + BC^2 = 2ABC + BCD$.
And moreover that

A C D B

$(AB + AD) \times CD + CD^2 = 2ADC + BCD$.

PROP. XLV.

If from the vertical angle of any triangle two lines be drawn to make equal angles with the ſides containing it, and to cut the baſe; then I ſay that the ſquare of one ſide is to the ſquare of the other ſide as the rectangle under the ſegments of the baſe contiguous to the firſt ſide is to the rectangle under the ſegments contiguous to the other ſide.

PROP. XLVI.

If in AB the diameter of a ſemicircle any point C be taken, and from thence any line as CD drawn to meet the circumference in D, and a perpendicular DE be demitted; then I ſay that the the ſquare of the line AC is equal to the ſquare of the line CD together with the rectangle under the ſum of the diſtances of C from A and C from B and the line AE, when C is taken in the diameter AB produced; but equal to the ſquare of CD together with the rectangle under the difference of the diſtances of C from A and C from B and the ſame line AE, when C is taken in the diameter itſelf.

ARTICLE

ARTICLE XLVI.

DAWSON ON THE DISTANCE OF THE SUN.

(Continued from page 234.)

PROP. IV. Fig. 457. Pl. 25.

LET ADE be a circle described round the centre T, and [illegible] body describes it by a force, tending [illegible] is inversely as the square of the [illegible]ce [illegible]; if another force, which is as its [illegible] on the body directly from the centre, [illegible]mpound force, make it describe the [illegible]t is required, from having the propo[illegible]e two forces given, to determine the motion [illegible]me apses, and *e contra*, from having the motion [illegible]e apses given, to determine the proportion of the two forces.

Put TA $= r = 1$, TB $= x$, GB $= y$, AB $= z$, Cd $= \dot{s} = \dot{y}$, BC $= \dot{z}$, and BD $= \dot{u}$. Let the force of the body T, at the distance TA, be represented by r or 1, and the disturbing force by n. Then, at the distance TB or x, the two forces that act upon the body, moving at B, will become $\frac{r^3}{x^2}$ and $\frac{nx}{r}$. The velocity of the body in the circle AGD, is the same as would be acquired by falling from a height above A $=$ AT or x, which therefore is easily found to be $= r$. [See prop. 3, tract 3, of Tracts *Physical and Mathematical*.] It is likewise commonly known, that the central force, multiplied into $\dot{x}$, is equal the velocity into the fluxion of the velocity. Let the velocity, at B, be put $= v$, then, we have this equation,

ion, $\frac{r^3}{x^2} - \frac{nx}{r} \times \dot{x} = -v\dot{v}$, and, taking the fluents, $-\frac{r^3}{x} - \frac{nx^2}{2r} = -\frac{v^2}{2}$. But, at A, the equation becomes $-\frac{r^3}{r} - \frac{nr^2}{2r} = -\frac{r^2}{2}$; therefore, by correction and reduction, we have $\frac{r^2}{x} \times \overline{2r - x} + \frac{n}{r} \times \overline{x^2 - r^2} = v^2$. Suppose AN to be described, in the same time as BC; then, from the triangle ATN, being equal the triangle BTC, we gain the following proportions, viz.

$$\begin{aligned}
&&\mathrm{AN}^2 : \mathrm{BC}^2 &:: r^2 : v^2,\\
\text{and} \quad &-\quad - & \mathrm{B}u^2 : \mathrm{AN}^2 &:: r^2 : x^2.\\
\text{therefore, } &ex\ æquo & \mathrm{B}u^2 : \mathrm{BC}^2 &:: r^4 : x^2 v^2,\\
\text{that is,} \quad &-\quad - & \dot{u}^2 : \dot{z}^2 &:: r^4 : x^2 v^2.\\
\text{By division} \quad &-\quad - & \dot{u}^2 : \dot{x}^2 &:: r^4 : x^2 v^2 - r^4,\\
\text{hence} \quad &-\quad - & \dot{u} : \dot{x} &:: r^2 : \sqrt{x^2 v^2 - r^4},
\end{aligned}$$

consequently $\dot{u} = \frac{r^2 \dot{x}}{\sqrt{v^2 x^2 - r^4}}$. In the value of v^2, found above, substitute $r + y$ for x, and, neglecting all the terms where y is found above the second power, we shall have, when reduced, $\dot{u}$

=

$$= \frac{r^2 \dot{y}}{\sqrt{2nr^2 y - r^3 - 5nr \times y^2}} = \frac{r^{\frac{3}{2}} y - \frac{1}{2} \dot{y}}{\sqrt{2nr - r - 5n \times}}$$

Therefore $\overline{r + y} \times \dot{u} = \frac{r^{\frac{5}{2}} y^{-\frac{1}{2}} \dot{y}}{\sqrt{2nr - r - 5n} \times y}$

$\frac{r^{\frac{1}{2}} y^{\frac{1}{2}} \dot{y}}{\sqrt{2nr - r - 5n} \times y}$ = twice the area of the fluxi

ary triangle BTC. When the body comes to t apogee, $\dot{u} = \dot{z}$ or $\dot{y} = 0 = \dot{x}$, that is, the denom

nator $\sqrt{2nr - r - 5n} \times y = 0$; therefore, fu

posing $r = 1$, y will be found $= \frac{2n}{1 - 5n}$. Tak

the fluent of the above fluxion, at that time, and fi

supposing $r = 1$, we have twice the area described while the body passes from peregee to apogee or the area described from apogee to apoge

$$= \frac{c}{\sqrt{1 - 5n}} + \frac{nc}{\overline{1 - 5n}^{\frac{3}{2}}}, \text{ (c being $= 3{\cdot}14159$)}$$

Now, the area of the circular orbit is $= c$; therefore the above found area is to the area of the circle,

$$\text{as } \frac{1}{\overline{1 - 5n}^{\frac{1}{2}}} + \frac{n}{\overline{1 - 5n}^{\frac{3}{2}}} \text{ to } 1 :: 1 - 4n : \overline{1 - 5n}^{\frac{3}{2}}.$$

Hence the time of moving from apogee to apogee, is to the time of describing the circle ADE with the force r or $1 :: 1 - 4n : \overline{1 - 5n}^{\frac{3}{2}}$.

The

The periodic time of the body, in the curve ABC, with the compound force, will be nearly the ſame as in a circle, at the mean diſtance, and with the compound force proper to that diſtance. The leaſt diſtance is r, or 1, the greateſt diſtance $= 1 + \frac{2n}{1 - 5n}$, half the ſum of theſe $= 1 + n$, nearly, equal the mean diſtance. The compound force, at this diſtance, will be eaſily found $= \frac{1 - n \times \overline{1 + n}^3}{\overline{1 + n}^2}$. It is a property of central forces commonly known, that the periodic times in circles, are in the ſubduplicate ratio of the radius of the circle directly, and of the central force inverſly: Therefore, the time of deſcribing the circle ADE, is to the time of deſcribing a circle at the mean diſtance, or the time of a revolution, as $\sqrt{1 - n \times \overline{1 + n}^3}$ to $\overline{1 + n}^{\frac{3}{2}}$. Hence, compounding this proportion with that found above, we ſhall have the time from apogee to apogee, to the time of a revolution round the body T, as $1 - 4n \times \sqrt{1 - n \times \overline{1 + n}^3}$ to $\overline{1 - 5n}^{\frac{3}{2}} \times \overline{1 + n}^{\frac{3}{2}}$, or as $1 - \frac{9}{2}n + \frac{3}{8}n^2$ to $1 - 6n - \frac{3}{2}n^2$, nearly. Therefore if n be given, the motion of the apſes may be found; and, if the motion of the apſes be given, n is eaſily found: For, taking the ſame numbers as are made uſe of in prop. 9, ſect. 1. Supplement to Tracts Phyſical and Mathematical, 58091 : 57600 :: $1 - \frac{9}{2}n + \frac{3}{8}n^2 : 1 - 6n - \frac{3}{2}n^2$, or, $\frac{57609}{57600} = 1 - \frac{3}{2}n - \frac{9}{8}n^2$; hence n is eaſily found.

The ſame by a different method.

We found before that $\dot{u} = \frac{y^{-\frac{1}{2}}\dot{y}}{\sqrt{2n - 1 - 5^n \times y}}$:

Then by ſimilar triangles TB : TG :: Bd : Gg, that is, $1 + y : 1 :: \frac{y^{-\frac{1}{2}}\dot{y}}{\sqrt{2n - 1 - 5^n \times y}} : \frac{y^{-\frac{1}{2}}}{1 + y} \times \frac{\dot{y}}{\sqrt{2n - 1 - 5^n \times y}} = Gg$, which is the meaſure of the angle BTC. Throw $\frac{y^{-\frac{1}{2}}}{1 + y}$ into a ſeries, then, multiplying and finding the fluent, we have $\frac{c}{\overline{1 - 5n}^{\frac{1}{2}}} - \frac{nc}{\overline{1 - 5n}^{\frac{3}{2}}} + \frac{3n^2c}{2.1 - \overline{5n}^{\frac{5}{2}}} - \frac{5n^3c}{2.1 - \overline{5n}^{\frac{7}{2}}}$ &c. = arc deſcribed in paſſing from perigee to apogee; therefore, from what has been ſaid above, it is evident, that 58091 : 57600 :: $\frac{c}{\overline{1 - 5n}^{\frac{1}{2}}} - \frac{nc}{\overline{1 - 5n}^{\frac{3}{2}}} + \frac{3n^2c}{2.1 - \overline{5n}^{\frac{5}{2}}}$ &c. : c, from which proportion, n will eaſily be found.

COROL-

COROLLARY.

It is a thing commonly known, and which we ſhall not ſtay here to demonſtrate, that the diſturbing force of the ſun upon the moon is, *cæteris paribus*, in the direct ſimple ratio of the moon's diſtance from the earth; therefore n may very fitly be put equal the mean diſturbing force found in Cor. 1ſt, Prop. 3d. Hence, for determining a, or the ſun's diſtance, there is this equation $\frac{p^2}{P^2} \times \frac{8a^4 + 30a^2 + 297}{16a^4} = n$, which reduced gives $a =$

$$\sqrt{\frac{15p^2}{16P^2n-8p^2} + \sqrt{\left|\frac{15p^2}{16P^2n-8p^2}\right|^2 + \frac{297p^2}{16P^2n-8p^2}}}.$$

But if Dr. Stewart's mean force be put $= n$, we ſhall have, when reduced as above, $a =$

$$\sqrt{\frac{45p^2}{16P^2n-8p^2} + \sqrt{\left|\frac{45p^2}{16P^2n-8p^2}\right|^2 + \frac{729p^2}{16P^2n-8p^2}}}.$$

Theſe two equations will give the values of a much different, let n be what it will, as is evident from inſpection.

SCHOLIUM.

We ſhall now endeavour to point out the principal miſtakes, which, we apprehend, Dr. Stewart has made, in attempting to ſolve this curious problem.

And firſt, let us examine his calculations, ſuppoſing all the principles, he has gone upon, to be right.

In this propoſition we had the following proportion, viz. the time of the body moving from apogee to apogee, is to the time of the body, deſcribing the circle ADE, with the force of the body T alone

$:: 1 - 4n : \overline{1-5n}^{\frac{3}{2}}$. Then, if we ſuppoſe, with Dr. Stewart, that the time of a revolution of the body, in the curve ABC, is equal to the time of a revolution, in the circle AGD, with the compound force at A, we ſhall have the following proportion, which is the ſame with his; ſee page 381, Tracts Phyſical and Mathematical, viz. The time of deſcribing the circle, with the force of the body T alone, is to the time with the compound force ::

$\overline{1-n}^{\frac{1}{2}} : 1$. Therefore, compounding this proportion with the above, the time of moving from apogee to apogee, is to the time of revolving with

the compound force $:: 1 - 4n \times \overline{1-n}^{\frac{1}{2}} : \overline{1-5n}^{\frac{3}{2}}$;

that is, $58091 : 57600 :: \overline{1-4n} \times \overline{1-n}^{\frac{1}{2}} : \overline{1-5n}^{\frac{3}{2}}$; or (putting $a = 58091$, $b = 57600$) $a^2 : b^2 :: 1 - 9n + 24n^2 - 16n^3 : \overline{1-5n}^3 = 1 - 15n + 75n^2 - 125n^3$. From dividing the conſequents by the antecedents, we get $\frac{b^2}{a^2} = 1 - 6n - 3n^2$, nearly; whence n will be found $= \cdot 0028015$.—Dr. Stewart, for finding the value of n, has this proportion, a^2 is to b^2 as the cube of the greateſt diſtance is to the cube of the leaſt diſtance.

diſtance. [See page 381, Tracts, &c.] But 1 is the leaſt diſtance, and $1 + \frac{2n}{1-5n}$ the greateſt diſtance; hence this proportion, $a^2 : b^2 :: \overline{1 + \frac{2n}{1-5n}}^3 : 1^3 :: \overline{1-3n} : \overline{1-5n}^3 :: 1 - 9n + 27n^2 - 27n^3 : \overline{1-5n}^3$. Compare this with the former proportion, and it will be eaſy to ſee where they differ. For, if n^3 in each be neglected, they only vary in the third term, where there is $27n^2$ in the laſt proportion, inſtead of $24n^2$ in the former. The difference is, indeed, very ſmall, yet it occaſions a very great alteration in the ſun's diſtance, as will appear immediately. From this laſt proportion, the value of n is ·00279770 &c. whoſe reciprocal is 357·43365, the very ſame number found in prop. 9, ſect. 1, ſupplement to Tracts, &c. Now if this value of n be ſubſtituted in the ſecond equation for a, page 31, and p^2 be put $= 1$, and $P^2 = 178{\cdot}725$, a will come out $= 495{\cdot}932$ &c. which falls within the limits the Doctor has preſcribed, and agrees with his number in every place, except the laſt. But if the other value of n, viz. ·0028015, be ſubſtituted in the ſame equation, a will come out $=$ 89·8, nearly.

The difference of the two values of n, here made uſe of, is only ·0000038 &c. which is ſcarce the 737th part of the whole, and yet the difference in the ſun's diſtance occaſioned by this extremely ſmall variation in n, or the mean diſturbing force, is no leſs than 406 times the moon's diſtance; which is even more than the ſun's diſtance has generally been thought to be. But if n were taken $=$ ·00279759,

and ſubſtituted in the ſame equation, a would come out infinite, though the variation here, from Dr. Stewart's value of n, is only ·00000011 &c. which is leſs than the 25433d part of it. This laſt value of n is $= \frac{1}{357\cdot45} = \frac{1}{2P^2}$, hence in the equation for the value of a, the denominator $16P^2n - 8P^2 = 0$, therefore a muſt be infinite.

To prevent any objections, that may be made to the above method of calculation, we ſhall examine in what manner the value of n is affected, by rejecting all the terms in the value of v^2x^2, in which y is found above the ſecond power. [See page 381.]

Our value of $\dot{u}$ is $\frac{\dot{y}}{\sqrt{2ny - 1 - 5n \times y^2}}$; but if the term where y^3 is found be taken in, then $\dot{u}$ will be $= \frac{\dot{y}}{\sqrt{2ny - 1 - 5n \times y^2 + 4ny^3}}$. Extracting the root, the denominator becomes $=$

$$\frac{}{2ny - 1 - 5n \times y^2}\Big|^{\frac{1}{2}} \frac{1}{2} \times \frac{4ny^3}{2ny - 1 - 5n \times y^2\Big|^{\frac{1}{2}}} - ,$$

&c. Dividing $\dot{y}$ by this ſeries, $\dot{u}$ will $=$

$$\frac{\dot{y}}{\sqrt{2ny - 1 - 5n \times y^2}} - \frac{2ny^3\dot{y}}{2ny - 1 - 5n \times y^2\Big|^{\frac{3}{2}}}$$

&c. Multiply both ſides by $1 + y$, and take the fluent

luent when $y = \frac{2n}{1 - 5n}$, and we have the whole

area $= \frac{c}{\overline{1 - 5n}^{\frac{1}{2}}} + \frac{nc}{\overline{1 - 5n}^{\frac{4}{2}}}$ — ſome ſmall quantity, which put $= s$. This area therefore is to the area of the circle, as $\frac{c}{\overline{1 - 5n}^{\frac{1}{2}}} + \frac{nc}{\overline{1 - 5n}^{\frac{3}{2}}} - s$ is to c, or as $1 - 4n - s \times \overline{1 - 5n}^{\frac{3}{2}}$ is to $\overline{1 - 5n}^{\frac{3}{2}}$. And the time from apogee to apogee, is to the time of a revolution, in the circle AGD, with the force of the body T alone, as $\overline{1 - 4n} - s \times \overline{1 - 5n}^{\frac{3}{2}}$ is to $\overline{1 - 5n}^{\frac{3}{2}}$. By proceeding in the ſame manner as above, it will eaſily appear, that n will come out greater than before, though s be ever ſo ſmall; and if y^4 be taken into the account, n will ſtill be greater.

From what has been ſaid, it is evident, that the value of n, determined above, is not greater than it ought to be, and conſequently, that Dr. Stewart's value is too little, allowing all the principles, he has gone upon, to be right.

And by the way, does it not appear probable, from the Doctor's being ſatisfied with a proportion for determining n, which, though very near the truth, is nevertheleſs inaccurate; that he has not been well apprized of the *great* alteration in the ſun's

ſun's diſtance, which is occaſioned by a very ſmall one in the in the diſturbing force?—This is mentioned, is a circumſtance which leads to the moſt favourable apology, that can be made, for the confidence with which our author gives us a meaſurement of the ſolar ſyſtem, ſo much different from that which has been hitherto held, by our moſt eminent aſtronomers, as not egregiouſly wide of the truth.—To proceed:

But if our value of n, determined above, be ſubſtituted in the equation for determining a, deduced from what we call the true mean diſturbing force, we ſhall have a, or the diſtance of the ſun, ſtill much leſs than before; and, indeed, much leſs than the ſun's diſtance was ever thought to be; for it is only $= 50{\cdot}1$ times the diſtance of the moon, nearly.

Here, then, is a very large error, ariſing from a miſtake in the Doctor's *calculus;* it now remains, that we examine the principles themſelves, which he has aſſumed, and ſee, whether we have any thing to expect from this method of finding the ſun's diſtance or not. Thoſe objections only will be mentioned which ſeem to be of the greateſt weight, nor ſhall we give any calculations, as it will be ſufficient to point out where we apprehend he has erred.

In the firſt place, Dr. Stewart ſuppoſes the body to deſcribe a revolution in the curve ABC, in the ſame time that another body would deſcribe the circle AGD, with the compound force at A; but that this is not ſo, has been proved, page 382.

2dly, Is it not eſſentially neceſſary in this enquiry, that the proportion of the ſolar force to that of the earth upon the moon, be exactly found? And, in determining this, is not the periodic time of the moon, round the earth, taken into the account, and the earth ſuppoſed to be the centre, round which the moon moves? This, therefore, muſt occaſion a conſiderable error, the earth and moon, as it is well known,

known, moving round their common centre of gravity; and conſequently, a period of the moon will be accompliſhed in leſs time, than if the earth were the centre about which it revolved. To correct this properly, it will be neceſſary to know exactly the proportion of the quantities of matter in the earth and moon; a particular, I am afraid, we ſhall never be able perfectly to come at.

3dly, We have anticipated another capital objection, where we obſerved, that though the mean diſturbing force were ever ſo exactly found, yet the diſtance of the ſun could not be determined from it to any *great* degree of accuracy. [See page 234.]

4thly, Since the centre of gravity of the earth and moon moves forward, in the ecliptic, ſeveral degrees every revolution of the moon round the earth, the motion of the apſes occaſioned by this diſturbing force, will be conſiderably different from what it would be, ſuppoſing the centre of gravity at reſt, as we have done in this propoſition. This requires no demonſtration, being evident as ſoon as mentioned.

5thly, The true figure of the moon's orbit being an ellipſis, it muſt, with the ſame diſturbing force, have a different motion of the apſes to that of our ſuppoſed circular one; and to anſwer the queſtion right, the motion of the apſes ought to be determined in general for any excentricity whatever.—To ſee how the above conſiderations would ſucceed in calculation, I made a rude trial, but the reſult varied greatly both from Dr. Stewart's numbers, and thoſe commonly adopted by aſtronomers.

6thly, But if the motion of the moon's apogee cannot be accounted for, upon the common principles of central forces, (as is the opinion of Mr. Machine) will not this be a more diſheartning circumſtance than any pointed out above?

From

From what has been ſhewn, it plainly appears, that the method of determining the ſun's diſtance, pitched upon by our author, is very ill fitted for the purpoſe; it being ſcarce poſſible, I think, to accompliſh it this way (that is, by the motion of the apſes) though the calculations were made with the greateſt accuracy. For the principles are too complex to make it poſſible to take every thing into the account which belongs to it; and unleſs that be done, ſince the ſmalleſt neglect occaſions a very great error, the reſult will ever be much wide of the truth.

Neither is the motion of the nodes much better fitted than the motion of the apſes for the ſolution of this problem. In the firſt place, this motion is ſlower, and therefore requires a more accurate calculus. And beſides, the force MQ (ſee fig. 278, pl. 18.) which moſtly contributes to the motion of the nodes, affects the diſtance of the ſun very much, by a ſmall variation. For if the value of it be put into fluxions, and reduced as was ſhewn above, we ſhall have the variation of this force to the variation of the ſun's diſtance, as $-4y^2 + r^2$ to $\overline{a - 2y}^3$. Hence it is evident, that as the variation of the ſun's diſtance bears ſo very great a proportion to the variation of the diſturbing force; it is requiſite that this force be determined to the *laſt* degree of accuracy. But this will be very difficult, if not impoſſible to come at; the figure of the lunar orbit, its inclination to the ecliptic, &c. &c. being ſo variable.

From what has been ſaid, 'tis preſumed, it may be ſafely concluded, that the diſtance of the ſun will never be ſatisfactorily aſcertained by the THEORY of GRAVITY.

ARTICLE

ARTICLE XLVII.

Improved ſolution to Queſtion 109, *by Mr. LOWRY.*

THE ſolution of this problem depends on the following curious property of the circle, which I believe is not generally known, namely, If BKCL (Fig. 458, Pl. 25.) be a circle given in magnitude and poſition, and A be a given point without it, a point I may be found within the circle, ſuch, that if any ſtraight line BIC be drawn through it to interſect the circle at B and C, and AB, AC be drawn; the ſquare of AB will always be to the ſquare of AC as BI to IC.

To find this point, through O, the centre of the circle, draw AOL, and find the point E ſuch that the rectangle AOE may be equal to the ſquare of the ſemidiameter of the circle; biſect AE at D, and divide the diameter LK at I, ſo that LI : IK = LD : DK, ſo ſhall I be the point that was to be found.

Draw DB, DC and on KL demit the perpendiculars BG, CF; then, ſince LI : IK :: LD : DK by conſtruction, BD : DC = BI : IC, *by prop. VI. of Stewart's Propoſitiones Geometricæ;* therefore ID biſects the angle BDC, and the right angled triangles BDG, CDF are ſimilar; hence GD : DF = BD : DC = BI : IC. But *by Prop. VI. Stewart's General Theorems*, the ſquare of AB, is equal to twice the rectangle AO·DG, and the ſquare of AC equal to twice the rectangle AO·DF; therefore the ſquare of AB is to the ſquare of AC as DG to DF, that is, as BI to IC.

COROLLARY. The ſquare of AL is to the ſquare of AK as LI to IK.

Hence this CONSTRUCTION of the Problem propoſed.

Through the centre O draw AKOL and divide AL at I ſo that LI may be to IK as the ſquare of AL to the ſquare of AK; on AI deſcribe a ſemicircle and apply therein AQ equal to the given perpendicular. Draw BQIC, and join AB, AC, ſo ſhall ACB be the triangle required.

ARTICLE

ARTICLE XLVIII.

To the Editor of the Mathematical Repoſitory.

SIR,

THE advantage of transformation, previous to finding fluents, in many caſes, is well known to thoſe who are converſant in ſuch ſpeculations: for, by a judicious ſubſtitution, expreſſions of every complicated and perplexing appearance are brought to a more tractable form, whereby the labour of calculation is much facilitated. It is, therefore, a matter of importance, in this branch of the analytic art, to diſcover ſuch a convenient ſubſtitution for ſome part of any fluxional expreſſion, as ſhall tend to reduce it to the moſt ſimple form, which it is capable of aſſuming;—to point out a method for effecting this purpoſe, in certain caſes, is the principal deſign of this eſſay. Beſides, it has always appeared to me difficult for young fluxioniſts to comprehend the rationale of ſome of thoſe fluents, which are commonly placed among the firſt in the table and, on that account, reckoned amongſt the eaſieſt: now, the want of perſpicuity, in the commencement of any ſcience, muſt have a great tendency to diſcourage young beginners, becauſe obſtacles preſent themſelves, which cannot eaſily be ſurmounted at ſo early a period.

I have ſelected a few examples of ſuch forms of fluxions as are moſt frequently met with, in the ſolution of problems, in order to illuſtrate a method of inveſtigating their fluents, which, I believe, is not very generally known, and which, when it firſt occurred to my mind, I imagined had ſome degree of novelty in it. If you think it deſerves a place in the Repoſitory, by inſerting it you will greatly oblige

Your moſt obedient Servant,

JAMES CUNLIFFE.

Bolton-le-Moors, Oct. 8, 1800.

EXAMPLE 1. To find the fluent of $\frac{\dot{x}}{\sqrt{(x^2 - a^2)}}$?

Aſſume $x^2 - a^2 = (av - x)^2 = a^2v^2 - 2avx + x^2$, from whence $x = \frac{a}{2} \times \frac{v^2 + 1}{v}$, $\sqrt{(x^2 - a^2)} = av - x = \frac{a}{2} \times \frac{v^2 - 1}{v}$;

$\frac{v^2-1}{v}$, and $\dot{x} = \frac{1}{2}a\dot{v} \times \frac{v^2-1}{v^2}$, and therefore $\frac{\dot{x}}{\sqrt{(x^2-a^2)}}$ $= \frac{\dot{v}}{v}$, which is the moſt ſimple of logarithmic forms, and its fluent is well known to be the hyp. log. of v. But $x = \frac{a}{2} \times$ $\frac{v^2+1}{v}$, as reduced above, therefore, by ſolving a quadratic equation, $v = \frac{x+\sqrt{(x^2-a^2)}}{a}$, and conſequently the fluent of $\frac{\dot{x}}{\sqrt{(x^2-a^2)}} =$ h. l. of $\frac{x+\sqrt{(x^2-a^2)}}{a}$.

EXAMPLE 2. To find the fluent of $\frac{\dot{x}}{\sqrt{(x^2-2ax)}}$?

Aſſume $x^2 - 2ax = (av - x)^2 = a^2v^2 - 2avx + x^2$, from hence $x = \frac{a}{2} \times \frac{v^2}{v-1}$, $\sqrt{(x^2-2ax)} = av - x = \frac{a}{2} \times$ $\frac{2v}{v-1}$, and $\dot{x} = \frac{a\dot{v}}{2} \times \frac{v^2-2v}{(v-1)^2}$, and therefore $\frac{\dot{x}}{\sqrt{(x^2-2ax)}}$ $= \frac{\dot{v}}{v-1}$. Now the fluent of $\frac{\dot{v}}{v-1}$, is well known to be the hyp. log. of $(v-1)$. But $x = \frac{a}{2} \times \frac{v^2}{v-1}$, as examined above, whence $v = \frac{x+\sqrt{(x^2-2ax)}}{a}$, or $v - 1$ $= \frac{x-a+\sqrt{(x^2-2ax)}}{a}$; and therefore the fluent of $\frac{\dot{x}}{\sqrt{(x^2-2ax)}}$

$\frac{\dot{x}}{\sqrt{(x^2 - 2ax)}}$ = fluent of $\frac{\dot{v}}{v - 1}$ = h. l. $(v - 1) =$ h. l. $\left(\frac{x - a + \sqrt{(x^2 - 2ax)}}{a}\right)$.

The preceding fluent might have been eaſily obtained, by writing $x - a$ in the place of x in the firſt example; but I rather choſe to inveſtigate it at full length, as the concluſions will be found of uſe, in what follows.

The preceding example may be done as follows.

Aſſume $x^2 - 2ax = x^2z^2$, from whence $x = \frac{2a}{1 - z^2}$, $\sqrt{(x^2 - 2ax)} = xz = \frac{2az}{1 - z^2}$, and $\dot{x} = \frac{4az\dot{z}}{(1 - z^2)^2}$; therefore $\frac{\dot{x}}{\sqrt{(x^2 - 2ax)}} = \frac{2\dot{z}}{1 - z^2} = \frac{\dot{z}}{1 + z} + \frac{\dot{z}}{1 - z}$; the fluents of which are h. l. $(1 + z)$ — h. l. $(1 - z) =$ h. l. $\left(\frac{1 + z}{1 - z}\right)$.

But $x = \frac{2a}{1 - z^2}$, whence $z = \frac{\sqrt{(x^2 - 2ax)}}{x}$; conſequently the fluent of $\frac{\dot{x}}{\sqrt{(x^2 - 2ax)}} =$ h. l. $\left(\frac{1 + z}{1 - z}\right) =$ h. l. $\left(\frac{x + \sqrt{(x^2 - 2ax)}}{x - \sqrt{(x^2 - 2ax)}}\right)$.

EXAMPLE 3. To find the fluent of $\frac{\dot{x}}{x\sqrt{(x^2 - a^2)}}$?

By making uſe of the values of x &c. as determined in the firſt example, there will be had $\frac{\dot{x}}{x\sqrt{(x^2 - a^2)}} = \frac{a}{2} \times \frac{\dot{v}}{1 + v^2}$; the

fluent

at of which is known to be $\frac{a}{2} \times$ circular arc to radius 1

$\&$ tangent $v = \frac{x + \sqrt{(x^2 - a^2)}}{a}$.

EXAMPLE 4. To find the fluent of $\frac{\dot{x}}{x\sqrt{(x^2 - 2ax)}}$?

Taking the values of x &c. as determined in the firſt part of the ſecond example, and there will be had $\frac{\dot{x}}{x\sqrt{(x^2 - 2ax)}}$ $= \frac{2\dot{v}}{av}$; the fluent of which is $-\frac{2}{av} = -\frac{2a}{x + \sqrt{(x^2 - 2ax)}}$ $= \frac{\sqrt{(x^2 - 2ax)} - x}{x}$, by reſtoring the value of v, as determined in the ſaid example.

The ſame otherwiſe. Making uſe of the values of x &c. as determined in the ſecond operation of the ſecond example, there will be had $\frac{\dot{x}}{x\sqrt{(x^2 - 2ax)}} = \frac{\dot{z}}{a}$; the fluent of which is $\frac{z}{a}$ $= \frac{\sqrt{(x^2 - 2ax)}}{ax}$.

EXAMPLE 5. To find the fluent of $\frac{x\dot{x}}{\sqrt{(x^2 - 2ax)}}$?

Taking the values of x &c. as determined in the former part of the ſecond example, and there will be had $\frac{x\dot{x}}{\sqrt{(x^2 - 2ax)}} = \frac{a^2\dot{v}}{(v^2 - 1)^2} = \frac{a\dot{z}}{2} \times \frac{(z + 1)^2}{z^2} = \frac{a\dot{z}}{2} + \frac{a\dot{z}}{z} + \frac{a\dot{z}}{2z^2}$; the fluents of which are $\frac{az}{2} + a \times$ h. l. $(z) - \frac{a}{2z}$, in which expreſſion

reflowing $z = v - 1 = \frac{x - a + \sqrt{(x^2 - 2ax)}}{a}$, and the fluent of $\frac{xz}{\sqrt{(x^2 - 2ax)}}$ will be obtained.

EXAMPLE 6. Required the fluent of $\frac{2ax}{x\sqrt{(a^2 + x^2)}}$?

Assume $a^2 + x^2 = (av - x)^2 = a^2v^2 - 2avx + x^2$, from whence $x = \frac{a}{2} \times \frac{v^2 - 1}{v}$, $\sqrt{(a^2 + x^2)} = av - x = \frac{a}{2} \times \frac{v^2 + 1}{v}$, and $z = \frac{av}{2} \times \frac{v^2 + 1}{v^2}$; therefore $\frac{2az}{x\sqrt{(a^2 + x^2)}} = \frac{4z}{v^2 - 1} = \frac{2v}{v - 1} - \frac{2v}{v + 1}$. Now the fluent of this laſt expreſſion is known to be $z \times$ h. l. $(v - 1) - z \times$ h. l. $(v + 1)$ $= z \times$ h. l. $\left(\frac{v - 1}{v + 1}\right)$. But $v = \frac{av}{a} = \frac{v^2 - 1}{x}$, as deduced above, from whence, by ſolving a quadratic equation, there will be had $v = \frac{x + \sqrt{(a^2 + x^2)}}{a}$, and conſequently the fluent of $\frac{2az}{x\sqrt{(a^2 + x^2)}} = z \times$ h. l. $\left(\frac{v^2 - 1}{v + 1}\right) = z \times$ h. l. $\left(\frac{v - 1}{v + 1}\right) = z \times$ h. l. $\left(\frac{x - a + \sqrt{(a^2 + x^2)}}{x + a + \sqrt{(a^2 + x^2)}}\right)$.

The [illegible] Example done ſomething differently.

Aſſu[illegible] $xu - a)^2 = x^2u^2 - 2axu + a^2$, whence

[illegible] $x^2) = xu - a = a \times \frac{u^2 + 1}{u^2 - 1}$, and x

$=$

$= -2axx \times \frac{x^2+1}{(x^2-1)^2}$. Therefore $\frac{2ax}{x\sqrt{(a^2+x^2)}} = -$

, the fluent of which is $2 \times$ h. l. $(\frac{1}{u})$. But $x =$

$\frac{}{-1}$, as found above, whence $u = \frac{a+\sqrt{(a^2+x^2)}}{x}$,

d therefore the fluent of $\frac{2ax}{x\sqrt{(a^2+x^2)}} = 2 \times$ h. l. $\frac{1}{u} = 2$

h. l. $\left(\frac{x}{a+\sqrt{(a^2+x^2)}}\right)$.

EXAMPLE 7. Required the fluent of $\frac{2ax}{x\sqrt{(a^2-x^2)}}$?

Assume $a^2 - x^2 = (xu - a)^2 = x^2u^2 - 2axu + a^2$, whence

$= \frac{2au}{u^2+1}$, $\sqrt{(a^2-x^2)} = xu - a = a \times \frac{u^2-1}{u^2+1}$, and x

$= 2au \times \frac{u^2-1}{(u^2+1)^2}$; therefore $\frac{2ax}{x\sqrt{(a^2-x^2)}} = -\frac{2u}{u}$,

e fluent of which is well known to be $2 \times$ h. l. $(\frac{1}{u})$. Now

$= \frac{2au}{u^2+1}$, as is found above, from whence, therefore, by a

dratic equation, $u = \frac{a+\sqrt{(a^2-x^2)}}{x}$, and consequently the

ent of $\frac{2ax}{x\sqrt{(a^2-x^2)}} = 2 \times$ h. l. $(\frac{1}{u}) = 2 \times$ h. l.

$\left(\frac{x}{a+\sqrt{(a^2-x^2)}}\right)$.

EXAMPLE 8. To find the fluent of $\frac{2a\dot{x}}{x\sqrt{(2ax-x^2)}}$?

Aſſume $2ax - x^2 = v^2x^2$, whence $x = \frac{2a}{1+v^2}$, $\sqrt{(2ax-x^2)} = vx = \frac{2av}{1+v^2}$, and $\dot{x} = -\frac{4av\dot{v}}{(1+v^2)^2}$; therefore $\frac{2a\dot{x}}{x\sqrt{(2ax-x^2)}} = -2\dot{v}$, the fluent of which is evidently $-2v$. But $x = \frac{2a}{a^2+1}$, as deduced above, whence $v^2 = \frac{2a}{x} - 1 = \frac{2ax-x^2}{x^2}$, or $v = \frac{\sqrt{(2ax-x^2)}}{x}$. Conſequently the the fluent of $\frac{2a\dot{x}}{x\sqrt{(2ax-x^2)}} = -2v = \frac{2\sqrt{(2ax-x^2)}}{x}$.

EXAMPLE 9. To find the fluent of $\frac{x\dot{x}}{\sqrt{(2ax-x^2)}}$?

Taking the values of x &c. as determined in the preceding example, and there will be had $\frac{x\dot{x}}{\sqrt{(2ax-x^2)}} = -\frac{4v\dot{v}}{(1+v^2)^2} = -2\dot{v} \times \frac{1-v^2}{(1+v^2)^2} - 2\dot{v} \times \frac{1+v^2}{(1+v^2)^2}$, the fluents of which are $\frac{2v}{1+v^2} - 2A$; — where A denotes the length of a circular arc to radius 1 and tangent $v = \frac{\sqrt{(2ax-x^2)}}{x}$; or, by reſtoring the value of v, the expreſſion becomes $\frac{\sqrt{(2ax-x^2)}}{a} - 2A$, which is, therefore, the fluent of $\frac{x\dot{x}}{\sqrt{(2ax-x^2)}}$.

Diophantine Algebra has generally been eſteemed a more curious than uſeful ſpeculation; but from the foregoing examples it appears, that it is capable of being employed with ſucceſs in the calculation of fluents, one of the moſt important branches of Mathematical [illegible]ning.

ARTICLE

ARTICLE XLIX.

ATWOOD's INVESTIGATIONS ON WATCH BALANCES.

(Concluded from page 246.)

NOW let Q (fig. 82, pl. 5. and fig. 86, pl. 6.), or the point of quieſcence of the auxiliary ſpring deviate from O, the point of quieſcence of the balance ſpring, by an arc OQ; ſuppoſe this arc OQ to be $=1°$; and let the point Q be ſituated in the firſt ſemiarc of vibration between O and B. The time of deſcribing the ſemi-arc BO will be aſcertained on theſe conditions, by referring to the inveſtigation (page 129), and making the following ſubſtitutions:

$a=1{\cdot}5707963=$ an arc of 90° to radius $=1$*

$b=2{\cdot}3561945=$ an arc of 135°

$c=2{\cdot}3387412=$ and arc of 134°

$b-c=d=0{\cdot}0174533=$ an arc of 1°

$l=193$

$f=0{\cdot}9562754$

$n=\frac{1}{2\,7}$.

ſhewed twelve minutes by the motion of the hands in one hour of mean time, which correſponds to an interval of 12 ſeconds of time, ſhewn by the watch in 60 ſeconds, or one minute of mean time. According to the calculation, 13 ſeconds of time are ſhewn by the watch in one minute. A nearer agreement between the theory and matter of fact could ſcarcely be expected in the circumſtances of the experiment.

* In all the following calculations the radius is alſo $=1$.

Then,

Then, $\sqrt{\frac{2a}{lf \times n+1}}$ × into a circular arc,

Pts. of a ſec.

of which the ſine is $\sqrt{\frac{c \times n+1}{2b+2nc}} = 0{\cdot}0995507$

$\sqrt{\frac{a}{2lf}}$ × into a circular arc, of which

the ſine is $\frac{d}{\sqrt{b^2+nc^2}}$ - $= 0{\cdot}00047174$

Time of a ſemivibration in the arc BO $= {\cdot}10002244$

The time ſhewn by the watch in $24^h = \frac{24^o}{1{\cdot}0002244}$

$= 23^h\ 59'\ 40''.\ 60$, giving a daily rate of $19''.\ 40$ ſlow.

This variation of the daily rate is not to be conſidered as effecting the regularity of the watch, as it is either compenſated by adjuſtments when the watch is regulated to mean time, or taken as the eſtabliſhed rate. A more material point is next to be determined; admitting the deviation of O and Q (fig. 82, and 86,) to be OQ $= 1°$, the ſame as in the former caſe; ſuppoſe the ſemiarc of vibration to be diminiſhed from 135° to 125°. If the points of quieſcence O and Q were coincident, this diminution of the arc of vibration would cauſe no alteration (page 164, Vol. I.) in the time of a ſemivibration, becauſe the forces of acceleration of both ſprings would be as the angular diſtances from the ſame quieſcent point O; but ſince theſe points are ſeparated by an arc of 1°, a diminution of 10° in the

the ſemiarc of vibration, will cauſe a change in the daily rate, which will be obtained from the general theorem (page 129) by making the following ſubſtitutions:

$a = 1{\cdot}5707963 =$ an arc of 90°
$b = 2{\cdot}1816616 =$ an arc of 125°
$c = 2{\cdot}1642083 =$ an arc of 124°
$b - c = d = 0{\cdot}0174533 =$ an arc of 1°.
$l = 193$
$f = 0{\cdot}9562754$
$n = \frac{1}{10}$.

$\sqrt{\frac{2a}{lf \times n+1}} \times$ into a circular arc,

Pts. of a ſec.

of which the ſine is $\sqrt{\frac{c \times n+1}{2b+2nc}} = 0{\cdot}09951472$

$\sqrt{\frac{a}{2lf}} \times$ a circular arc, of which the

ſine is $\frac{d}{\sqrt{b^2+nc^2}} =$ - - 0·00050949

The time of a ſemivibration ·10002421

Time ſhewn by the watch in $24^h =$

$\frac{24^h}{1{\cdot}0002421} =$ $23^h\ 59'\ 39''\ {\cdot}08$

giving a daily rate of - - 20″ ·92 ſlow.

Daily rate, when the ſemiarc of vibration was - - 135° 19″ ·40 ſlow.

Retardation of the daily rate in conſequence of the diminution of the ſemiarc of vibration from 135° to 125° 1″ ·52.

In

In theſe examples, the point of quieſcence of the auxiliary ſpring Q is ſituated in the firſt ſemiarc of vibration between B and O; and in conſequence of this poſition, the daily rate of the time-keeper is retarded by a ſeparation of the points of quieſcence, while the ſemiarc of vibration remains the ſame; *i.e.* 135°; ſecondly, the ſeparation of the points of quieſcence remaining unaltered, a diminution of the arc of vibration cauſes a further retardation of the rate. Oppoſite effects on the daily rate take place when the point of quieſcence Q is ſituated in the latter ſemiarc of vibration, (fig. 83.) between O and E. The arc OQ remaining = 1° as before, this caſe produces an acceleration inſtead of a retardation of the rate.

Suppoſe the ſemiarc of vibration BO to be 135°
Separation of the points of quieſcence = OQ 1°

Arc BQ in fig. 83, or BN in fig. 86, 136°

The time of deſcribing the ſemiarc BO will be determined by referring to the theorem (page 135),

making $a = 1{\cdot}5707963 =$ an arc of 90°
$b = 2{\cdot}3561945 =$ an arc of 135°
$c = 2{\cdot}3736478 =$ an arc of 136°
$d = 0{\cdot}0174533 =$ an arc of 1°
$l = 193.$
$f = 0{\cdot}9562754.$
$n = \frac{1}{20}.$

The time of deſcribing the ſemiarc BO = $\sqrt{\frac{2a}{lf \times n + 1}}$ × a circular arc of which the

ſine

Pts. of a ſec.

ine is $\sqrt{\frac{b-d \times n+1}{2b+2nc}}$ = 0·09950617

$\sqrt{\frac{a}{2lf \times 1+2n}}$ × a circular arc of

which the ſine is $\sqrt{\frac{d^2 \times 1+2n}{b^2+nc^2-2nd^2}}$ = 0·00047141

Time of a ſemivibration in the arc BO = 0·09997758

Time ſhewn by the watch in 24^h =

$\frac{24^h}{99977\cdot58}$ = - - - 24^h 0′ 19″·38,

giving a daily rate of - 19.″38 faſt.

Every thing elſe remaining, let the ſemiarc of vibration be diminiſhed from 135° to 125°, and make

b = 2·1816616 = an arc of 125°

c = 2·1991149 = an arc of 126°

The time of deſcribing BO will now be

$\sqrt{\frac{2a}{lf \times n+1}}$ × a circular arc of which the ſine is

Pts. of a ſec.

$\sqrt{\frac{b-d \times n+1}{2b+2nc}}$ - = 0·09946662

$\sqrt{\frac{a}{2lf \times 1+2n}}$ × a circular arc of

which the ſine is $\sqrt{\frac{d^2 \times 1+2n}{b^2+nc^2-2nd^2}}$ = ·00050912

Time of a ſemivibration in the arc BO = ·09997574

Time

Time ſhewn by the watch in 24^h =

$\frac{24^h}{\cdot 9997574}$ = - - $24^h\ 0'\ 20''\cdot 91$,

giving a daily rate of - - $20''\cdot 91$ faſt.

Daily rate when the ſemiarc of vibration was 135° - - $19''\cdot 38$ faſt.

Acceleration of the rate in conſequence of the diminution of the ſemiarc of vibration from 135° to 125° = $1''\cdot 53$

The theorem from which this latter reſult has been calculated is founded on a ſuppoſition, that the arc BN (fig. 83.) is deſcribed by the accelerative forces of the balance ſpring and the auxiliary ſpring *u*, and that the arc NO is deſcribed by the accelerative forces of the two ſprings juſt mentioned, combined with the retarding force of the other auxiliary ſpring *v*, of which the point of quieſcence is ſituated at N (fig. 83.). It may poſſibly be thought, that this ſuppoſed action of the auxiliary ſpring which retards the balance while it deſcribes the arc from N to O, cannot take place, becauſe by the peculiar conſtruction of Mr. Mudge's invention, when the balance vibrates from B towards E, the retarding force of the auxiliary ſpring is removed from acting on the balance while it deſcribes the arc N*k* = 27° (fig. 83.), including the ſaid arc NO; but it is to be conſidered, that this removal of the retarding force from acting on the balance while it deſcribes the arc from N to *k*, is merely a mechanical expedient for ſupplying the power which is loſt by friction, and as it is thus wholly employed and counteracted, it appears requiſite, in making calculations of the times in which balance vibrates, that the entire forces of the ngs are to be taken into the calculation in the e manner as if friction did not exiſt, and no chanical contrivance was neceſſary for compenſating

ing the loſs of motion in conſequence of it. But ce, in point of fact, no retarding force acts on the lance while it deſcribes the arc NO, according to e conditions under conſideration, it may be ſatis-ſtory to calculate the time of a ſemivibration in e arc BO on this ground alſo, that is, on a ſup-ſition that the entire ſemiarc BO is deſcribed by e joint accelerative forces of the balance ſpring, d the auxiliary ſpring u only; it will appear that is ſlight variation of the conditions does not at all ſect the concluſions deduced from the preceding lculations, and very little alters the reſults them-lves.

The time of deſcribing the ſemiarc BO, on theſe nditions, will be determined by referring to the heorem (page 130), and making the following otations.

$a = 1{\cdot}5707963 =$ an arc of 90° to radius $= 1$
$b = 2{\cdot}3561945 =$ an arc of 135°
$c = 2{\cdot}3736478 =$ an arc of 136°
$d = 0{\cdot}0174533 =$ an arc of 1°.
$l = 193$
$f = 0{\cdot}9562754$
$n = \frac{1}{256}$.

The time of a ſemivibration is $= \sqrt{\frac{2a}{lf \times n + 1}}$

x into a circular arc, of which the fine $=$

$$\sqrt{\frac{b \times n + 1}{2b + 2nc}} \quad = \quad \text{Pts. of a ſec.} \quad = 0{\cdot}9997752.$$

Time ſhewn by the watch in 24^h. $=$

$$\frac{24}{a} \quad = 24^h\ 0'\ 19''{\cdot}42,$$

daily rate of 19″·42 faſt.

Every thing elſe remaining, let the ſemia vibration be diminiſhed from 135° to 125°; make

$b = 2{\cdot}1816616 =$ an arc of 125° to radius
$c = 2{\cdot}1991149 =$ an arc of 126°.

The time of deſcribing the ſemiarc BO wil be =

$$\sqrt{\frac{2a}{lf \times n + 1}} \times \text{a circular arc, of whic}$$

Pts. o

$$\text{ſine} = \sqrt{\frac{b \times n + 1}{2b + 2nc}} \quad - = 0{\cdot}0999$$

Time ſhewn by the watch in $24^h =$

$$\frac{24^h}{{\cdot}9997570} \quad - \quad - \quad - = 24^h\ 0'\ 2c$$

giving a daily rate of - 20″·9

daily rate, when the ſemiarc of vibration was 135° - = 19″·4

Acceleration of daily rate in conſequence of the diminution of the ſemiarc of vibration from 135° to 125° - - 1″·54,

ſcarcely differing from the former determinat page 406.

By having recourſe to the figure, we may diſt perceive the three quieſcent poſitions of the GO and IO, (fig. 86.) in reſpect to the rods ZW, which correſpond with the ſeveral conc aſſumed for the calculation of the precedin amples. In the firſt place, the crank AXYD the arms GO, IO, being in their quieſcent po it is evident from the preceding obſervations when the arms GO, IO are adjuſted ſo as ju touch the rods LM, ZW, the points of quieſ of the auxiliary ſprings u and v coincide wi point of quieſcence of the balance ſpring O,

84.) and in this caſe, the balance making 5 vibrations in a ſecond when adjuſted to mean time, the daily rate is = 0. (page 136).

Secondly, ſuppoſe the arms GO and IO to be ſo affixed to the axis TR, FS, (fig. 86 and 82.) that inſtead of juſt touching the rods LM and NW when quieſcent, they are inclined * to that poſition at any angle OGQ = OIN: the ſituation of the pallets p and q not being altered: in this caſe the point of quieſcence of the auxiliary ſpring u, will be ſeparated from the point of quieſcence of the balance ſpring by the angular diſtance OGQ = OCQ. In like manner the point of quieſcence of the auxiliary ſpring v will deviate from the point of quieſcence of the balance ſpring by the angular diſtance OIN = OCN. Suppoſe that the arc OQ = ON is = 1°: the point of quieſcence Q of the auxiliary ſpring u is in the firſt ſemiarc of vibration, that is, between B and O, (fig. 82.) while the balance vibrates from B to E; and the point of quieſcence of the auxiliary ſpring v, is in the firſt ſemiarc of vibration between E and O, while the balance vibrates from E to B; on theſe conditions, the ſemiarcs of vibration BO and OE being 135°, the daily rate of the time-keeper will be 19″ ·40 ſlower than mean time, (p. 402) and if the ſemiarcs BO, OE ſhould be diminiſhed from 135° to 125°, the daily rate will be 20″ ·92 ſlow (p. 403); the retardation of the daily rate in conſequence of the diminiſhed ſemiarc of vibration being 1″ ·52.

To conſider the remaining caſe; if the arms GO, IO, (fig. 86.) are adjuſted ſo as to preſs equally by the force of the auxiliary ſprings againſt the rods LM, WZ, when quieſcent, the equal and contrary preſſures prevent any apparent effect on the poſition

* The force of the main ſpring not being ſuppoſed to act on the balance wheel, or pallets p and q.

of the balance or crank AXYD; if the rod LM ſhould be removed (by turning the balance in its plane), ſuppoſe that the arm GO reſts in a poſition GN, at a diſtance beyond O, which is meaſured by the arc ON = 1°; in this poſition the point of quieſcence of the auxiliary ſpring *u* will be ſituated in the latter ſemiarc of vibration at N, between E and O, and by a ſimilar adjuſtment of the quieſcent poſition of the arm IO, (fig. 83 and 86.) the point of quieſcence of the auxiliary ſpring *v* will be ſituated at Q in the latter ſemiarc of the vibration, between O and B; in conſequence of this poſition of the points of quieſcence, the daily rate of the watch will be accelerated 19″·38, (p. 405) while the ſemiarc of vibration continues 135°, and when the ſemiarc of vibration is diminiſhed to 125°, the daily rate will be further accelerated; the rate being 20″·91 faſt, (page 406) or 1″·53 faſter than when the ſemiarc of vibration was 135°.

From theſe ſeveral * reſults it may be concluded, that although the rate of going of Mr. Mudge's time-keeper depends materially on the quieſcent poſition of the arms GO, IO, (fig. 86.), that is, on the poſition of the points of quieſcence of the auxiliary

* It is ſcarcely neceſſary to obſerve, that although theſe reſults have been inveſtigated from ſuppoſing the elaſtic forces of the ſpiral ſprings to be in the preciſe law of the tenſions, or diſtances from quieſcence; yet if the ſpring's forces ſhould deviate ſomewhat from that law, the general concluſions deduced from the preceding calculations, reſpecting the acceleration or retardation of the rate ariſing from an alteration in the poſition of the points of quieſcence, will ſtill be true, although the degree in which theſe effects take place may not be exactly the ſame as when the ſpring's forces are in the preciſe law aſſumed in the inveſtigations.

Suppoſing, as in the former examples, the points of quieſcence, of the auxiliary ſprings to be at the diſtance of 1° from the point of quieſcence of the balance ſpring, the variations of rate from [illegible]e, when the ſemiarcs of vibration are 135°, 125°, 60°, [illegible]verally, will be as expreſſed underneath:

Points

auxiliary ſprings, yet while that poſition remains unaltered, whatever it may be, the regularity of the time-keeper will not be affected in conſequence of the ſaid poſition, provided the ſemiarc of vibration continues the ſame; but when this arc is diminiſhed, an acceleration or retardation of the daily rate will take place, according to the ſituation of the points of quieſcence of the auxiliary ſprings referred to the ſemiarcs of vibration in which theſe ſprings reſpectively act. If the points of quieſcence of the auxiliary ſprings ſhould be ſituated in the firſt ſemiarcs of vibration, a diminution of theſe arcs will cauſe a retardation of the rate; but if the points of quieſcence ſhould be ſituated in the latter ſemiarcs of the reſpective vibrations, the daily rate will be accelerated.

Points of quieſcence in the firſt ſemiarc of vibration.

Semiarcs of vibration.	Variation of the daily rate from mean time.
135°	— 19″·40
125°	— 20″·92
60°	— 44″·33
10°	— 4′ 24″·24

Points of quieſcence in the latter ſemiarcs of vibration.

135°	+ 19″·38
125°	+ 20″·91
60°	+ 43″·60
10°	+ 4′·23″·60

If the auxiliary ſprings ſhould be applied for the ſole purpoſe of making the vibrations of the balance iſochronal, it may probably be found convenient to adjuſt the points of quieſcence of the auxiliary and balance ſprings at a greater diſtance than 1°; ſuppoſe the diſtance to be 10°. Let the arc of vibration be 450°, or 1¼ of a revolution, and ſuppoſe the balance to make four vibrations in a ſecond; on theſe conditions two auxiliary ſprings, each of which is about $\frac{1}{45}$ part of the force of the balance ſpring, or perhaps a ſingle auxiliary ſpring proportionally ſtronger, will, in moſt caſes, be ſufficient to compenſate for the want of iſochroniſm in the balance ſpring.

By means of Mr. Mudge's conſtruction, we may apply the principles deduced from the preceding inveſtigations to correct ſuch alterations in the daily rate as may ariſe from a diminiſhed or increaſed arc of vibration. For if it ſhould be known, from any ſatisfactory mode of trial, that the properties * of the balance ſpring are ſuch as cauſe the longer arcs of

* Different opinions have been entertained reſpecting the times in which a balance, vibrating freely by the action of a ſpiral ſpring, deſcribes the longer and ſhorter arcs of vibration. Mr. Harriſon ſays, "large arcs are naturally performed in leſs time than ſmall ones."—Notes taken at the diſcovery of Mr. Harriſon's time-keeper, p. vii. In this opinion he is followed by M. Berthoud; "J'ai appris par des experiences ſûres, que les grand arcs et les petits arcs d'un balancier ne ſont pas iſochrones, et qu'en general dans un balancier libre, les grands arcs ſont plus prompts que les petits." Mr. Ludlam, in his report addreſſed to the commiſſioners of the board of longitude, and intituled, a Short View of the Improvements made or attempted in Mr. Harriſon's Watch, has the following remark: "The principle on which Mr. Harriſon forms the alteration of the third part (before deſcribed) is, that the longer vibrations of a balance moved by the ſame ſpring are performed in leſs time. This is contrary to the received opinion among philoſophers and workmen."

Theſe contradictory opinions may poſſibly have ariſe nfrom experiments, in which all theſe circumſtances capable of influencing the times of vibration in longer or ſhorter arcs, were either not noticed, or omitted to be properly allowed for; this will ſeem the more probable if it be allowed, that a balance ſpring may be adjuſted in various ways ſo that either the longer or ſhorter arcs ſhall be performed in the leaſt time; not only by altering the thickneſs and ſtrength of the ſpring in different parts, but, if we ſubſcribe to the opinion of M. Berthoud, in a ſpring uniform in every reſpect throughout, by altering the length and number of turns. He infers, that a certain length and number of turns may be given to an uniform ſpiral ſpring which will make it perfectly iſochronal. This latter principle, however, does not appear to have been verified by any ſatisfactory experiments. According to the inferences deduced from the preceding inveſtigations, three ſpiral ſprings, which are not iſochronal when acting ſingly, may be ſo united by properly adjuſting their points of quieſcence, that their combined action ſhall cauſe the balance to perform its vibrations in any two arcs of unequal lengths in the ſame time.

vibration

n to be deſcribed in leſs time than the ſhorter
whenever the arc of vibration is diminiſhed,
-keeper will loſe; and this error in the rate
e corrected by adjuſting the points of quieſ-
f the auxiliary ſprings in the reſpective latter
; of vibration at ſuch a diſtance ON = OQ
and 86.) from the point of quieſcence of
ince ſpring, as correſponds with the error
l to be rectified.* In like manner, if the
r of the balance ſpring ſhould be ſuch that
ution of the arc of vibration cauſes an acce-
of the daily rate, this error will be corrected
ing the points of quieſcence of the auxiliary
in the firſt ſemiarcs of vibration, (fig. 82 and
heir proper diſtances from the point of quieſ-
f the balance ſpring. It is, however, to be
d, that if the balance ſpring ſhould be
latter kind here aſſumed, and the points of
ice of the auxiliary ſprings, either by any
erangement in their poſition, or by an ad-
t of them for the purpoſe of experimental
ion, ſhould be placed in the latter ſemiarcs
ation, the effect of this poſition would be
leration of the rate, whenever the ſemiarc of
n is diminiſhed: and this effect would be
d on a double account: firſt, from the aſ-
iature of the balance ſpring, which diſpoſes
ſcribe the ſmaller arcs in leſs time than the
ircs of vibration: and ſecondly, from the
of the points of quieſcence of the auxiliary

But it is evident from the preceding
rations, that although the balance ſpring
not be iſochronal, yet the regulary of the

poſition of the points of quieſcence of the auxiliary
here underſtood to be altered, by affixing the arms GO,
rently on the axes TR, FS; the quieſcent poſition of the
ing no ways changed.

time-

the annuity x, in the ſame time t. Hence, by the queſtion, $x \times \frac{r^t - 1}{r - 1} = p \times r^t$, and $x = 864{\cdot}09973$l. $= 864$l. 1s. 11d$\frac{1}{2}$, the 865th part of which is 2l. 7s. 4d. nearly the daily expenditure required.

According to one or other of theſe methods was the anſwer given by Meſſrs. Collins, Lowry, Marrat, May, Merones Minor, *and* Thornoby.

II. QUESTION 184, *anſwered by Mr.* John Johnſon, *Birmingham.*

Put $a = 88$ yards the verſed ſine of the ſegment, $b = 9680$, the ſquare yards in two acres, and $x =$ the diam. of the circle. Then, by Dr. Hutton's Menſuration, Rule 6th, for finding the area of a circular ſegment, $\frac{2}{3}a\sqrt{(ax - \frac{1}{4}a^2)} = b$, this reduced gives $x = 130{\cdot}14375$, therefore $131{\cdot}14375$ is the diam. of a circle whoſe circumference is the outward boundary of the circular part of the wall, the verſed ſine of which is $88 + 1 = 89$; hence the area is found, by the above rule, $= 9868{\cdot}989$, from which deducting 9680, the area of the garden, we have $188{\cdot}989$ ſquare yards, area of the baſe of the wall, $= 1700{\cdot}901$ ſquare feet, which multiplied by 7, the height, gives the content of the wall $= 11948{\cdot}307$ ſolid feet.

Other anſwers were alſo received from Meſſrs. Barron, Blackwell, Buffham, Davis, Lowry, Marrat, May, Merones Minor, Squire, *and* Thornoby.

III. QUESTION 185, *anſwered by Miſs* Suſan May.

It is evident, from Art. 3388 of Martin's Inſtitutions, that the force of gravity, at the equator, is to that, at the pole, as 230 to 231, and that the increaſe, in any latitude, is nearly as the ſquare of the ſine of that latitude; therefore, if the force, at the equator, be 1, the increaſe of gravity, at the pole, will be $\frac{1}{230}$, and $(\text{radius})^2 : (\text{ſine})^2\ 30°$, or $1 : \frac{1}{4} :: \frac{1}{230} : \frac{1}{920}$ the increaſe, in latitude 30°. Hence the force, in latitude 30°, is to that, at the equator, as 921 to 920.

Similar to this is the propoſer's ſolution.—By Emerſon's Geography, Sec. I. Prop. X. Cor. 2nd, the force of gravity at the pole is $\frac{231}{230}$, when that at the equator is 1, and the diminution thereof at the equator $= \frac{1}{230}$. Hence, by Cor. 1ſt, to the ſame propoſition, $1^2 : (\sqrt{3} \div 2 = \text{cos. lat.})^2 :: \frac{1}{230} : \frac{1}{920}$, the diminution in the lat. 30°, which ſubtracted from $\frac{231}{230}$ leaves $\frac{921}{920}$ for the force required.

The

The ſame, anſwered by Mr. John Barron, *Schoolmaſter, Spilſby.*

If g denotes the gravity at the equator ($= 1$), and y the gravity under any other latitude λ (80°), then $y = (1 + 0{\cdot}00528\,48 \text{ fine}^2 \lambda) \times g$. See Dr. Hutton's Dictionary, Art. Gravity.

Ingenious ſolutions were received from Meſſrs. Blackwell, Buffham, Johnſon, Lowry, Marrat, Squire, *and* Thornoby.

IV. QUESTION 186, *anſwered by Mr.* Thomas Hewit, *Teacher of the Mathematics, Featherſtone-ſtreet, London.*

Let $r =$ the radius of the earth, and $R =$ that of the moon's orbit. Put $p = 3{\cdot}14159$ &c. and $d = 16\frac{1}{12}$ the ſpace, through which a heavy body, at then ſurface of the earth, deſcends in the firſt ſecond of time, the $2d$ will be the meaſure of the force of gravity at the ſurface, therefore the velocity, per ſecond, in a circle at the ſurface of the earth will be $\sqrt{2rd}$, and the time of a revolution $= p \times 2r \div \sqrt{2rd} = p\sqrt{(2r \div d)}$ ſeconds. Becauſe the force of gravitation, above the ſurface, is known to vary according to the ſquare of the diſtance inverſely, we have $r^{-\frac{1}{2}} : R^{-\frac{1}{2}} :: \sqrt{2rd}$, the velocity per ſecond at the ſurface, to $(r^{\frac{1}{2}} \div R^{\frac{1}{2}}) \sqrt{2rd} = r\sqrt{2d} \div R)$ the velocity in a circle whoſe radius is $R = 5189{\cdot}3$ feet, nearly, per ſecond. Alſo, becauſe the ſquares of the periodic times vary as the cubes of the diſtances directly, the periodic times will vary as the ſquare roots of the cubes of the diſtances, therefore $r^{\frac{3}{2}} : R^{\frac{3}{2}} :: p\sqrt{(2r \div d)}$, the periodic time, at the ſurface, to $(R^{\frac{3}{2}} \div r^{\frac{3}{2}}) \sqrt{(2r \div d)} = p \times (R \div r) \times \sqrt{(2R \div d)}$, the time of a revolution in a circle, whoſe radius is $R = 640041{\cdot}9$ ſeconds. See Simpſon's Fluxions, Vol. I. pa. 240, 241.

Neat ſolutions were alſo received from Meſſrs. Barron, Blackwell Buffham, Johnſon, Hill, Marret, May, Merones Minor, *and* Squire.

V. QUESTION 187, *anſwered by* Merones Minor.

Fig. 459, Pl. 25. Let NBG be the baſe of the cone, CE the length of the ſhadow, C being the centre; draw EB a tangent to the circle at B, and CB, CA $\perp$ to EB, EC reſpectively. Then NA is a quadrant $= 3{\cdot}75$, and $CA = r = 7{\cdot}5 \div 3{\cdot}14159$; put

put x = arch AB, then AB + BE = arch + its cotangent = $18 - 3{\cdot}75 = 14{\cdot}25 = b$. Now, by Emerſon's Trig. B. I. Prop. 16. Cor. 2nd, the cotan. of AB = $\frac{r^2}{x} - \frac{x}{3} - \frac{x^3}{45r^2} - \frac{2x^5}{945r^4}$ &c; therefore $\frac{r^2}{x} - \frac{x}{3} - \frac{x^3}{45r^2} - \frac{2x^5}{945r^4}$ &c. $= b$; and, by reverſion of ſeries, $x = \frac{r^2}{b} + \frac{2r^4}{3b^3} + \frac{13r^6}{15b^5} + \frac{1318r^8}{945b^7}$ &c. = ·40772; Then BE $= b - x = 13{\cdot}84228$, and EC = $\sqrt{(BE^2 + BC^2)} = 14{\cdot}04664$.

Again, by Mayer's Tables, the ſun's declination, at the given time, was 14° 11′ 22·5″ N, hence, per ſpheries, the ſun's altitude was 29° 36′ 2·1″, and then, radius: tang. ſun's alt. :: EC : 7·9798, the height of the cone; and finally, $\frac{15^2}{4\rho} \times \frac{\text{height}}{3}$ (putting ρ = 3·14159 &c.) = 47·626 feet, nearly, the content required.

The ſame, anſwered by Mr. Thomas Squire, *Baldock.*

Let C be the centre of the cone's baſe, ABD a quadrant of its circumference, ABE part of the ſtring = 14·25 feet, B the point where the ſtring quits the circumference, and E the extremity of the cone's ſhadow.—Make Cα = Cβ = 1, and draw the tangent line $\beta\varepsilon$ parallel to BE. Then, as 15 feet, the circumference of the cone's baſe, is to 3·14159 × 2, ſo is ABE = 14·25, to $\alpha\beta\varepsilon$ = 5·960805 = the ſum of the arc $\alpha\beta$ and its cotang. $\beta\varepsilon$ to radius 1. Now, I think the beſt though indirect method to find the ∠ ACB is, to aſſume ſome ∠ near the true one, then by the 10th and 12th of Dr. Hutton's Mathematical Tables, after a few trials, the ∠ ACB (= CEB) will be found = 9° 47′ $7\frac{2}{3}$″, hence, as ſine of 9° 47′ $7\frac{2}{3}$″ : BC = 2·3873262 :: radius 1 : CE = 14·04655 the length of the ſhadow from the centre of the cone's baſe. Then, by having the latitude day and hour given, the ſun's altitude is found = 29° 36′ $1\frac{1}{2}$″ or by allowing for refraction, ſemidiameter, and parallax, the true altitude of the ſun's upper limb = 29° 53′ $25\frac{1}{2}$″. Then, as coſine of 29° 53′ $25\frac{1}{2}$″ : CE :: ſine of 29° 53′ $25\frac{1}{2}$″ : 8·0769 the altitude of the cone. Hence the content = [illegible]·2558 feet.

The

The ſame, anſwered by Mr. John Johnſon, *Birmingham.*

Let ABHGN repreſent the circular baſe of the cone, and EBANGHE the chord; let AG be a diam. drawn ‖ to HB, the line joining the points of contact, and let C be the centre of the circle. Join CB and CE and make BC ⊥ to AC. Then the circumf. GNA is $= 7\frac{1}{2}$ feet, which taken from 36 feet, the length of the cord, leaves $28\frac{1}{2}$ feet, the half of which is $14\frac{1}{4} =$ the ſum of the arc AB and tangent BE. Put $x =$ BE, and $r =$ rad. of the circle $= 2{\cdot}39$, then $\sqrt{(x^2 + r^2)} =$ CE, and by ſim. △s, CE : BE :: CB : BF $= xr \div \sqrt{(x^2 + r^2)} =$ Cc, and Ac $= r - xr \div \sqrt{(x^2 + r^2)}$. Hence, by rule 5th of Dr. Hutton's Menſuration, 8vo. Ed. the length of the arc AB is $= 2r \times \sqrt{\frac{3\sqrt{(x^2+r^2)} - 3x}{5\sqrt{(x^2+r^2)} + x}}$. Therefore $2r \times \sqrt{\frac{3\sqrt{(x^2+r^2)} - 3x}{5\sqrt{(x^2+r^2)} + x}} + x = 14\frac{1}{4}$. By the method of trial and error x is found $= 13{\cdot}81$, and CE $= \sqrt{(BE^2 + BC^2)} = 14{\cdot}05$. Now, from the data, I find the ſun's altitude $= 29°\ 36'$, and thence the content of the cone 47·5 feet nearly.

The ſame, anſwered by Mr. Wm. Francis.

Let NE repreſent a right line drawn from the ends of the chord, through the centre of the cone's baſe C, and terminating with its edge at N. Then will BE be a tangent to the cone's baſe, which ſuppoſe $= 10$. Now the circumference being 15, the radius of the baſe will be 2·38736; whence CE $= 10{\cdot}281$. And as CE : rad. :: BE : ſine ∠ BGE $= 76°\ 34'\ 20''$ Therefore ∠ BCN $= 103°\ 25'\ 40''$ conſequently the length of the arch BAN $= 4{\cdot}30949$. But $4{\cdot}30949 + 10 = 14{\cdot}30949$, which ſhould be 18, therefore the error is 3·69051, too little. Again, if we ſuppoſe BE $= 15$, the error will be 1·126736, too much; whence, by the rule of Poſition, BE $= 13{\cdot}83$ very nearly, and CE $= 14{\cdot}084$. Now, the ſun's alt. corrected for the given time, was $29°\ 56'$; hence, as coſine $29°\ 56' : 14{\cdot}034 ::$ ſine $29°\ 56' : 8{\cdot}08$ feet; therefore the ſolidity was 48·223 feet.

According to one or other of theſe methods the queſtion was anſwered by Meſſrs. Barron, Blackwell the propoſer, Cunliffe, Hill, Lowry, Marrat, May, *and* Thornoby.

VI. QUESTION 188, *anſwered by Mr.* James Cunliffe, *Bolton.*

Let the fractions be denoted by x, y, and z. Then, by the queſtion, $1 - xy = a^2$, and $1 - xz = b^2$, whence $x = (1 - a^2) \div y = (1 - b^2) \div z$, or $z = y \times (1 - b^2) \div (1 - a^2)$; by means of which, $1 - yz = 1 - y^2 \times (1 - b^2) \div (1 - a^2) = a$ ſquare, per queſtion. Aſſume $1 - ry$ for its root, that is, make $1 - y^2 \times (1 - b^2) \div (1 - a^2) = (1 - ry)^2 = 1 - 2ry + r^2y^2$, which being reduced gives $y = 2r \times (1 - a^2) \div (r^2 \times (1 - a^2) + (1 - b^2))$, where a, b, and r may be taken at pleaſure.

Or the ſolution may be as follows. Put $1 - xy = a^2$, $1 - xz = b^2$, and $1 - yz = c^2$; by proceeding as before, $1 - y^2 \times (1 - b^2) \div (1 - a^2) = c^2$, which gives $y^2 = (1 - a^2) \times (1 - c^2) \div (1 - b^2)$. Whence it is plain that a, b, c may be expounded by any proper rational fractions of ſuch a nature as that $(1 - a^2) \times (1 - c^2) \div (1 - b^2)$ may be a rational ſquare, or that $1 - a^2$, $1 - b^2$, $1 - c^2$, may all be rational ſquares. It therefore appears, that three fractions may be found of ſuch ſort, that if the product of every two be ſubtracted from unity, the remainders ſhall be equal, reſpectively, to given ſquares. But the given ſquares muſt be ſubject to ſome ſuch reſtrictions as thoſe above ſpecified.

Example. Put $a = \frac{3}{5}$, $b = \frac{4}{5}$ and $c = \frac{5}{13}$: then $y^2 = \frac{(1 - a^2) \cdot (1 - c^2)}{(1 - b^2)} = \frac{256}{169}$, or $y = \frac{16}{13}$; whence $x = \frac{13}{25}$, and $z = \frac{9}{13}$, being three fractions that will anſwer the queſtion.

The ſame, anſwered by Miſs Suſan May.

Let x, y, and z repreſent the fractions required; then, by the queſtion, $1 - xy$, $1 - xz$, and $1 - yz$ muſt be ſquares. Put $1 - xz = m^2$, and $1 - yz = n^2$. From the firſt of theſe equations $x = (1 - m^2) \div z$, and from the ſecond $y = (1 - n^2) \div z$; therefore $1 - xy = 1 - (1 - m^2) \times (1 - n^2) \div z^2$, which muſt be a ſquare, that is, $z^2 - (1 - m^2) \times (1 - n^2)$ muſt be a ſquare. Aſſume its root $= z - r$, then $z^2 - (1 - m^2) \times (1 - n^2) = z^2 - 2rz + r^2$, and, by reduction, $z = \frac{(1 - m^2) \times (1 - n^2) + r^2}{2r}$, where m, n, and r muſt be fractions leſs than unity.

If $m = \frac{1}{2}$, $n = \frac{1}{3}$, and $r = \frac{1}{2}$, then $z = \frac{33}{56}$, $x = \frac{27}{33}$, and $y = \frac{32}{33}$; hence $1 - xy = (15 \div 33)^2$, $1 - xz = (3 \div 6)^2$ and $1 - yz = (2 \div 6)^2$.

The

The ſame, anſwered by Mr. W. Marrat, *Boſton.*

Let x, y, and z, denote the required fractions, and put $1 - xy = r^2$, $1 - xz = v^2$, and $1 - yz = w^2$; then, by tranſpoſition,

$$\left.\begin{aligned} -r^2 &= xy \\ -v^2 &= xz \\ -w^2 &= yz \end{aligned}\right\}$$ and by reſolving into factors according to Leſlies' method $$\left\{\begin{aligned} (1+r)\cdot(1-r) &= x \times y \\ (1+v)\cdot(1-v) &= x \times z \\ (1+w)\cdot(1-w) &= y \times z \end{aligned}\right.$$

that is, by aſſumption $$\left\{\begin{aligned} 1 - r &= my \text{ and } 1 + r = x \div m \\ 1 - v &= nz \text{ and } 1 + v = x \div n \\ 1 - w &= pz \text{ and } 1 + w = y \div p \end{aligned}\right\}$$ Hence

$$\left.\begin{aligned} 1 - my &= r = x \div m - 1 \\ 1 - nz &= v = x \div n - 1 \\ 1 - pz &= w = y \div p - 1 \end{aligned}\right\}$$ and $$\left\{\begin{aligned} x &= 2n - m^2y \\ x &= 2n - n^2z \\ y &= 2p - p^2z \end{aligned}\right.$$ From theſe equations, by proper reduction, we get

$$\left.\begin{aligned} y &= (2n^2p + 2mp^2 - 2nm^2p^2) \div (n^2 + m^2p^2) \\ z &= (2m^2p - 2m + 2n) \div (n^2 + m^2p^2) \\ x &= (2nm^2p^2 - 2n^2m^2p + 2mn^2) \div (n^2 + m^2p^2) \end{aligned}\right\}$$ If $m = 1$, $n = 3$, $p = 4$ then $y = 8 \div 25$, $z = 12 \div 25$, and $x = \frac{44}{25}$.

The ſame, anſwered by Mr. I. H. Swale.

Aſſume $v \div x$, $v \div y$, and $v \div z$ for the required fractions. Then the conditions of the queſtion are, that $1 - v^2 \div xy$, $1 - v^2 \div xz$, and $1 - v^2 \div yz$ ſhall be perfect ſquares. That is, $(xy - v^2) \div xy$, $(xz - v^2) \div xz$, and $(yz - v^2) \div yz$ muſt be perfect ſquares, in order to which it is neceſſary that xy, xz, and yz muſt be ſquares. Hence, then, we have to determine three ſquares xy, xz, and yz ſuch that ſubtracting the given ſquare, v^2, from each, the remainders ſhall be perfect ſquares. Let $xy - v^2 = m^2$, then $xy = m^2 + v^2$, a ſquare alſo; let it be equal $r^2 + m^2 + 2rm$, then will $m = (v^2 - r^2) \div 2r$, $m^2 = (v^2 - r^2)^2 \div 4r^2$, and $xy = m^2 + v^2 = (v^2 - r^2)^2 \div 4r^2 + v^2 = (v^2 + r^2)^2 \div 4r^2$. In like manner we have $xz = (v^2 + s^2)^2 \div 4s^2$, and $yz = (v^2 + t^2)^2 \div 4t^2$, all ſquares, where v, r, s, and t may be taken at pleaſure.

But, it yet remains to determine the reſpective values of x, y, and z. For brevity, put $(v^2 + r^2)^2 \div 4r^2 = a^2$, $(v^2 + s^2)^2 \div 4s^2 = b^2$, and $(v^2 + t^2)^2 \div 4t^2 = c^2$. Then $x = a^2 \div y = b^2 \div z$; that is, $y = z \times (a^2 \div b^2) = c^2 \div z$; or $z^2 = b^2c^2 \div a^2$, and $z = bc \div a$; hence $x = b^2 \div z = ab \div c$, and $y = c^2 \div z = ac \div b$. That is, by reſtoring the values of a, b, and c, $x = t \times (v^2 + r^2) \times (v^2 + s^2) \div 2rs \times (v^2 + t^2)$, $y = s \times (v^2 + r^2) \times (v^2 + t^2) \div 2rt \times (v^2 + s^2)$, and $z = r \times (v^2 + s^2) \times (v^2 + t^2) \div 2st \times (v^2 + r^2)$. Therefore, by proper ſubſtitution, the fractions become

$$\frac{2rsv \times (v^2 + t^2)}{t \times (v^2 + r^2) \times (v^2 + s^2)}, \quad \frac{2rtv \times (v^2 + s^2)}{s \times (v^2 + r^2) \times (v^2 + t^2)}, \text{ and}$$

$$\frac{2stv \times (v^2 + r^2)}{r \times (v^2 + s^2) \times (v^2 + t^2)}.$$

Let $v = 1$, $r = 2$, $s = 3$, and $t = 4$ and the fractions will be $51 \div 50$; $32 \div 51$; and $6 \div 17$.

Meſſrs. Barron, Hill, Lowry, M'Donald, *and* Merones Minor, *ſent neat ſolutions to this queſtion.*

VII. QUESTION 189, *anſwered by Mr.* Wm. Peacock, *Birmingham.*

Analyſis. Let CF (fig. 460, Pl. 25.) repreſent the meridian, AC, CB the courſes, and A, B the poſition of the ſhips, at the time when the rock at F is in a direct line between them. In the $\triangle$ ACB there are given $\angle$ ACF = BCF = 16° 52½', AC — CB = 3·75, and CF = 9 leagues.—About the $\triangle$ ACB let a circle be deſcribed and continue CF till it meets it at Q; draw the diam. QSD, which will be $\perp$ to the baſe AB, and join DC, DB, and DF; and upon AC demit the $\perp$ DE. Then, it is well known that, CE is = to half the diff. of the ſides, and DC is $\perp$ to QC, therefore the $\triangle$ DCE is given in magnitude, conſequently D is a given point, but F is a given point and the $\angle$ FBD = ACD is given, being the complement of half the given $\angle$: Hence this

CONSTRUCTION. On the line biſecting the given $\angle$ take CF of the given length, and EC = half the diff. of the ſides; draw DC, DE $\perp$ to CF, CA reſpectively, interſecting in D, and upon the line joining the points D, F, deſcribe a ſegment of a circle to contain an $\angle$ = the complement of half the given one, and let it meet CB in B; draw BF and it is done.

CALCULATION. Let O be the centre of the circle and draw the radii OD, OF, and OB and on DF demit the $\perp$ OP and produce it to meet CB in P' and CF in Y. Then in the right $\angle$d $\triangle$ CED, EC and $\angle$ECD are given to find DC, and in the right $\angle$d $\triangle$ DFC, FC and DC are given to find DF and the $\angle$ DFC. Again, in the right $\angle$d $\triangle$s FPO, FPY, PF and the $\angle$s FOP, PFY are known to find the ſides FO, PO, FY, PY and the $\angle$ FYP, hence YO becomes known, and alſo CY. In the $\triangle$ YCP, CY and all the $\angle$s are known to find CP' and YP', then in the $\triangle$ BOP', OP' and OB are known to find BP' and the $\angle$ BOP', hence AC, CB are known, and in the iſoſceles $\triangle$ BOF, $\angle$ BOF, and BO, FO are known to find BF, and becauſe

cauſe CB : CA :: BF : AF, AB is found = 6·70956, and the rates of ſailing, per hour, = 4·656 and 3·156 leagues reſpectively.

The ſame, anſwered by Mr. James Cunliffe.

Fig. 461, Pl. 25. Let P repreſent the port, R the rock, A the place of the ſwifter veſſel, B that of the other. It is evident, from the queſtion, that in the △ APB, there are given PR = 9 leagues, being the length of the line biſecting the vertical ∠ APB, AP — BP = $3\frac{3}{4}$ leagues, and ∠ APR = BPR = 16° $52'\frac{1}{2}$. Draw R*m* parallel to the ſides, alſo *mr* ⊥ to PR. The △ P*m*R is iſoſceles, wherefore, as radius : ſecant of ∠ *m*P*r* :: P*r* = ½ PR : P*m* = R*m* = 4·7024874, which put = *a*, alſo put AP — BP = A*m* — B*m* = 3·75 = *d*, and B*m* = *x*, then A*m* = *d* + *x*. The △s B*m*R, A*m*R are ſimilar, therefore B*m* : R*m* :: R*m* : A*m*, whence B*m* × A*m* = (R*m*)², or $x^2 + dx = a^2$, which gives $x = \sqrt{(a^2 + \frac{1}{4}d^2)} - \frac{1}{2}d = 3{\cdot}1875105$; whence PB = 7·8899979, and PA = 11·6399979. And it is eaſily found that the ſwifter veſſel muſt have ſailed at the rate of 4·656 leagues per hour, and the other at the rate of 3·15 leagues in the ſame time. AB or the diſtance of the two veſſels at the time mentioned may be found by trigonometry and is 6·70927 leagues.

Or the ſolution might have been as follows :

Draw RE, R*e* ⊥ to the ſides PB, PA, and upon PA take P*b* = PB, alſo take *eb'* = *eb*, and draw R*b'*, R*b*. The △ *b*R*b'* is iſoſceles, by the conſtruction, theref. ∠ A*b'*R = P*b*R = PBR, and conſeq. ∠ AR*b'* = APB, the given vertical ∠. In the right ∠d △ PR*e*, the hypothenuſe PR is given, together with the ∠ RP*e*, theref. R*e* becomes known ; wheref. in the △ A*b*R, there are given the ⊥ R*e*, and A*b* = PA — PB = PA — P*b*, the given difference of the ſides, and ∠ AR*b'* = A*b*R — *b*AR = APB, which is the 15th problem in the Appendix to Simpſon's Algebra, and the calculation may be performed, as there directed, without the aid of Algebra.

From the latter method of ſolving the preceding problem, it appears that, $\sqrt{}$, S — *s*, and L is reducible to A — *a*, B and P, or $\sqrt{}$, P and *m* — *n*; the former of which takes place when the ∠ B exceeds a right angle ; and the latter data take place when both the angles A and B are acute.

The ſame, anſwered by Mr. John Barron, *Spilſby*.

Fig. 461, Pl. 25. Since both ſhips had been out $2\frac{1}{2}$ hours, and the rate of ſailing of the one exceeded that of the other by

1¾ leagues per hour, if x = PB, the diſtance ſailed by the leaſt, the diſtance ſailed by the other, that is, AP will be $= x + 3{\cdot}75$, and the diſtance between the port and the rock, that is, PR is = 9 leagues; alſo half the vertical $\angle$, or half the $\angle$ contained between the courſes of the ſhip, is given $= 16^{\circ}\ 52'\ 30''$, the coſine of which call c. Then, by Prop. 6, Cor. 5, pa. 105 of Emerſon's Trigonometry, as $2AP \times PB : PR \times (AP + PB) ::$ radius : coſine of half $\angle$ APB, that is, as $2x^2 + 7{\cdot}5x : 18x + 33{\cdot}75 :: 1 : c$, or, by multiplying means and extremes $2cx^2 + 7{\cdot}5cx = 18x + 33{\cdot}75$; hence $x = 7{\cdot}889985$ = PB and AP is = $11{\cdot}889985$, conſequently the rates of ſailing are $3{\cdot}15599$, and $4{\cdot}65599$ leagues, per hour, and their diſtance, at the end of 2½ hours, is eaſily found = 6·70988 leagues.

The ſame, anſwered by Mr. John Johnſon.

Having, like Mr. Peacock, found that D is a given point, Mr. Johnſon then draws the lines S, E, which is well known to be ∥ to CF, and therefore given by poſition, alſo the $\angle$ FSD being a right angle, the following *Conſtruction* is evident.

Take EC = ½ diff. of the ſides, and draw ED, CD ⊥ to AC, FC reſpectively, to meet at D; draw ES ∥ to CF, and on FD deſcribe a ſemicircle meeting ES at E and draw FS to meet AC, CB, at A and B, and it is done; for AC, CB are the diſtances ſailed by the two ſhips, and AB is their diſtance aſunder.

Ingenious ſolutions were alſo received from Meſſrs. Blackwell, Buffham, Davis, Hill, Lowry, Marrat, May, Merones Minor, Squire, *and* Swale.

VIII. QUESTION 190, *anſwered by Mr.* Geo. Buffham, *Boſton.*

Let AD (Fig. 462, Pl. 25.) be a portion of the horizon, AB a part of the plane making the $\angle$ BAD = 33½°. Draw BC ⊥ to AB, and AC ⊥ to AD, meeting in C, which will be the centre of gravity of the pyramid, as is evident from the principles of mechanics. It is alſo evident that BC = ¼ of the height of the pyramid = 2½, and $\angle$ ACB = BAD, therefore, by trigono. radius : BC : : tan. ACB : AB = 1·489243. In fig. 463, pl. 25, let EFG repreſent the baſe of the pyramid, *b* its centre, *a* the point where the perpendicular from the centre of gravity meets the ſide, agreeing with the point A in fig. 462, and *de* a right line in the horizon parallel to the interſection of that plane and the plane of the horizon. Then *ab* = AB, and *bc* = the radius of the inſcribed

ſcribed circle: whence by trig. *ab* : *bc* : : rad. : cof. ∠ *abc* = 4° 15′½ = ∠*eaG* (ſince by the nature of the queſtion *ba* is ⊥ to *de*) = the angle the ſide EG made with *ed*.

The ſame, anſwered by Mr. Wm. Marrat, *Boſton.*

Fig. 464, Pl. 25. Let EPF repreſent the pyramid (whoſe axis OP = 9 feet, and each ſide of its baſe EL, LF, FE = 5 feet,) ſtanding on the plane ABCD, which makes an angle (BAY) of 33°½ with the horizon; alſo let G be its centre of gravity, which is known to be at ¼ of its height, and let GA be ⊥ to the horizon, which, in the poſition of the ſolid, will paſs through the edge of its baſe as at S. Join OS, OL, OE and through S draw QR parallel to the horizon, or at right angles to OS.

Then, as radius : OS (2·25) : : tan. ∠ OGS (33°½) : OS; as ſine ∠ EOL (120°) : EL (5) : : ſine OEL (30°) : OL; and as OS : ſine ∠ OLS (30°) : : OL : ſine ∠ OSL = 75°¾; hence OSQ — OSL = 90° — 75° ¾ = 14° ¼ = ∠LSQ, the angle which one ſide of the pyramid's baſe made with a horizontal line QR.

The ſame, anſwered by Mr. Thomas Squire, *Baldock.*

Let ACB (Fig. 465, Pl. 25.) be the baſe of the pyramid ſtanding on the inclined plane, D its vertex, and DO the axis drawn perpend. to the baſe, the point O is equally diſtant from the points A, B, C, and, by the property of the equilateral △, AO = 2·8866. Let G be the centre of gravity of the fruſtum and draw GF ⊥ to the horizon to meet the inclined plane at S and draw OSE. Now it is evident that the pyramid will ſupport itſelf on the inclined plane while the line GF falls within its baſe, and therefore when it can juſt ſtand, the point S will fall in the ſide of the baſe AC. By a known property OG = ¼OD = 2·25 and ∠ OGS = FEO = 33°½, therefore SO is found = 1·4892, and in the △ AOS, AO, SO and the ∠ SAO (= 30°) are known; hence the ∠ ASO = 75° 45′, and its complement 14° 15′ is the angle of inclination required.

True ſolutions to this queſtion were received from Meſſrs. Barron, Blackwell, Burdon, Cunliffe, Johnſon, Hill, Lowry, May, M'Doneld, Merones Minor, *and* Thornoby.

IX. QUESTION 191, *anſwered by Miſs* Suſan May.

Fig. 466. Pl. 25. Let BE be the upright piece and AB the the horizontal one, and let AD be the oblique piece required.

Com-

Compleat the parallelogram ABDF and on AD drop the ⊥ FG. Then if AD repreſent the abſolute ſtrength of the oblique piece, AF will be that part of it which is exerted againſt AB in the direction AF. Now, by Prop. 55, Emerſon's Mechanics, 8vo. the ſtrength of AD is reciprocally as its length; therefore, by the property of the leaver, AB × AF ÷ AD muſt be a maximum: but AB × AF = DF × AF = AD × GF, therefore GF muſt be a max. which it evidently will be when AF = FD = AB = $4\frac{1}{2}$; therefore AD = AB $\sqrt{2}$ = 6·364 feet the length required.

Alſo, by Mr. Wm. Francis.

Let BE repreſent the upright piece, and AB the horizontal. Then, ſince by the principles of Mechanics the forces on each are directly as the ſines of their oppoſite angles, it follows, that theſe angles muſt be equal for AD to ſuſtain the greateſt weight poſſible. But equal angles are ſubtended by equal ſides. Hence, make BD = to AB, then will AD be the piece whoſe length is required.

And $\sqrt{AB^2 + AB^2}$ = 6·36396 = AD.

The ſame, anſwered by Merones Minor.

Suppoſe AB the horizontal piece, BE the upright piece, and AD the required oblique piece. Draw BG, GT ⊥ to AD, AB. Then, by Em. Mec. 8vo, Prop. 45, Cor. 2, the force acting at A in the direction DA, to turn AB about the point B, is as BG, and ibid. Prop. 55, the ſtrength of AD is as 1 ÷ AD; that is, the force required will be as AB × BG ÷ AD, AB being given.

By ſim. △ˢ, AD : AB :: BG : GT, a max. by the queſtion:

Hence this CONSTRUCTION. On AB deſcribe a ſemicircle and from the centre T demit the ⊥ TG to meet the ſemicircle in G, then through G draw AGD, the poſition of the oblique piece required. Then AD = AB $\sqrt{2}$ = 6·364 feet, nearly.

This queſtion was alſo ingeniouſly anſwered by Meſſrs. Blackwell, Buſſham, Cunliffe, Johnſon, Hill, Lowry, Marrat, *and* Thornoby.

X. QUESTION 192, *anſwered by* James Cunliffe.

Fig. 467, Pl. 25. Let CSB be the ſemicycloid, CF its axis, ESD a tangent to the curve at the point S, meeting CF, FB in E and D reſpectively. Draw the ſemiordinate GS perpendicular to the axis, cutting the periphery of the circle in I, alſo draw

SW

SW perpend. to FD. Put CF $= 2a$, CG $= x$, and arch CI $= z$. Draw the chord CI, which will be parallel to the tangent ES, per prop. 3. Emerſon on the Cycloid, and per corol. to prop. 2. ibid. GS $=$ GI $+$ arch CI $= \sqrt{(2ax - x^2)} + z$. Then, by reaſon of the parallels CI and ES, GI : GC :: GS : GE $=$ (GC $\times$ GS) $\div$ GI $= x(\sqrt{(2ax - x^2)} + z) \div \sqrt{(2ax - x^2)}$; alſo GC : GI :: SW : WD $=$ (GI $\times$ SW) $\div$ GC $= (2a - x)\sqrt{(2ax - x^2)} \div x$. Now it appears *from the Scholium to Theorem* 19, *Simpſon on the maxima et minima*, that the length FW of the greateſt cylinder that can be inſcribed in the ſolid formed by the revolution of the ſemicycloidal ſpace CSB about the baſe FB will be $= \frac{1}{3}$WD, that is, $\sqrt{(2ax - x^2)} + z = \frac{2a - x}{2x}\sqrt{(2ax - x^2)}$, whence $z = \frac{2a - 3x}{2x}\sqrt{(2ax - x^2)}$;

from which equation, by the method of trial and error, z, in this caſe will be found to be an arc of $31°\cdot5055 = 51° 30'\frac{1}{3}$ nearly. By means whereof GS $=$ FW $= 1\cdot681609a$, the length of the cylinder, and GF $=$ WS $= 1\cdot6224384a$, the ſemidiameter of its baſe, and therefore its ſolid content is $10\cdot764785a^3$ nearly.

Moreover the length GF of the greateſt cylinder that can be inſcribed in the ſolid formed by the revolution of the ſemicycloidal ſpace CSB about its axis CF will be $= \frac{1}{3}$GE; that is, $a - x = \frac{x(\sqrt{(2ax - x^2)} + z)}{2\sqrt{(2ax - x^2)}}$, whence $z = \frac{4a - 3x}{x}\sqrt{(2ax - x^2)}$,

from which equation, by the method of trial and error, z in this caſe will be found to be an arc of $89°\cdot9287 = 83° 55'\cdot722$, by means of which the length FG of the inſcribed cylinder will be found to be $1\cdot105766a$, and GS $= 2\cdot4592032a$, the ſemidiameter of its baſe, and from theſe dimenſions its ſolidity is found to be $21\cdot0088824a^3$, which is almoſt double the content of the other. Therefore the greateſt cylinder that can be cut out of the ſolid formed by the revolution of the ſemicycloid about its axis, exceeds the greateſt cylinder that can be cut out of the ſolid formed by the revolution of the ſame ſemicycloid about its baſe by $10\cdot2441574a^3$, where a denotes the radius of the cycloid's generating circle. If the propoſer means that the diameter of the generating is 16, then $a = 8$, and $10\cdot2441574a^3 = 5245\cdot0085888$ the required difference of the cylinders.

☞ If any one ſhould be inclined to examine the approxima- in the preceding ſolution, the following obſervations may to facilitate the labour, and be of uſe in other caſes of the when they occur in the ſolution of problems.

In the firſt caſe we have ſhewn that $x = \frac{2a - 3x}{2x}\sqrt{(ax - x^2}$ or arc CI $= \frac{2a - 3x}{2x} \times$ fine GI, when x denotes the verſed ſine and a the radius of the arc CI. Now it is very obvious that any arc is greater than its fine, wherefore $\frac{2a - 3x}{2x}$ muſt be greater than unity; put it equal to $1 + v$, viz. $\frac{2a - 3x}{2x} = 1 + v$, this being reduced gives $x = \frac{2a}{5 + 3v} = \frac{4a}{10 + 6v}$; from whence it appears that becauſe v is poſitive, x muſt be leſs than $\frac{4a}{10}$. And, in the ſame manner, in the ſecond caſe, it may be ſhewn, that x muſt be leſs than a.

The ſame, anſwered by Miſs Suſan May.

Fig. 467. Pl. 25. Let ACB repreſent a ſection of the ſolid formed by the revolution of the cycloid about its axis CF, and TSWN a ſection of the greateſt inſcribed cylinder. At S draw the tangent SE, to meet the axis produced at E; then, by B. II. Prop. 5, Cor. Emerſon on curve lines, when the cylinder is a max. the ſubtangent EG is $= 2GF$, and by B. I. Prop. 3, Cor. 2, ibid. EG $=$ (CG·GS) $\div$ GI: but, by the nature of the cycloid IS is $=$ the arc IC; theref EG is (CG $\div$ GI) $\times$ (GI $+$ arc IC) $=$ CG $+$ (CG $\div$ GI) $\times$ arc IC; conſeq. CG $+$ (CG $\div$ GI) $\times$ arc IC $= 2GF = 2FC - 2CG$; theref. $3CG + (CG \div GI) \times$ arc IC $= 2FC =$ twice the diam. of the given circle.—Now to find the value of CG it is obvious that the arc IC divided by GI muſt be between 1 and 2: ſuppoſe it $= 1\frac{1}{2}$, then $4\frac{1}{2} \times CG = 2FC$, or CG $= \frac{6}{7}$ nearly. But to obtain a more accurate value of CG we muſt make uſe of ſome approximating theorem ſuch as that on page 126, Dr. Hutton's Menſuration, where it is ſhewn that the length of the arc IC is $= FC\sqrt{[3CG \div (2CF - CG)]}$, and GI is $\sqrt{(CF - CG) \times CG}$, theref. $3CG + \frac{CG}{\sqrt{(CF - CG) \times CG}} \times FC\sqrt{\frac{3CG}{3CF - CG}} = 2CF$, this reduced and put into numbers gives $3CG + \frac{16\sqrt{3} \times CG}{\sqrt{(768 - 64CG + CG^2)}} = 32$.

Again, let QLR (fig. 468, pl. 25.) be a ſection of the ſolid formed by the revolution of the cycloid about its baſe OL and NH on of the cylinder. Then it appears as before that X ζ, and conſq. by ſim. $\triangle$s ON is $= 2NY$, and $\sqrt{} + (2QN \div NZ) \times$ arc QZ; hence OQ $-$ QN

$\frac{2}{3}$N is $= 2QN + (2QN \div NZ) \times$ arc QZ, or $OQ = 3QN + (2QN \div NZ) \times$ arc QZ. And by aſſuming the arc QZ divided by $NZ = 1\frac{1}{8}$ as before, we have, by uſing Dr. Hutton's Theorem, and proceeding as before,

$$3QN + \frac{32\sqrt{3} \times NQ}{\sqrt{(768 - 64NQ + NQ^2)}} = 16.$$

From theſe two equations, by means of trial and error, Miſs May finds the dimenſions of the cylinders very near the ſame as Mr. Cunliffe.

XI. QUESTION 193, *anſwered by Mr.* I. T. M'Donald.

Firſt. To determine the centre of gravity of a conic fruſtum ABCD (fig. 469, pl. 25.). Put $AB = 2b$, $CD = 2d$, $d - b = h$, $SQ = a$, $SV = x$, $MN = 2b + 2hx \div a$, and $3{\cdot}1416 = c$. Then the fluxion of the ſolid ABMN $\times$ SV the diſtance of the centre of gravity of the generating plane from the axis of ſuſpenſion, is $= b^2cx\dot{x} + (2bchx^2\dot{x}) \div a + (ch^2x^3\dot{x}) \div a^2$, whoſe fluent, divided by the ſolid, (when $x = a$), is $a \times (6b^2 + 8bh + 3h^2) \div (12b^2 + 12bh + 4h^2)$, which may ſerve as a general theorem for aſcertaining the centre of gravity of a conic fruſtum. In applying it to the preſent queſtion, we have $b = 1\frac{1}{2}$, $d = 2$, $h = \frac{1}{2}$, and $a = 20$, whence we find $SG = 10\frac{5}{7}$ as required.

Secondly. To determine the centre of oſcillation in the fruſtum of a right cone. Let P be the point of ſuſpenſion in the axis produced, then retaining the above ſymbols, and putting $PS = e$, $VR = y$, and $RT = [(b + hx \div a)^2 - y^2]^{\frac{1}{2}}$, we ſhall have $[(e + x)^2 + y \times 4\dot{y} \times [(b + hx \div a)^2 - y^2]^{\frac{1}{2}}$ for the fluxion of the generating plane into $(PR)^2$, the ſquare of its diſtance from the axis of motion, the whole fluent of which (x being conſtant) is $c \times (e + x)^2 \times (b + px \div a)^2 + \frac{1}{4}c \times (b + hx \div a)^4$, and this expreſſion being drawn into SV (x), the fluent thereof, (when $x = a$), is

$(e^2b^2 + be^2h + \frac{1}{3}e^2h^2 + \frac{1}{4}d^4 + \frac{1}{2}d^3h + \frac{1}{4}d^2h^2 + \frac{1}{4}dh^3 + \frac{1}{20}h^4) \times ca + (b^2e + \frac{4}{3}beh + eh^2) \times ca^2 + (\frac{1}{3}a^2 + \frac{1}{2}hd + \frac{1}{4}h^2) \times ca^3$, which call F.

Again the fluxion of the ſolid into PV is

$$b^2ce\dot{x} + \frac{2bcehx\dot{x}}{a} + b^2cx\dot{x} + \frac{c\dot{h}^2x^2\dot{x}}{a^2} + \frac{2bchx^2\dot{x}}{a} \times \frac{ch^2x^3\dot{x}}{a^2}$$

whoſe fluent, (when $x = a$), is $(b^2e + 2beh + eh^2) + ca + (\frac{1}{2}b^2 + \frac{2}{3}bh + \frac{1}{4}h^2) \times ca^2$, which put $= M$; alſo put the fruſtum

ABMN

ABMN = B, and the diſtance of the centre of gravity from S, found as above, = g. Then, vide Emerſon's Fluxions, pa. 311,

$$\frac{F}{M}\left(=\frac{F}{(e+g)B}\right)=\frac{+(e^5b^5+be^4h+\frac{1}{3}e^3h^2+\frac{1}{3}d^2+\frac{1}{2}d^2h^2+\frac{1}{2}dh^3+\frac{1}{10}h^4)\times ca}{(b^5e+2beh+eh^2)\times ca^2+(\frac{1}{2}d^2+\frac{1}{2}dh+\frac{1}{3}h^2)\times ca^3}$$
$$+(b^5e+\frac{2}{3}beh+eh^2)\times ca^2+(\frac{1}{2}b^2+\frac{2}{3}bh+\frac{1}{2}h^2)+ca^3$$

which is a general theorem for aſcertaining the diſtance PO of the centre of oſcillation in the fruſtum of a right cone from the point of ſuſpenſion. In the caſe before us, when the fruſtum is ſuſpended by the centre of the leſs end, e being = 0, all the terms in which it is found will vaniſh, and by ſubſtituting the values of the other letters as above, we get PO = 14 very nearly. Hence $\sqrt{14} : \sqrt{39{\cdot}2} :: 60 : 100$, the number of vibrations made by the given fruſtum in a minute.

XII. QUESTION 194, *anſwered by* Mr. Robert Wallace, *the Propoſer.*

Fig. 470. Pl. 25. Let the velocity of the point n be to that of the point m in the given ratio of a to b. Take AC a 4th proportional to a, b, and BC. Join any two contemporary poſitions of the points n and m. Then, becauſe the motions are uniform, the ſpaces paſſed over will be as the velocities, that is, $Bn : Cm :: a : b$, but $a : b :: BC : CA$, by conſtr. Therefore $CB : CA :: Bn : Cm$, or $CB : Bn :: CA : Cm$. Conſequently $CB - Bn : Bn :: CA - Cm : Cm$, or $Bn : nC :: Cm : mA$. Therefore the curve required is a parabola, touching CB, CA in the points B and A. Q. E. F.

Cor. 1. Let CA, CB be given in magnitude and poſition, and let it be required to deſcribe a parabola that ſhall touch CB, CA in the points B and A.

Take $CB : CA :: Bn : Cm :: Bn' : Cm'$. Join nm, $n'm'$; the circles which circumſcribe the triangles nCm, $n'Cm'$, will interſect each other in F, the focus of the parabola; the curve may therefore be deſcribed by the common methods.

Cor. 2. Let there be found in CB, CA, any number of points n, n'; m, m'; &c. ſuch that $Bn : Cm$ (and $Bn' : Cm'$) $:: CB : CA$. The parabola which touches CB, CA in the points B, A, will alſo touch each of the ſtraight lines ſo found by joining the points nm, $n'm'$, &c. and will divide them at the points of ſection and contact in a ſimilar manner, that is,

$$Bn : nC :: nD : Dm, \text{ &c.}$$

The ſame, anſwered by Mr. I. T. M'Doneld.

Let the given lines CB, CA be of ſuch lengths that while *n* deſcribes BC, *m* deſcribes CA, then will they be tangents to the curve at B and A; and let *n* and *m* be contemporaneous poſitions of thoſe points, ſo will the right line *nm*, which joins them, be likewiſe a tangent to the curve, from the conditions of the problem. Now it is evident that BC : CA — B*n* : C*m*, which is a property of the parabola. For, let F be its focus, and draw AF, BF, CF, *m*F, and *n*F; then *(vide ſolution to queſtion* 88, *vol.* 2, *page* 54, *Repoſitory,)* the triangles BCF, ACF, are ſimilar, as are the triangles B*n*F, C*m*F. Hence

BC : CA : : BF : CF : : B*n* : C*m*.

Neat ſolutions were alſo received from Meſſrs. Cunliffe, *and* Lowry.

XIII. QUESTION 195, *anſwered by Mr.* Tho. Hewitt, *London.*

ANALYSIS. Fig. 471, Pl. 25. Suppoſe ABC the △ required. Produce BC to meet a ſemicircle deſcribed on the baſe AB in D, and join AD. Produce, alſo, the ſides till CL, CM are each = to half the given perimeter. Now LO, MO, being made ⊥ to CL, CM, will evidently meet CE, the line biſecting the vertical ∠, produced, in the ſame point O. Demit the ⊥s CG, OF, and FH. Then, by Eu. II. 12, $AB^2 = AC^2 + BC^2 + 2BCD$, but, by hyp. $AB^2 = AC^2 + BC^2 + ACB$; theref. rect. ACB = 2 rect. BCD, or AC = 2CD; hence the ∠ ACD is given, being ⅓ of two right angles, and conſeq. its ſupp. the vertical ∠ ACB is given; theref. the equal radii OL, OM, OF are given, and, by hyp. the ratio of CE : EG = OE : EF = OF : FH is given: but OF is given, theref. FH is given, and ſo is, OH, and AB is ⊥ OF: Whence the *conſtruction* is manifeſt.

COR. When the vertical ∠ of a △ is 120°, the ſum of the ſquares of the ſides together with their rectangle, is equal to the ſquare of the baſe.

The ſame, anſwered by Mr. James Whalley, *Bolton.*

CONSTRUCTION. Fig. 471, Pl. 25. Make the ∠ LCM = 120°, and CL, CM each = half the given perim. and make LO, MO ⊥ to CL, CM; from O deſcribe the arch LM and join CO. Take CE′ : CO in the given ratio, and with the centre C and radius CE′ deſcribe the arch *ab*, and draw OE′ a tangent

thereto, draw alſo AB parallel to CE' to touch the arch LM, and ACB is the △ required.

For draw CG ⊥ to AB, then by conſtr. we have CO : CE' :: CE : EG, alſo, by drawing AD ⊥ to BC produced, the ∠ ACD = ∠CAD, therefore CA = CD. By Euclid's Geom. II. 12, AC² + BC² + BC × CD is = AB², therefore AC² + BC² + BC × AC = AB²; the reſt is evident from the conſtruction.

The ſame, anſwered by Mr. Louis Hill.

CONSTRUCTION. Fig. 472, Pl. 25. Take EF = the given perimeter and on it conſtitute the equilateral △ EGF, and on GH, drawn ∥ to EF, take GH : EG in the given ratio of the ſegment to the biſecting line. Draw HC ⊥ to GH, and from G, to HC, apply GC = GE; join EC, FC, and draw CA, CB, making the ∠ ACE = AEC, and the ∠ BCF = BFC, ſo will ACB be the △ required.

Let GC meet AB in D, and on AC produced drop the ⊥ BK, then becauſe GC = GE, by conſtr. the ∠s GEC, GCE are equal, and taking from theſe the equal ∠s AEC, ACE, there remains the ∠ GEF = GCA. And in the ſame way it is ſhewn that the ∠ GFE = GCB; but the ∠ GEF = GFE, by conſtr.; therefore the ∠ GCA = GCB, and therefore GC biſects the vertical ∠ ACB; wherefore, by parallels, CD : DP :: CG (EG) : GH, that is, in the given ratio. Again, becauſe of the = ∠s, AC is = AE and BC = BF, therefore AC + CB + AB = EF = the given perimeter. Moreover, ſince it is ſhewn above that the ∠ ACE is = to twice the ∠ GEE, that is = to two thirds of two right angles, the ∠ BCS is = to one third of two right angles, that is, the △ BCS is one half of an equilateral one, therefore CB = 2CS; and by Euc. II. 12, AB² = AC² + CB² + AC · 2CS, but 2CS · AC = CB · AC, therefore AB² = AC² + CB² + AC · CB.

Very neat ſolutions were alſo received from Meſſrs. Cunliffe, Lowry, M'Donald, Peacock, *and* Swale.

XIV. QUESTION 196, *anſwered by Mr.* James Whalley, *Bolton.*

ANALYSIS. Fig. 473, Pl. 25. Let ACB repreſent the △, about which ſuppoſe a circle, and draw the diam. EF ⊥ to AB. Let FC, which produce till it cuts the circle again in G, join GC.

By

By the queſt. the ratio of $CD^2 : ADB = CD^2 : CDH = CD : DH$ is given. Moreover, rect. $ACB = EF \cdot CD$ and $AGB = EF \cdot DG$, by Simpſon's Geometry, III. 25. Wherefore, the ratio of $EF \cdot CD : EF \cdot DG$, that is, the ratio of $CD : DG = ACB : AGB$, but the ratio of CD : DG has been ſhewn to be given, therefore the ratio of the rect. ACB : AGB is given, and the rect. ACB is given, theref. the rect. AGB is given. Again $MG^2 - BG^2 = AD^2 - DB^2 = AC^2 - CB^2$, by Simp. Geom. II. 9; whence, the rectangle and the diff. of the ſquares of AG and BG being given, AG and BG will each be given. Finally $CD^2 - DG^2 = CB^2 - BG^2 =$ a given magnitude, becauſe CB and BG, are both given, whence as the ratio and the diff. of the ſquares of the two lines CD, DG are both given, theſe lines will each be given, by means of which the problem may be readily and eaſily conſtructed.

The ſame, anſwered by Mr. John Lowry.

ANALYSIS. Suppoſe ACB the $\triangle$ required, having the ſides AC, BC equal to the given ones. Produce the $\perp$ CD to meet the circle circumſcribing the $\triangle$ at G, and draw GP $\parallel$ to AB to meet AC produced in P. Join AG, BG; let CQ be a diam. of the circle, and $a : b$ the given ratio of the rectangle of the ſegments of the baſe to the ſquare of the perpendicular. Then $AD \cdot DB = CD \cdot DG : CD^2 :: GD : CD :: GD \cdot CQ = AG \cdot GB : CD \cdot CQ = AC \cdot CB$ (Euc. Prop. c, VI.) $:: a : b$. Again by parallels $DG : CD (:: a : b) :: AP : AC$, theref. AP is given, whereſ. $AG \cdot GB : AC \cdot CB :: AP \cdot BC : AC \cdot BC$; theref. $AG \cdot GB = AP \cdot BC =$ a given rectangle, and $AG^2 - BG^2 = AD^2 - BD^2 = AC^2 - BC^2 =$ a given ſpace, and theref. AG and BG are given by the 87th Prop. of the Data. Now the *locus* of the point G is a ſemicircle, whoſe diam. is the given line CP, and A is a given point, and AG a given line, therefore the point G is given: Hence this

CONSTRUCTION. Produce the given ſide AC to P, ſo that $PA : AC :: a : b$, and on PC, as a diam. deſcribe a ſemicircle PCG. Find, by Prop. 87 of the data, two lines AG, BG, ſo that their rectangle $= CB \cdot AP$, and the difference of their ſquares $= AC^2 - BC^2$, and from A, to the circle, apply AG (it being the greater or leſs of the two lines according as AC is the greater or leſs of the given ſides); draw CG and perpendicular to it draw ADB, to which, from C, apply CB = the other given ſide, and it is done, as is manifeſt from the Analyſis.

The same, answered by Mr. I. T. M'Donald.

ANALYSIS. Let the required △ ACB be circumscribed by a circle, in which draw a diam. EF ∥ to the ⊥ CD, which produce to meet the circle in G, and demit the ⊥s GH, FK; produce FK to meet the circle in M and draw AM, AF. Now the rect. ACB = CD · EF is a given magnitude, also the ratio CD²: AD · DB or CD · DG = CD : DG is given; therefore DG · EF = HL · EF = LF · FE — HF · FE = AF² — AM² = FK² — KM², is a given magnitude. Again, CK is = ½ the sum, and AK = ½ the difference of the given sides; therefore AK · KC = FK · KM = a given rectangle. Whence the following

CONSTRUCTION. On any right line FM, take FK, KM of such magnitudes, that their rectangle may be equal to one fourth of the diff. of the squares of the sides, and the diff. of their squares to the rectangle of the sides in the given ratio of the rectangle of the segments of the base to the square of the perpendicular (which is a well known problem); then ⊥ to FM draw CKA, making KC = ½ the sum, and KA = ½ the diff. of the given sides, and having described a circle take FB = FA; draw AB, CB and it is done.

This question was elegantly constructed by Messrs. Cunliffe, Hill, Peacock, Swale, *and* Thornoby; *and Mr.* Barron *sent an algebraical solution.*

XV. QUESTION 197, *answered by Mr.* W. Wallace, *Perth Academy.*

Fig. 495. pl. 26. Let AB and CD be the straight lines given by position, take any points E and F in them and join EF, draw FG parallel to AB, draw LH perpendicular to the plane passing through FG and CD, meeting it in H, and draw HK parallel to FG, then HK is also parallel to AB, and since HL is perpendicular to the plane passing through FG, CD, a plane passing through EB, HK is perpendicular to the plane. Join EF, the triangle EHF is right angled at H, therefore EF² = EH² + HF²; but since FH is a constant quantity, viz. the distance between the parallel lines AB, HK, the line EF will be the least possible when EH is the least possible. Therefore if HK, the common section of the planes EK, KD, be produced to meet CD in *f* and *f* be drawn ⊥ perpendicular to AB meeting it in *e*, it is evident that *fe* is the least distance between the given lines: for in this position FH [illegible] es and thus EF is perpendicular to both the given lines AB

Other correspondents who answered this question referred to Geometry.

XVI.

I. QUESTION 198, *answered by Mr.* R. Simpſon, *Croxdale, Durham.*

Let ES (fig. 492, pl. 86.) be the ſurface of the water, and ILI the ſquare aperture, each ſide of which is 15 inches. Now, as ſhewn by the writers on Hydraulics (ſee Hutton's Courſe, vol. ii. pa. 226.) that the quantity flowing through ILHG is to which would flow through an equal orifice placed horizontally be depth EG, as the parabolic fruſtum IKHG is to ILHG. $a^2 = 15^2$, the area of the ſquare hole, $g = 193$ inches, and EG, then $a^2\sqrt{gx}$ will repreſent the quantity of water diſ- from the equal horizontal aperture in a ſecond: and, by nature of the parabola, we ſhall have RG : GH^2 :: EI : IK^2, $a : x :: a^2 : (a^2x \div a^2) \div x$. Then $\frac{2}{3}ax =$ the area the parab. EGH, and $\frac{2}{3}(x - p)\sqrt{(a^2x - a^3) \div x} =$ the of the parab. EIK; the difference of theſe is the fruſtum EG. Then ſay, as $a^2 : \frac{2}{3}ax - \frac{2x - 2a}{3}\sqrt{\frac{a^2x - a^3}{x}} :: 2a^2$ $: 2\sqrt{gx} \cdot \frac{ax - x - a}{3}\sqrt{\frac{a^2x - a^3}{x}} =$ by the queſtion to 9988887 gallons, or 46894·876653 cubic inches. After ſubſtituting the values of a and g in this equation, we ſoon, by trial and error, find $x = 64$ inches $= EG$, and conſequently EI $=$ inches.

The ſame, unſolved by Metrones Minor.

Let $g = 16\frac{1}{12}$, $b =$ the breadth or depth of the aperture, $h =$ the depth of the water above the top of the aperture, $x =$ variable depth from the top of the water, and $q = 166$ gallons $= \frac{47}{1728}q$ feet. Then, by Hutton's Courſe, vol. ii. art. 494, $\sqrt{x} : \sqrt{x} :: 2g : 2\sqrt{gx}$, the velocity at the depth x: alſo $bx =$ the fluxion of the aperture, and $2bx\sqrt{gx}$ the fluxion of the quantity, the correct fluent which, when $x =$ $h + b$, is $= \frac{4b}{3}(h + b)^{\frac{3}{2}}\sqrt{g} - h^{\frac{3}{2}} = \frac{47}{1728}q$, by the queſtion. Hence $h = 4{\cdot}071$, and $h + b = 5{\cdot}321$ feet the depths required.

And thus the queſtion was anſwered by Mr. James Cunliffe. *All other anſwers on received were falſe.*

XVIII. QUESTION 200, *answered by Mr.* John Lowry.

ANALYSIS. Fig. 475, Pl. 25. Let ACB be the given $\triangle$, and FGHE the inſcribed rectangle, and let its perimeter be equal to a given line inſtead of a *maximum*. On AB demit the $\perp$ CD, and let it meet the FG in R, then, by ſim. $\triangle$s, AB : CD :: FG : CR = CD — DR, and by diviſion AB $\sim$ CD : CD :: FG + DR $\sim$ CD : CR, but FG + DR = FG + GH is given, being equal to half the perimeter, and AB and CD are given; theref. CR is given, and the conſtruction is evident.

Now it is plain that FG + DR increaſes as CR increaſes, therefore if AB be greater than CD, the perimeter will continue to increaſe till FG coincides with AB; but, if CD be greater than AB, the perimeter will continue to increaſe till the points R, C, coincide; and when CD is = AB the perimeter will always be equal to the conſtant quantity 2AB or 2CD. Wherefore it is evident, that the perimeter cannot be greater than twice the longeſt ſide of the $\triangle$ nor leſs than twice the $\perp$ demitted on that ſide from the oppoſite angle.

The ſame, anſwered by Mr. J. H. Swale.

GENERALLY. When, inſtead of a maximum, the perimeter is equal to a given line.

Demit, from the vertex C of the given $\triangle$ ACB, the $\perp$ CD, make DQ = DB, QK = KA, and join C, K. In DC produced take DP = $\frac{1}{2}$ the given perim.; at I, any point in DP, erect the $\perp$ IL = $\frac{1}{2}$ IP and join PL, and produce it to meet CK at N. Through N. parallel to AB, draw FG, and demit upon AB the $\perp$s FE, GH: the required rectangle is EFGH.

Join C, Q, and let CQ, FG meet FG, CD in T and R. By the conſtr. AK = KQ, and QD = DB, theref. FN = NT and TR = RG; theref. FT + TG = 2NT + 2TR = 2NR = = FG: But, ſince PI = 2IL, PR will be = 2RN, that is, DR + 2RN = DR + RP = DP. Now DR = EF = GH, FG = EH, therefore EF + FG + GH + HE = 2EF + 2FG = 2DR + 2RP = 2DP = the given line, by conſtruction.

When PL produced does not meet CQ, the problem becomes impoſſible, hence the utmoſt limit for the point N will be its coincidence with K, therefore FG will coincide with AB; whence it appears that the perimeter cannot be greater than twice the longeſt ſide of the triangle.

Solutions were received from Meſſrs. Hill, May, M'Donald, Nicholas Minor, and Peacock.

XIX.

XIX. QUESTION 201, *answered by Mr.* James Cunliffe, *Bolton*

Analysis. Fig. 476, Pl. 25 Having described a figure and marked the angular points, &c. as in question 158, draw MN ‖ to GC meeting CE in N. The ∠s MEN, HCG are obviously similar, therefore HC : ME : : HG : MN = GC, but HC and ME are homologous parts of the similar trapezia ACBH, and DEFM, and HG is the diameter of the circle circumscribing the trapezium ACBH, consequently GC must be = to the diameter of the circle circumscribing the trapezium DEFM, or the △ FED. Then because of the sim. △s ABC, FED, HG : GC : : GC : $(GC)^2 \div HG$ = the diameter of the circle circumscribing the △ formed from FED; and in like manner $(GC)^3 \div (HG)^2$ will express the diameter of the circle circumscribing the △ formed from the preceding, &c. Therefore the expressions for the diameters of the several circles in succession will be

$$GC, \frac{(GC)^2}{HG}, \frac{(GC)^3}{(HG)^2}, \frac{(GC)^4}{(AG)^3}, \frac{(GC)^5}{(HG)^4} \text{ \&c.}$$

which are evidently a series of lines in geometric proportion. Now it is well know that $(HG \cdot GC) \div (HG - GC)$ will express the sum of the said series continued *ad infinitum*, which put = L, the given line, or make $HG \cdot GC = L \times (HG - GC)$, whence $GC \times (L + HG) = L \times HG$, which points out the following method of determining the position of the vertex of the △ ABC, viz. apply the chord GC = a fourth proportional to L + HG, L, and HG.

The same, answered by Mr. Lowry.

Let MN be the diam. of the circle circumscribing the △ EDF, drawn from M the middle of AB at right angles to the base DF and meeting it in *m*; and from the points E, D, F, on DF and ME demit the ⊥s E*e*, F*f*, and D*d*, and draw *mn* parallel to GH to meet E*e*, produced, in *n*. Produce CE to meet the circle MEFD which, by reason of the right angle at E, it will do at N, the extremity of the diameter MN. Also on MN and EN, demit the perpendiculars EK, KL, LS, SQ, &c. alternately, and join GC. Then it is evident, from Prop. XV. Art. XXXI, of the Repository, that MN is equal and parallel to CG, and it may be shewn in the same way as in that proposition, that EN is equal and parallel to *mn* the diameter of the circle circumscribing the △ formed by joining the points *e*, *d*, *f*, and that KN is equal and parallel to the diameter of the circle circumscribing the next △ formed in a similar manner, LN equal and parallel to the next,

SN

SN to the ſucceeding one, and ſo on. Hence, by ſim. figures HG : (GC) MN : : MN : EN : : EN : KN : : LN : : LN : SN and ſo on: therefore Euc. V. 12, HG : MN as the ſum of the antecedents HG, MN, EN, KN, LN, &c. to the ſum of the conſequents MN, EN, KN, LN, &c. that is, HG : GC : : HG + the given ſum : the given ſum, and by diviſion HG — GC : GC : : HG : the given ſum. *Ergo Solutum.*

Divide HG at Z ſo that HZ : GZ : : HG : the given ſum, make GC = GZ, and C is the vertex of the △ required.

Cor. When GC = the radius of the given circle, the ſum of all the diameters is equal to twice GC.

Anſwers were alſo received from Meſſrs. Hill, May, M'Doneld, *and* Swale.

XX. QUESTION 202, *anſwered by Mr.* Cunliffe.

Let SQRV (fig. 484, pl. 25.) be the given conic ſection, C its centre, and P the given point. Through P draw the diam. SV of the given curve, and upon P as a diam. deſcribe another conic ſection ſimilar to the given one and in a ſimilar poſition, which will be the locus of the point of biſection, and the axes of this latter conic ſection will manifeſtly be parallel to thoſe of the given curve. For, draw the right line PQR to cut the given curve in Q and R and interſect the other curve in *m*; alſo through *m* draw the ſemidiameter C*m*T, and parallel to CT, through O the centre of the other curve, draw the ſemidiameter O*t*, cutting PR in *n*. It is evident enough that P*m* is biſected in *n*, therefore P*m* is an ordinate to the ſemidiameter O*t*, by the well known property of conic ſections; and becauſe the ſemidiameters O*t* and CT are parallel and in curves ſimilar and ſimilarly ſituated it is a plain that QR will be an ordinate to the ſemidiameter CT, and will therefore be biſected by it in *m*. *Q. E. D.*

The above demonſtration is founded on properties that are common to all the conic ſections, and is therefore general, though the figure is only drawn for the ellipſis.

The ſame, anſwered by Mr. W. Wallace, *Perth Academy.*

Fig. 494, Pl. 26. Let P be the given point, EHF the given conic ſection, which any ſtraight line drawn through P cuts in A and B; let AB be biſected in D; it is to be proved that the point D is alſo in a given conic ſection ſimilar to EHF and ſimilarly ſituated. Let C be the centre of the given conic ſection, join PC meeting the curve in E and F, draw the ſemi-diameter CDH, draw EG parallel to AB, meeting CH in K, and the curve in G and join FG. Becauſe AB is biſected at D, it follows that AB is

a-

an ordinate to the ſemidiameter CH, and ſince EG is parallel to AB, EG is alſo an ordinate to the ſame ſemidiameter, and is therefore biſected at K; now the diameter EF is biſected at C, therefore CK or CD is parallel to FG; thus it appears that the triangles EGF, PDC are ſimilar, and ſimilarly ſituated: and ſince EF, PC, homologous ſides of theſe triangles, are given in poſition and magnitude, and EFG one of them, is inſcribed in a given conic ſection EHF, the other triangle PDC is alſo inſcribed in a given conic ſection ſimilar to EHF, and ſimilarly ſituated; therefore the locus of D is a given conic ſection, &c. as was to be demonſtrated.

XXI. QUESTION 203, *anſwered by Mr.* James Whalley.

ANALYSIS. Fig. 477, Pl. 25. Let the figure be conſtructed by the given data, and produce AD to meet BG, draw ‖ to OD, in G, and join CD. Then, becauſe ∠ FDE = FCD, and ∠ F common to the △s FDE, and FCD, theſe △s are ſimilar, theref. FE : FD : : FD : FC, and, by the parallel lines OD, BG, FD² : FE² : : BG² : BE², theref. FE · FC : FE², that is, FC : FE : : BG² : BE²: but the ratio of FC : FE is given, and BG is given, being equal to the given diam. AB; theref. BE will be given: Hence the following

CONSTRUCTION, *by Mr.* James Cunliffe.

Let the given ratio of CE to EF be denoted by that of m to n. Apply BE to AD, ſuch that $m + n : n$: : BA² : BE², and produce BE to meet the circle at C, and the thing is done, as is evident from the analyſis.

From the preceding analyſis the following theorem is evident. If two chords AD and BC, drawn from the extremes A and B of the diameter of a circle, intersect in E, and the radius OD be drawn cutting BC in F; then will FC : FE : : AB² : BE².

The ſame, anſwered by Mr. J. H. Swale.

CONSTRUCTION. Divide AD at G′ in the given ratio, and apply, to AB, DP, the ſide of a ſquare equal to the rect. AD·DG′; draw DC, making the ∠ ODC = DPB, to meet the circle in C and join BC meeting DA, DO, in E and F and it is done.

DEMONSTRATION. The △s DFC, DFE are ſim. for the ∠ DCF = DAB = EDF, and the ∠ CFD and the ſide DF are common to both, theref. CF : FD : : FD : FE, and theref. CF · FE = FD². The △s DFC, PDA are alſo ſim. for the ∠ CDF = DPA, by conſtr. theref. CF² : FD² = AD² : DP², that is, CF² : FC · FE = AD² : AD · DG′, or, CF : FE = AD · DG′, theref. dividendo, CE : EF = AG′ : G′D = the given ratio by Conſtr.

When the ratio is that of equality, biſect AD in G′.

The

The same, answered by Mr. L. T. M'Donald.

CONSTRUCTION. Let ADB be the given semicircle, and AD the chord given by position. Draw BG ‖ to the radius DO meeting AD produced in G, and through the points A, B, G describe a circle. Also in BD produced take DH : DB equal to the given ratio, and draw HI ‖ to DA meeting the circle at I; join BI, cutting the given semi-circle at C, the given chord in E, and the radius in F, so shall BC be the line required to be drawn.

DEMONSTRATION. Demit, upon AG, the ⊥ IL, and join CA, CL. Then, because ICA, ILA are right angles, the points A, C, L, I, are in a circle, also ∠ ADO = AGB = AIB = supp. of ALC = CLE; theref. CL, DF are parallel; whereof. by sim. △s, HD : DB = IE : EB = LE : ED = CL : DF = CE : EF = the given ratio.

COR. If BH be produced to meet the circle at K, BK will be the diameter, for it bisects the chord AG perpendicularly: moreover it appears that the ratio of CE to EF will always be less than that of KD to DB.

The same, answered by Mr. Louis Hill.

CONSTRUCTION. Let $m : n$ be the given ratio of CE ‖ EF, and cut AC at H', so that AH' : H'O :: $m + n : n$; draw HT' ⊥ to AO to meet a semicircle described on AO at T, and draw AT to meet OK', drawn ⊥ to AO, at K'. From O, to the chord AD, apply OR = OK' and draw EFC ‖ to OR and it is done.

For, by constr. Eu. cor. 8. VI, and sim. △s, $AO^2 : (DO^2) : OK'^2 : (OR^2) :: AI' : IK' :: AH' : H'O :: m + n : n$; but by sim. △s, DF : FE :: DO : OR, or, $DF^2 : FE^2 :: DO^2 : OR^2 :: m + n : n$. Again, since OD = OA, ∠ ADO is = BAD = BCD (Euc. III. 21,); theref. the △s FCD, EDF are similar; wherefore DF : EF :: CF : DF, theref. $DF^2 = EF \cdot CF$, and theref. $EF \cdot CF : EF^2 :: CF : EF :: m + n : n$, theref. by division, CE : FE :: $m : n$, that is, in the given ratio.

Mr. Lowry *also answered this question.*

XXII. QUESTION 204, *answered by Mr.* J. H. Swale.

Fig. 478. Pl. 25. *Construction.* Divide the base AB in the ratio $m : n$ I, and demit, from D, upon AB, the ⊥ DP, in the PS : PD = BT : BO, the given line; draw SH ‖ to AC meet AC in H, join BH meeting DE in G, and make GF = BO, and join BF meeting AC in I, and it is done.

Demon-

Demonstration. Demit upon AB the ⊥s HL, GN, and produce NH to meet BI at K, and join HT. Then, since FG = BO = the given line, by conſtr. it only remains to prove that AH : HI = the given ratio. Now by conſtr. PS = LH : PD = NG :: BT : BO = FG = , per parallels, KH : FG, that is, HK = BT : but HK, BT are alſo parallel, therefr. BI, TH are parall. Conſequently AT : TB = AH : HI = the given ratio, by conſtr.

The ſame, aſſerted by Mr. Lowry.

Analyſis. Suppoſe the problem ſolved. Draw IK' ‖ to BH to meet AB produced in K and DE in M. Then, by parallels, AB : BK' :: AH : HI, i.e. a given ratio, but AB is given in magnitude, therefr. BK' is given, and conſequently GM is given. Alſo, by parallels, BK' : EK :: AB : DF, therefr. DF is given. *Ergo* Solvitur.

On AB produced take BK' : AB in the given ratio of HI to AH, and take BL = the given length of GF; on DE take DF the diſt. DF : AB :: LK' : BK', and draw BF to meet AC in I and join FL', then if BGH be drawn ‖ to FL' it is done, as is evident from the analyſis.

The ſame, aſſerted by Mr. James Whalley.

Fig. 4th Pl. 25. *Analyſis.* Through F draw Fe ‖ to AC meeting BH, and AB in *f* and *n*, alſo draw *Bd* ‖ to AC meeting DE in *d*. The △s Bde, and GfF are ſim. whence Ff : *fe* :: Gf : Be = Fd, and, by the parallels Fn, and AC, Ff : *fe* :: HI : AH, therefr. HI : AH :: Gf : Fd; but GF is given, and the ratio of HI : AH is alſo given, therefr. Fd is given; hence the conſtr. is evident.

Elegant conſtructions to the ſame were alſo received from Meſſrs. Carlile, Hill, *and* M'Donald.

XXIII. QUESTION 1023, *aſſerted by* A.B. *the propoſer.*

Becauſe the nth differences of *a*, *b*, *c*, &c. are equal, by the differential method, the ſeries = $a + (a + a')x + (a + 2a' + a'')x^2 + \&c.$ Let the required ſcale = $Ax^{n+1} + Bx^n + Cx^{n-1}$ &c. $+ Gx^3 + Hx^2 + Ix + Kx$, then by the nature of the problem we ſhall have,

$$a \times Ax^{n+1}$$
$$+ (a + a')x \times Bx^n$$
$$+ (a + 2a' + a'')x^2 \times Cx^{n-1}$$
&c. &c.

$$
\left.\begin{array}{l}
\left\{\begin{array}{l}
a+(n-3)d+\dfrac{(n-3)(n-4)}{1\cdot 2}d''+\dfrac{(n-3)(n-4)(n-5)}{1\cdot 2\cdot 3}d'''+\&c.\\
\qquad +(n-3)d^{n-3}
\end{array}\right\} x^{n-3}\times Gx^{4}\\
\left\{\begin{array}{l}
a+(n-2)d+\dfrac{(n-2)(n-3)}{1\cdot 2}d''+\dfrac{(n-2)(n-3)(n-4)}{1\cdot 2\cdot 3}d'''+\&c.\\
\qquad +\dfrac{(n-3)(n-2)}{2\cdot 1}d^{n-4}+(n-2)d^{n-3}+d^{n-2}
\end{array}\right\} x^{n-2}\times Hx^{3}\\
\left\{\begin{array}{l}
a+(n-1)d+\dfrac{(n-1)(n-2)}{1\cdot 2}d''+\dfrac{(n-1)(n-2)(n-3)}{1\cdot 2\cdot 3}d'''+\&c.\\
\qquad +\dfrac{(n-3)(n-2)(n-1)}{3\cdot 2\cdot 1}d^{n-4}+\dfrac{(n-2)(n-1)}{2\cdot 1}d^{n-3}+(n-1)d^{n-2}+d^{n-1}
\end{array}\right\} x^{n-1}\times Ix^{2}\\
\left\{\begin{array}{l}
a+nd+\dfrac{n(n-1)}{1\cdot 2}d''+\dfrac{n(n-1)(n-2)}{1\cdot 2\cdot 3}d'''+\&c.\\
\qquad +\dfrac{(n-3)(n-2)(n-1)n}{4\cdot 3\cdot 2\cdot 1}d^{n-4}+\dfrac{(n-2)(n-1)n}{3\cdot 2\cdot 1}d^{n-3}+\dfrac{(n-1)n}{2\cdot 1}d^{n-2}+nd^{n-1}+d^{n}
\end{array}\right\} x^{n}\times Kx\\
\left\{\begin{array}{l}
a+(n+1)d+\dfrac{(n+1)n}{1\cdot 2}d''+\dfrac{(n+1)n(n-1)}{1\cdot 2\cdot 3}d'''+\&c.\\
\qquad +\dfrac{(n-3)(n-2)(n-1)n(n+1)}{5\cdot 4\cdot 3\cdot 2\cdot 1}d^{n-4}+\dfrac{(n-2)(n-1)n(n+1)}{4\cdot 3\cdot 2\cdot 1}d^{n-3}+\dfrac{(n-1)n(n+1)}{3\cdot 2\cdot 1}d^{n-2}+\\
\qquad \dfrac{n(n+1)}{2\cdot 1}d^{n-1}+(n+1)d^{n}
\end{array}\right\} x^{n+1}
\end{array}\right.
$$

which is the next term in the feries. The above will be evident from the nature of recurring feries. By comparing the co-efficients

of the ſame differences it will appear that $K = n + 1$; $I + nK = \frac{n(n+1)}{2 \cdot 1}$, therefore $I = \frac{n(n+1)}{1 \cdot 2} - n(n+1) = -\frac{n(n+1)}{1 \cdot 2}$; $H + (n-1)I + \frac{(n-1)n}{1 \cdot 2}K = \frac{(n-1)n(n+1)}{1 \cdot 2 \cdot 3}$, therefore $H = (n-1)\left(\frac{n(n \times 1)}{1 \cdot 2 \cdot 3} - \frac{n}{2}K - I\right) = (n-1)\left(\frac{n(n+1)}{2 \cdot 3} - \frac{n(n+1)}{1 \cdot 2} + \frac{n(n+1)}{1 \cdot 2}\right) = \frac{(n-1)n(n+1)}{1 \cdot 2 \cdot 3}$; $G + (n+2)H + \frac{(n-1)(n-2)}{1 \cdot 2}I + \frac{(n-2)(n-1)n}{1 \cdot 2 \cdot 3}K = \frac{(n-2)(n-1)n(n+1)}{1 \cdot 2 \cdot 3 \cdot 4}$, therefore by tranſpoſition &c. $G = -\frac{n-2)(n-1)n(n+1)}{1 \cdot 2 \cdot 3 \cdot 4}$, &c. &c. Wherefore, $Kx + Ix^3 + Hx^3 + Gx^4 + \&c.\ldots\ldots Cx^{n-1} + Bx^n + Ax^{n+1} = (n+1)x - \frac{(n+1)n}{1 \cdot 2}x^2 + \frac{(n+1)n(n-1)}{1 \cdot 2 \cdot 3}x^3 - \frac{(n+1)n(n-1)(n-2)}{1 \cdot 2 \cdot 3 \cdot 4}x^4 - \&c. = 1 - (1-x)^{n+1} =$ ſcale of relation required.

Hence it is evident that if the nth differences be equal, the number of terms in the ſcale of relation $= n + 1$.

Mr. Cunliffe *alſo anſwered this queſtion.*

XXIV. QUESTION 206, *anſwered by Mr.* James Cunliffe.

Fig. 480, Pl. 25. By the hyp. $\angle$ DAF = DAB, alſo AF = AB, and the ſide AD is common to both the $\triangle$s AFD and ABD, theref. Eu. I. 4, FD=BD, and $\angle$ AFD = ABD = a right $\angle$. But FG is a tangent to the arc FB at F, wherefore the $\angle$ AFG is a right $\angle$ (Euc. III. 18), for which reaſon FG will paſs through D.

Through e, the middle of FD, draw the line Ah, to meet Ch, drawn $\perp$ to AC, in h, and interſect DE in g. The $\triangle$s AED, and AeD, are by the conſt. mutually equal in every reſpect, therefore the $\angle$s ADG, and ADG, are alſo equal in every reſpect, and therefore Bg = FG.

Becauſe AD biſects the $\angle$ BAC, by Eu. VI. 3,

$AC : AB = AF :: DC : BD$, and by the parallels De, Ch,
$Ch : De = Fe :: gC : Dg$: but
$Ch : Fe :: AC : AF :: DC : BD$; therefore
$DC : BD :: gC : Dg$, whence, by compoſition,
$BC : BD :: DC : Dg$, that is, $BC : DC : BD : DG$. Now
$AC : AB :: DC : BD$, as ſhewn above, whence, Eu. VI. 16,
$BD \times AC = DC \times AB$; and by Simpſon's Geom. III. 26,
$AC \times AB - DC \times BD = AD^2$, whence
$BD \times AC \times AB - DC \times BD^2 = BD \times AD^2$, that is,
$DC \times AB^2 - DC \times BD^2 = BD \times AD^2$, that is
$DC \times (AB^2 - BD^2) = BD \times (AB^2 + BD^2)$. Therefore
$AB^2 - BD^2 : AB^2 + BD^2 :: BD : DC$, and by compoſition
$2AB^2 : AB^2 + BD^2 :: BC : DC :: BD : DG$, as was ſhewn before,
$2AB^2 \times DG = BD \times (AB^2 + BD^2)$, that is*
$4AB^2 \times DG = BD \times (2AB^2 + 2BD^2)$: but
$4AB^2 \times FD = BD \times 4AB^2$, therefore, by adding,
$4AB^2 \times FG = BD \times (6AB^2 + BD^2) = 6BD \times AB^2 + 2BD^3$. But
$(AB + BD)^3 - (AB - BD)^3 = 6BD \times AB^2 + 2BD^3$, conſequently
$4AB^2 \times FG = (AB + BD)^3 - (AB - BD)^3$. *Q. E. D.*

Cor. $AB^2 : AD^2 :: DE : DG$. See the expreſſion above, marked *.

The ſame, anſwered by Mr. John Lowry.

Let the line joining the points F and B, interſect AD at H, then, ſince AD biſects the $\angle$ BAE, FB will be $\perp$ to AD, and $BH =$ to HF. Again, ſince DB and FG are tangents to the circle at B and F, and $\angle$ BAD $=$ DAF, it is evident that FG paſſes through the point D, and that ED is $=$ DB, therefore, ſince $DE = EB$, and $FH = HB$, the ſtraight line which joins the points H and E, will be parallel to FG, and the $\angle$ EHD $=$ HDF $=$ EDH, theref. EH is $=$ to ED. Wherefore, by ſim. $\triangle$s, $DG : HE :: AD : AH :: AD^2 : AD \cdot AH = AB^2$ (Eu. VI. 8.), or
$2DG : BD :: AD^2 = AB^2 + BD^2 : AB^2$, therefore
$BD \times (AB^2 + BD^2) = 2DG \times AB^2$, or
$2BD \times (AB^2 + BD^2) = 4DG \times AB^2$: but
$4DB \times AB^2 = 4DG \times AB^2$; theref. by addition,
$6 \times BD \times AB^2 + 2BD^3 = 4FG \times AB^2$. But it is evident from Propoſition E, Art. XXII. of the Repoſitory that,
$6 \times BD \times AB^2 + 2BD^3 = (AB+BD)^3 - (AB-BD)^3$; theref.
$4 \times FG \times AB^2 = (AB + BD)^3 - (AB - BD)^3$. *Q. E. D.*

The ſame, anſwered by Mr. Richard Nicholſon, *Private Teacher of the Mathematics, at Liverpool.*

Lemma. The cube on the ſum of two lines leſs the cube on their difference is equal to that ſolid, the baſe of which is ſix

times the ſquare of the greater and the height the leſs, together with twice the cube on the leſs. For,
$(AB+BD)^2=AB^2+3AB^2\,BD+3AB\cdot BD^2+BD^3$, and
$(AB-BD)^3=AB^3-3AB^2\cdot BD+3AB\cdot BD^2-BD^3$; therefore
$(AB+BD)^3-(AB-BD)^3=6AB^2\,BD+2BD^3$.
This being premiſed we have only to prove that
$4AB^2\cdot FG=6AB^2\cdot BD+2BD^3$.

DEMONSTRATION. By Euc. I. 26, FG interſects BC in D, and FD is $=BD$. Perpendicular to AD draw BH, and join HE, then will $HE=BE=BD$, theref. Euc. I. 32, twice the $\angle$ HBE $(=BAE)=HED$, and, by Simſon's Geometry III. 19, and Euc. III. 22, the $\angle$ BAE $(=HED)=EDG$, theref. HE is $\parallel$ to DG, conſequently
$AD:AH\ (::AD^2:AD\cdot AH=AB^2)::DG:DE$; hence
$2AD^2\cdot 2DE\ (=2AB^2\cdot BD+2BD^3)=4AB^2\cdot DG$,
and by adding $4AB^2\cdot DB$, we have
$6AB^2\cdot BD+2BD^3=4AB^2\cdot FG$. *Q. E. D.*

Demonſtrations were likewiſe received from Meſſrs. Hill, M'Donald, Swale, *and* Thomoby.

XXV. QUESTION 207, *anſwered by Miſs* May.

To render the ſolution of this queſtion as plain as poſſible, it will be neceſſary to point out the method of graduating a thermometer according to Fahrenheit's ſcale, which is thus: the tube being fixed to a frame, upon which the graduations are to be made, is put into water juſt freezing, and againſt the ſurface is marked 32; the tube being then put into boiling water and againſt the ſurface of the fluid is marked 212. Then the interval between theſe two marks being divided into 180 equal parts and marked 33, 34, &c.—55 will be temperate, 76 ſummer heat, and 98 blood heat. Now in Mr. Bulmer's thermometer the interval between freezing and boiling is divided into 205 equal parts. Therefore it is evident that

$$\begin{array}{rrl} 55-32 & & 26\frac{7}{16} \\ 180:76-32 & :: 205: & 50\frac{1}{9} \\ 98-32 & & 75\frac{5}{8} \end{array}$$

Hence $25+26\frac{7}{16}=51\frac{7}{16}$, Temperate
$25+50\frac{1}{9}=75\frac{1}{9}$, Summer Heat.
$25+75\frac{5}{8}=100\frac{5}{8}$, Blood Heat.

Ingenious ſolutions to this queſtion were received from Meſſrs. Johnſon, Hill, Lowry, M'Donald, Merones Minor, *and* Nicholſon.

XXVI. QUESTION 208, *anſwered by the propoſer* A. B.

Fig. 481, 482, Pl. 25. Let PQA be the orbit deſcribed by the body, F, the centre of force, FY perpendicular to the tangent PY,

and Fy, to Qy, FQ being drawn indefinitely near to FP. S be the true place of the star, SA and Sa the aberrations being to the points P and Q; then it is well known that SA, Sa, respectively parallel to PY, Qy, and proportional to $\frac{1}{FY}$ and hence the angle YPy = YFy = ASa. Put SA = Y and $\frac{1}{PY} = P$, then in the curve PQA, $QF = \frac{FY \times PT}{PY} = \frac{\dot{y}y}{(y^2 - \rho^2)}$, therefore the angular velocity of $PF = \frac{\rho\dot{y}}{y\sqrt{(y^2 - \rho^2)}}$ $\frac{y^{2n-1}\dot{y}}{\sqrt{(a^2y^2 - y^{2n})}}$. But $\frac{\dot{y}}{y} : \frac{\dot{\rho}}{\rho} :: \left(\frac{\dot{y}}{y} : \frac{n\dot{y}}{y}\right)$ 1 : n :: $\frac{y^{2n-1}\dot{y}}{\sqrt{(a^2y^2 - y^{2n})}} : \frac{ny^{2n-1}\dot{y}}{\sqrt{(a^2y^2 - y^{2n})}}$ = angular velocity of SA = $\frac{n\dot{Y}}{\sqrt{(Y^2 - P^2)}}$. Because SA = Y $\propto \frac{1}{\rho} \propto \frac{a}{y^n}$, let $Y = \frac{ma}{y^n}$, m being constant, then $y = \frac{(ma)^{\frac{1}{n}}}{Y^{\frac{1}{n}}}$ and $\dot{y} = -\frac{(ma)^{\frac{1}{n}}\dot{Y}}{nY^{\frac{n+1}{n}}}$. Hence by substitution and reduction $\frac{P\dot{Y}}{Y\sqrt{(Y^2 - P^2)}} = -\frac{\dot{Y}}{\sqrt{(a^2 \times m^{\frac{2}{n}} \times Y^{\frac{2n-2}{n}} - Y^2)}}$, therefore $\frac{P^2}{Y^2 - P^2Y^2} = \frac{1}{a^2 \times m^{\frac{2}{n}} \times Y^{\frac{2n-2}{n}} - Y^2}$, and $P^2 = \frac{Y^{\frac{2}{n}}}{a^2 \times m^{\frac{2}{n}}}$, there-

fore $P = \frac{Y^{\frac{1}{n}}}{a^{\frac{1}{n}} \times m^{\frac{1-n}{n}}}$, the equation of the curve required.

If $n = \frac{1}{2}$, then $P = Y^2 \div a^2 m$, an equation to a circle, the point S being in the circumference. If, therefore, a body move in a parabola, the centre of force being in the focus, the aberration curve will be a circle, the true place of the ſtar being in the circumference.

If a body move in a circle and the centre of force in the circumference, then $n = 2$, and $P = \frac{Y^{\frac{1}{2}}}{a^{\frac{1}{2}} \times m^{-\frac{1}{2}}} = \frac{(m)^{\frac{1}{2}}}{a} \times Y^{\frac{1}{2}}$, an equation to the parabola.

If $n = 1$, then the curve PQA is the log. ſpiral, and $P = Y \div a$, or $P : Y :: 1 : a$, therefore the aberration curve is a log. ſpiral ſimilar to the other.

The ſame, anſwered by Mr. Lowry.

Fig. 483, Pl. 25. Let NB*b* repreſent the curve in which the body is ſuppoſed to revolve, C the centre of force, B the place of the body at any aſſigned time, and S that of the ſtar. Draw the tangent PAB and take BA to BS as the velocity of the body when at B is to the velocity of light. Draw SQ equal and parallel to BA; then from the nature of aberration, (See Simpſon's Eſſays, Prop. 1 and 2,) it is plain, that Q will be the apparent place of the ſtar when the body is at B. Again, ſuppoſe the body to be at *b*, an indefinitely ſmall diſtance from B, and that *q* is the apparent place of the ſtar when the body is in that ſituation. Draw the tangent *bp* to meet CP, drawn perpendicular BP, at *p*, join B*b*, and CB demit the perpendicular *bb'*; join alſo S*q*, Q*q*, and draw QD perpendicular to S*q*. Let IQ be a tangent to the curve of aberration at Q, and SI a perpendicular to that tangent. Then it is evident, from the propoſitions above cited, that S*q* is parallel to *bp*, and therefore the triangles BP*p*, SQD, are ſimilar; moreover B*p* is ultimately equal to BP, and SQ to SD, wherefore BP (B*p*) : P*p* :: SQ : QD, and permutando, BP : SQ :: P*p* : QD, therefore BP · *bb'* : SQ · *bb'* :: P*p* · CB : QD · CB.

Again, the triangles CBP, *b*B*b'* are ſimilar, therefore PC : PB :: *bb'* : B*b'*, or PC · B*b'* = PB · *bb'*, conſequently P*p* · CB : PB · *bb'* :: P*p* · CB : PC · B*b'*, therefore by permutation and equality, SQ · *bb'* : QD · CB :: PC · B*b'* : P*p* · CB.

But, by the queſtion, CP is $= \frac{(CB)^n}{a}$, therefore its fluxion Pp is $= \frac{n}{a} \cdot (CB)^{n-1} \cdot B\dot{b}'$, hence $Pp \cdot CB = \frac{n}{a} \cdot (CB)^n \cdot B\dot{b}'$, and therefore $Pp \cdot CB = n \cdot PC \cdot B\dot{b}'$, and $QD \cdot CB = n \cdot SQ \cdot \dot{b}'$, or $QD^2 \cdot CB^2 = n^2 \cdot SQ^2 \cdot \dot{b}\dot{b}'^2$. Again, from what is done above and Euc. I. 47, $\dot{b}\dot{b}'^2 = \frac{PC^2 \cdot P\dot{b}'^2}{CB^2 - PC^2}$, and from the ſimilar triangles SQI, QqD, $QD^2 = \frac{SI^2 \cdot Dq^2}{QS^2 - SI^2}$; theſe values of $\dot{b}\dot{b}'^2$ and QD^2 being ſubſtituted in the above expreſſion, it becomes $\frac{SI^2 \cdot Dq^2 \cdot CB^2}{QS^2 - SI^2} = \frac{n^2 \cdot PC^2 \cdot B\dot{b}'^2 \cdot SQ^2}{CB^2 - PC^2}$, from which equation, the relation of SI to SQ may be eaſily determined. For, by Prop. XI. Cor. 2, Emerſon on Centripetal Forces; the velocity of the body in the curve at B is reciprocally as the perpendicular CP, therefore SQ (BA) will be reciprocally as CP or $\frac{a}{CB^n}$. Now let $SQ : \frac{a}{SB^n}$ be aſſumed as b to 1; then CP is $= \frac{b}{SQ}$, $SQ = \frac{ab}{CB^n}$, and $CB^2 = \left(\frac{ab}{SQ}\right)^{\frac{2}{n}}$; therefore the fluxion of CB ſquared, or $B\dot{b}'^2 = \frac{(ab)^{\frac{2}{n}}}{n^2} \cdot \frac{DQ^2}{SQ^{\frac{2n+2}{n}}}$, and theſe values of PC^2, CB^2, and $B\dot{b}'^2$ being ſubſtituted in the preceding equation, it becomes, after making the neceſſary reduction, $\frac{SI^2}{QS^2 - SI^2} = \frac{SQ^{\frac{2}{n}}}{a^{\frac{2}{n}} \cdot b^{\frac{2-2n}{n}} \cdot (QS^2 - SI^2)}$, from

from whence it is evident $SI = \frac{SQ^{\frac{1}{n}}}{a^{\frac{1}{n}} . b^{\frac{1-n}{n}}}$ the equation of the required curve.

XXVII. QUESTION 209, *anſwered by Mr.* James Cunliffe.

ANALYSIS. Fig. 485, Pl. 26. Suppoſe the thing done. To the centre O of the circle EF draw the line AO, and towards the centre S of the circle BC, take $\angle OAo = FAC$, the given angle, and $Ao = AO$. With the centre o deſcribe another circle equal to EF, and produce AC, if neceſſary, to cut the periphery of this circle in e and f. Draw the lines AS, Ao, the radii SB, oe, and upon AC let fall the $\perp$s Sn, om; alſo draw OM $\perp$ to AF; take away the $\angle$ OAC from the equal $\angle$s OAo, FAf, and there will remain the $\angle$ $oAm = OAM$, theref. by Euc. I. 26, $om = OM$, and $ef = EF$, by Euc. III. 14.

CASE I. When the ſum of the ſquares of the chords EF and BC is given.

By Euc. I. 47, $em^2 + mo^2 = oe^2$, and $Bn^2 + nS^2 = SB^2$, theref. $em^2 + Bn^2 + mo^2 + Sn^2 = oe^2 + SB^2 =$ a given magnitude: But $em^2 + Bn^2 = \frac{1}{4}(ef)^2 + \frac{1}{4}(BC)^2 = \frac{1}{4}(EF)^2 + \frac{1}{4}(BC)^2$ is $=$ to a given magnitude, by the queſtion; therefore $mo^2 + Sn^2$ is given. Again, by Euc. I. 47, $Am^2 + mo^2 = Ao^2$, and $An^2 + Sn^2 = AS^2$, theref. $Am^2 + An^2 + mo^2 + Sn^2 = Ao^2 + AS^2 =$ a given magnitude: But $mo^2 + Sn^2$ has been ſhewn to be given, therefore $Am^2 + An^2$ is given. Now the loci of the points m and n, are ſemicircles interſecting in A, whoſe diameters are Ao and AS; therefore this caſe of the problem is reduced to queſtion 783, Gentleman's Diary.

CASE II. When the difference of the ſquares of EF and BC is given.

$em^2 + mo^2 = oe^2$, and $Bn^2 + Sn^2 = SB^2$, and taking the diff. $em^2 - Bn^2 + mo^2 - Sn^2 = oe^2 - SB^2 =$ to a given magnitude. But $em^2 - Bn^2 = \frac{1}{4}(ef)^2 - \frac{1}{4}(BC)^2 = \frac{1}{4}(EF)^2 - \frac{1}{4}(BC)^2$ is a given magnitude, by the queſtion, therefore $mo^2 - Sn^2$ is given. Again $Am^2 + mo^2 = Ao^2$, and $An^2 + Sn^2 = AS^2$; taking the diff. $Am^2 - An^2 + mo^2 - Sn^2 = Ao^2 - AS^2 =$ a given magnitude: but $mo^2 - Sn^2$ has been ſhewn to be given, therefore $Am^2 - An^2$ is given; therefore this caſe of the problem is reduced to queſtion 803, Gentleman's Diary.

And thus the queſtion was anſwered by Mr. James Whalley.

The

The ſame, anſwered by Mr. Lowry.

ANALYSIS. Fig. 486. Pl. 26. Let O and Q be the centres of the given circles, and on the chords CB, EF demit the perpendiculars OI, QH, and produce OI to meet EF at G, draw the radii OB, EQ, and join AO, AQ.

Then, becauſe the ſum or difference of the ſquares of BC, EF is given, the ſum or difference of the ſquares of their halves IB, EH will be given: but $AO^2 - BO^2 = AI^2 - BI^2$, and $AQ^2 - EQ^2 = AH^2 - EH^2$, therefore the ſum or difference of the ſquares of AI, AH is given, and becauſe IAG is a given $\angle$ and GI is $\perp$ to AI, the $\angle$ AGI is given, and the ratio of AG to AI is alſo given. Therefore the ſum or difference of the ſquare of AH and the ſpace to which the ſquare of AG has a given ratio is given. Through the points A and O, deſcribe a ſegment of a circle AGO to contain an $\angle$ equal to the given one AGI, this circle is given by poſition. Let AK be its diameter, and join KG, alſo on AG take AR = AI, and draw RL $\parallel$ to GK to meet the diameter AK in L. Then by ſim. $\triangle$s, AG : AR = AI :: IK : AL; but AK and the ratio of AG : AI are given, therefore AL is given, and L is a given point. Now the loci of the points H and R are the ſemicircles QHA and LRA, deſcribed on the diameters AQ, AL, reſpectively. Hence, the problem is reduced to this, *viz.* To draw AH ſo that the ſum or difference of the ſquares of the chords AR, AH, may be given, which is a particular caſe of the general problem ſolved on page 362 of the Repoſitory.

The ſame, anſwered by Mr. Richard Nicholſon, *Liverpool.*

ANALYSIS. Fig. 487, Pl. 26. Draw AD, AG to the centres D and G of the given circles, upon the diameters AD, DG deſcribe the circles ADI, AGH interſecting each other in L, and cutting AF, AC in I and H. Join B, I; G, H; D, F; G, C; L, I; L, H; and $\perp$ to LI, LH draw AK, AP; join KP, and upon the diameter AL deſcribe the circle AKLP, which will paſs through the points K and P, becauſe the $\angle$s AKL, APL are right ones. Now, the $\angle$s AID, AHG, ſtanding in ſemicircles are right angles, theref. $AD^2 - DF^2 = AI^2 - IF^2$, and $AG^2 - GC^2 = AH^2 - HC^2$, conſequently, by the addition of equal ſpaces, we have $AD^2 + AG^2 + IF^2 + HC^2 - DF^2 - GC^2 = AI^2 + AH^2$, a given ſpace, for $IF^2 + HC^2$ is $= \frac{1}{4}$th of the given ſpace $EF^2 + BC^2$, and AD^2, AG^2, DF^2, GC^2 are ſquares upon given lines: Now, the $\angle$s AIL, AHL, ſtanding upon the given chord AL of the given circles AIL, AHL, are given, theref. the

right

right angled △s AIK, and APH are given in ſpecie, and
angle FAC is given by the problem, the ∠ KAP is al
and thence KP, being the chord on which the angle KA
in the given circle AKLP; moreover, the ratio of AI²
given, which ſuppoſe as $m : n$, and the ratio of AH² : A
given, which ſuppoſe as $r : n$; then $m \cdot AK^2 + r \cdot Al$
(AI² + AH²) a given ſolid; whence the △ AKP is eaſi
mined by the Lemma on page 174, of Simpſon's Select
Hence the poſition of the required lines becomes known.

When the diff. of the ſquares of BC and EF is given,
of $m \cdot AK^2$ and $r \cdot AP^2$ is given, in which caſe, as we
former, the locus of the vertex, A, of the △ APK is
which is proved in my ſolution to the 24th queſtion in th
matical Companion.

The ſame, anſwered by Mr. J. H. Swale.

CASE I. For the ſum of the ſquares. Fig. 488, Pl.

CONSTRUCTION. Let O and Q be the centres of
circles and UW half the ſide of a ſquare = the given ſpa
AO, AQ, and draw AH = AO, making ∠ OAH of
magnitude, and produce QA to I, making AI = AQ;
upon which as a diameter let a circle be deſcribed, and
taken = a quadrant thereof. Join AK, and let a circula
be deſcribed thereon, capable of containing an angle = t
plement of half a right angle, to which apply HP, the
ſquare = the difference between the ſum of the ſquares
AO, AQ, and OC, QF. Join H and S, the point w
produced, meets the circle on IH; draw AEF ∥ to SH,
making the ∠ FAC = HAO, and it is done.

DEMONSTRATION. Join OB, OC, QE, QF, a
upon BC, EF the ⊥s OD, QG, HL, join alſo PA
produce PA, SI to meet HS, FA produced, at T and
by the circle's property
FQ = QE, FG = GE, OC = OB, and CD = DB, a
$AQ^2 - QF^2 = AG^2 - GF^2$, and $AO^2 - OC^2 = AD^2 - D$
$FG^2 = AG^2 + QF^2 - AQ^2$, and $CD^2 = AD^2 + OC^2 - AO$
$FG^2 + DC^2 = AG^2 + AD^2 + QF^2 + OC^2 - AQ^2 - AO^2$.

Again, by conſtr. the ∠s TPS, TSP are each = h
∠, and PTS = a right angle; therefore, by Eu. I. 47,
$TP^2 = HP^2 = UW^2 + AO^2 + AQ^2 - OC^2$
by conſtr,

Again, by reaſon of = ∠s and parallels, PT = TS =
alſo to AG, for AI = IQ; and HT = LA; then
$HT^2 + TP^2 = HT^2 + TS^2 = LA^2 + AG^2 = UW^2 + AO^2$.
$OC^2 - QF^2$.

But, by constr. $\angle HAO = FAC$, or $\angle HAL = OAD$, and $AH = AO$, that is, $AL = AD$. Hence,
$AG^2 + AL^2 = AG^2 + AD^2 = UW^2 + AO^2 + AQ^2 - OC^2 - QF$; but
$AG^2 + AD = FG^2 + CD^2 + AQ^2 + AO^2 - OC^2 - QF^2$; theref.
$FG^2 + CD^2 = UW^2$, or $4FG^2 + 4CD = EF^2 + BC^2 = 4UW^2$, equal the given space by the construction. *Q. E. D.*

The *maximum* will obtain when HP passes through O′ the centre of the circle described on AK; the *minimum* will also be determined in the same circumstance, only it will be the opposite point of the circle to P.

Case II. For the difference of the squares.

Construction. Let O, Q, be the centres as in the first case, and YZ half the side of a square = the given space. Join AO, AQ, and draw $AH = AO$, making the $\angle OAH =$ the given one. Join HQ, and bisect it in R, and on HQ let a circle be described, to which apply AV the side of a square = the difference between the sum of the squares of HA, FQ and YZ, CO. Join QV, draw AF ‖ thereto, and AC making the $\angle CAF = OAH$, and AEF, ABC will be the lines required to be drawn.

Demonstration. Join HV, cutting AF in L, and draw the other lines as in the first case. Then
$CD^2 = CO^2 - OD^2$ and $FG^2 = FQ^2 - QG^2$, therefore
$FG^2 - CD^2 = OD^2 - QG^2 + FQ^2 - CO^2$. But $\angle OAH = CAF$, by const. from which take away the common $\angle OAF$, and the $\angle OAD = HAH$, theref. $HL = OD$, because $HA = AO$, by the constr. and by the semicircle, the angle HVQ, or HLA is a right angle, therefore $QG = VL$. Therefore,
$FG^2 - CD^2 = HL^2 - LV^2 + FQ^2 - CO^2 = HA^2 - AV^2 + QF^2 - CO^2$, hence
$AV^2 = HA^2 + FQ^2 - CO^2 - FG^2 + CD^2$, but by constr.
$AV^2 = HA^2 + FQ^2 - CO^2 - YZ^2$; therefore
$FG^2 - CD^2 = YZ^2$, and $4FG^2 - 4CD^2 = EF^2 - BC^2 = 4YZ^2$, equal to the given space by the construction. *Q. E. D.*

In this case the *maximum* obtains when AV and AL coincide, and the *minimum* when $AV = AH$.

XXVIII. *Or*, PRIZE QUESTION 210, *answered by the Proposer, Mr.* Wm. Wallace, *Perth Academy.*

Fig. 493, Pl. 26. Suppose PABC, &c. to be drawn as required.

Let the circle PA cut the remaining circles in the points R, S, &c. Draw PH touching the circle PBR and meeting the circle PAR in H. Join HA, HP, HR, RA, RB. Because

RP

PR is a chord in the circle PBR, and PH a tangent at the point P, the angle RBP = RPH = RAH, now the angle RPB = RHA, therefore the triangles RPB, RHA are similar, hence HR : PR : : HA : PB, now PH, being a tangent to a given circle at a given point, is given by position, therefore the point H is given, and since the point R is given, the lines HR, PR are both given in magnitude, therefore the ratio of HA to PB, and consequently of HA^2 to PB^2 is given, and $PB^2 = (PR^2 \div HR^2) \cdot HA^2$. In like manner if PK be drawn touching the circle PCS, and meeting the circle PARS at K, and KA, KS, SP be joined, it may be shewn that the point K is given, and also the ratio of KA^2 to PC^2, or $PC^2 = (PS^2 \div KS^2) \cdot KA^2$, and so on if there be more circles passing through the point P, therefore

$$PA^2 + \frac{PR^2}{HR^2} \cdot HA^2 + \frac{PS^2}{KS^2} \cdot KA^2 + \&c. = PA^2 + PB^2 + PC^2 + \&c. = Q$$ a given space (by hypothesis). Hence it appears that the *locus* of the point A is the circumference of a given circle (*Apolon. Loci Plani, Lib. II. Prop. 5, also Stewart's Gen. Theor. ſce Math. Rep. Vol. I. page* 248.). Now A is also in the circumference of the given circle PRS, therefore the point A is given and may be found by this *Conſtruction*.

From the given point P draw ſtraight lines PH, PK, &c. touching all the circles except one, and meetng the remaining circle in the points H, K, &c. Let the circle to which PH is a tangent meet the circle HKP in R. Join HR, PR, and conceive weights placed at P, and H proportional to 1 and $(PR^2 \div HR^2)$, or to a and b, putting $a = 1$ and $b = (PR^2 \div HR^2)$. Let the circle to which PK is a tangent meet the circle HKP in S, join KS, PS, and conceive a weight placed at K proportional to $(PS^2 \div KS^2 = c$, and so on if there be more circles. Find x the centre of gravity of the weights a, b, c, &c. join PX, HX, KX, &c. and on X as a centre, with radius equal to $\sqrt{\left(\frac{1}{a+b+c+\&c.}\left\{Q \cdot a \cdot PX^2 - b \cdot HX^2 - c \cdot KX^2 \&c.\right\}\right)}$ describe a circle MN meeting the circle PKH in A. Join PA cutting the remaining circles in B, C, &c. the line PABC ſhall be drawn as required. The Synthetic Demonſtration will readily follow from the preceding analyſis and from Dr. Stewart's 7th General Theorem as demonſtrated in the 1ſt Vol. of the Repoſitory, pa. 249.

Remarks 1. If it had been required to draw PBC ſo that $a \cdot PA^2 + b \cdot PB^2 + c \cdot PC^2$ &c. might be equal to a given ſpace, the leaſt

letters a, b, c, &c. denoting given quantities either positive or negative, it is evident from the preceding analysis that the *locus* of the point A would still have been a circle.

2. The preceding analysis suggests a number of curious propositions, in geometry, for example: Let any three circles given by position intersect each other at the same point P, from P draw any line whatever meeting all the circles in the points A, B, and C; Then $a \cdot PA + b \cdot PB = c \cdot PC$, where a, b, and c represent three invariable lines.

The same, answered by Mr. John Lowry.

ANALYSIS. Fig. 489, Pl. 26. Let the circles PB, PC, &c. intersect the circle PA at E, F, &c. and join EA, FA, &c. EB, FC, &c. PE, PF, &c.
and draw PP', PP'' &c. perp. to AE, AF, &c. Then, Eu. II. 4, $PB^2 = PA^2 + 2PA \cdot AB + AB^2$, $PC^2 = PA^2 + 2PA \cdot AC + AC^2$, and so on; $\therefore$
$AP^2 + PB^2 + PC^2$, &c. is $= n \cdot AP^2 + AB^2 + AC^2$, &c. $+ 2AP \cdot (AB + AC$, &c.)
where n represents the number of given circles. But because the points P, E, are given, the angles PAE, PBE, are given, therefore the triangle EAB is given in species; and in the same way it may be shewn that the triangle FAC is given in species; wherefore the ratios of AB to AE, AC to AF, and so on, are given. Let AB be to AE as a to b, AC be to AF as a to c, and so on; Then

$$AB = \frac{a}{b} AE,\ AC = \frac{a}{c} AF,\ AB^2 = \frac{a^2}{b^2} AE^2,\ AC^2 = \frac{a^2}{c^2} AF^2,$$

and so on; $\therefore$

$$AP^2 + BP^2 + PC^2 \text{ \&c.} = n \cdot AP^2 + \frac{a^2}{b^2} AE^2 + \frac{a^2}{c^2} AF \text{ \&c.} +$$

$$2AP \left(\frac{a}{b} AE + \frac{a}{c} AF \text{ \&c.}\right).$$

Now, by Euc. II. 13, $2AP' \cdot AE + PE^2 = AP^2 + AE^2$, $2AP'' \cdot AF + PF^2 = AP^2 + AF^2$, and so on, but, because the angles PAP', PAP'' &c. are given, the ratios of PA to AP', PA to AP'' &c. are given. Let PA be to AP' as d to a, PA to AP'' as e to a &c. Then
$2AP \cdot AE : 2AP' \cdot AE = AP^2 + AE^2 - PE^2 :: d : a$, therefore

$$2AP \cdot \frac{a}{b} \cdot AE = \frac{d}{b} \cdot (AP^2 + AE^2 - PE^2).$$

In the ſame way it is ſhewn that,

$2AP \cdot \frac{a}{c} \cdot AF$ is $= \frac{e}{c} \cdot (AP^2 + AF^2 - PF^2)$, and ſo on, that is,

$2AP \cdot \frac{a}{b} \cdot AE = \frac{d}{b} \cdot PA^2 + \frac{db}{b^2} \cdot AE^2 - \frac{d}{b} \cdot PE^2$, and

$2AP \cdot \frac{a}{c} \cdot AF = \frac{e}{c} \cdot PA^2 + \frac{ec}{c^2} \cdot AF^2 - \frac{e}{c} \cdot PF^2$. Therefore, it is evident that, $AP^2 + BP^2 + CP^2$ &c. is equal to the exceſs of

$$\left(n + \frac{d}{b} + \frac{e}{c} \text{ \&c.}\right) \cdot AP^2 + \left(\frac{a^2 + db}{b^2} \text{ \&c.}\right) \cdot AE^2 + \left(\frac{a^2 + bc}{c^2} \text{ \&c.}\right)$$

AF^2 &c. above $\frac{d}{b} \cdot PE + \frac{e}{c} PF^2$ &c. But PE and PF &c. are given lines, therefore, it is evident the ſum of the ſpaces to which the ſquares of AP, AE, AF, &c. have given ratios, is equal to a given ſpace. Wherefore, by Prop. D, Art. XXXI. Vol. I. of the Repoſitory, the *locus* of the point A is a circle given by poſition, and therefore the method of conſtruction is evident from that Propoſition.

From the above Analyſis the following Theorems are evident.

I. If the ſum of the ſolids contained under the ſquares of the chords, and given lines, be equal to a given ſolid, the *locus* of the point A will be a circle given by poſition.

II. If there be any number of given points, P, Q, R, S, &c. and a ſtraight line be drawn through P, ſo that the ſum of the ſquares of all the perpendiculars QA, RB, SC, &c. may be equal to a given ſpace; or that the ſum of the ſolids contained by theſe ſquares and given lines, may be equal to a given ſolid; the *locus* of the point A will in both caſes be a circle given by poſition.

III. If the difference between the ſum of the ſquares (or the ſpaces to which they have given ratios) of any aſſigned number of the chords, and the ſum of the ſquares (or the ſpaces to which they have given ratios) of the remaining chords be equal to a given ſpace; the *locus* of the point A will ſtill be a circle given by poſition.

IV. If PEF be a given circle, and PF, PE, &c. be chords given by poſition, and if from a point A, in the circumference, the perpendiculars Ap, Ap', &c. be drawn to the chords PE, PF, &c. and AG, AH, AI, &c. be drawn to the given points G, H, I, &c. ſo that the ſum of the ſquares of GA, HA, IA, &c. (or the ſpaces to which they have given ratios) together with the rectangles contained by Ap, Ap', &c. and given lines, may be equal

to

to a given ſpace; the *locus* of the point A will be a circle given by poſition.

It is alſo evident from the ſolutions to queſtions 180 and 209, and from what is done above, that ſolutions to the following problems may be readily had.

I. Let P be a given point, and AB, CD, EF, &c. be circles given in magnitude and poſition: It is required to draw a ſtraight line, through P, interſecting the circles in the points AB, CD, EF, &c. ſo that the ſum of the ſquares (or the ſpaces to which they have given ratios,) of all the chords AB, CD, EF, &c. may be equal to a given ſpace.

II. Let P be a given point and AB, CD, EF, &c. and *ab*, *cd*, *ef*, &c. be circles given in magnitude and poſition: It is required to draw two ſtraight lines, through P, interſecting the circles in the points AB, CD, EF, &c. and *ab*, *cd*, *ef*, &c. to contain a given angle at P, and ſo that the ſum of the ſquares of all the chords AB, CD, EF, &c. *ab*, *cd*, *ef*, &c. (or the ſpaces to which they have given ratios) may be equal to a given ſpace.

The ſame, anſwered by Mr. Richard Nicholſon, *Private Teacher of the Mathematics, at Liverpool.*

ANALYSIS. Fig. 490, pl. 26. Join ZA, Z being one of the points of interſection of the circles of which PA and PB are the chords, and produce ZA to meet the circle of which PB is the chord at Y, and join PY. Then the angles PAZ (= PYA + APY) and PYA are given, therefore the angle APY and the arch BY are given, and therefore the triangle PYA is given in ſpecies; wherefore $PA^2 : PY^2$ has a given ratio, which ſuppoſe as $a : b$; then $b \cdot PA^2 = a \cdot PY^2$, therefore $a \cdot PY^2 + b \cdot PB^2 = b \cdot PA^2 + b \cdot PB^2 =$ a given ſolid, when the number of circles is two, therefore the chord PB is given, by the Lemma on pa. 174, Simſon's Select Exerciſes.

Join PX, X being one of the points of interſection of the circles of which PB and PC are the chords, and upon PX, as a diameter, let a circle be deſcribed meeting XY, CX, and XB in W, U, and V. Join alſo PW, PU, and PV. Then ſince the angles PYX, PCX, and PBX, of the right angled triangles PYW, PCU, and PBV are given, as alſo the angle PBY, the arches VW, UW will be given, and therefore theſe triangles will be given in ſpecie; wherefore $PY^2 : PW^2$ has a given ratio, which ſuppoſe as $c : a$, and $PB^2 : PV^2$ has a given ratio, which ſuppoſe as $d : b$, and $PC^2 : PU^2$ has alſo a given ratio, which ſuppoſe as $e : b$. Then

$a \cdot PY^2 = c \cdot PW^2$, $b \cdot PB^2 = d \cdot PV^2$, and $b \cdot PC^2 = e \cdot PU^2$, therefore $c \cdot PW + d \cdot PV^2 + e \cdot PU^2 = b \cdot PA^2 + b \cdot PB^2 + b \cdot PC^2 =$ a given

 ſolid

ſolid when the number of circles is three, and therefore FU is given by the problem immediately following the above-mentioned lemma.

Join PT, T being one of the points of interſection of the circles of which PD is the chord of one, and PX the diameter of the other. Upon the diameter PT let a circle be deſcribed, meeting TW, TU, TV, and TD in S, Q, R, and O. Join PS, PQ, PR, and PO. Then by proceeding as above, we have $f \cdot TS^2 + g \cdot PR^2 + h \cdot PQ^2 + i \cdot PO^2 = b \cdot PA^2 + b \cdot PB^2 + b \cdot PC^2 + b \cdot PD^2$ = a given ſolid, when the number of circles is four, and the arches RS, RQ, and QO being given, PO will be given by the problem referred to above. And by thus proceeding may the problem be ſolved for any number of circles.

The ſame, anſwered by Mr. J. H. Swale.

ANALYSIS. Fig 491, Pl. 26. Suppoſe the line PAB, &c. to be drawn as required by the problem. Through P let the diameters PF, PG, PH, &c. be drawn, and join FA, GB, HC, &c. which will evidently be parallel to each other. In theſe diameters PF, PG, PH, &c. be taken, each equal to the diameter PT, and drawn upon PABC, &c. the perpendiculars LQ, MR, &c.

Then $PQ^2 = PG : PL^2 (PF^2)$, and $PC^2 : PR^2 = PH^2 : PM^2$: and ſo on.

Now $PS^2 =$ to the ſpace to which PG has the ſame ratio that PC has to PF, and PC is = to the ſpace to which PR has the ſame ratio that PH has to PF, and ſo on.

Now, perpendicular to PC, PH, &c. draw CA, HC', &c. each = PF, and join P with A, C, C', &c. and perpendicular to PA, PC, &c. draw CB, PA, &c. meeting AG, CH, &c. produced in B, D, &c.

Then $AG^2 (PF^2) : CH^2 : PH^2 = HD^2 : HC^2$, and ſo on.

That is, the ſpace to which PQ has the ſame ratio that PF has to GB is = HB, and the ſpace to which PR^2 has the ſame ratio to PF has to HD is = PC, and ſo on. Therefore the ſum of ſquares of PA, PB, &c. &c. is = to the ſquare of PA, together with the ſpace to which the ſquare of BQ has the ſame ratio that PF has to GB, and the ſpace to which the ſquare of PR has the ſame ratio that PF has to HD, and ſo on. Now GB, HD, &c. are given magnitudes, and F, L, M, &c. are given points, wherefore parallels FA, LQ, MR, &c. are equals. &c. by the Demonſtrations to Stewart's General Theorem. A point U may always be found [illegible] being drawn through U, and the perpendicular

dicular PV demitted thereon from P; the ſquare of PA, together with the ſpace to which the ſquare of PQ has the ſame ratio that BF has to GB', together with the ſpace to which the ſquare of PR has the ſame ratio that PF has to HD', and ſo on, ſhall be equal to the ſpace to which the ſquare of PV has the ſame ratio that 2PF has to the ſum of PF, HP', HD', &c. equal, in the preſent caſe, to a given ſpace.

Let U be thus determined for the given points P, F, L, M, &c. and the given magnitudes PF, GB', HD' &c. Join PU and produce it U', making $PU^2 : PU'^2 = 2PF : PF + GB' + HD'$, &c. and draw U'V'W' parallel to UVW.

Then, by parallels, $UP^2 : U'P^2 = PV^2 : V'P^2 = 2PF + GB' + HD'$ &c. that is, $V'P^2$ is equal to a given ſpace. Therefore it only remains to deſcribe a ſemicircle upon PU', apply therein PV' the ſide of a ſquare = to the given ſpace, and through the points V', P draw the line required meeting the circles in A, B, C, &c.

Hence the ſolutions of the following Problems are evident.

Prob. I. To draw a ſtraight line through a given point P ſuch, that if perpendiculars be demitted thereon from any number of given points, the ſum of the ſquares of theſe perpendiculars ſhall be equal to a given ſpace.

Prob. II. Let there be any number of given points in the circumference of a circle given in magnitude and poſition: It is required to draw a diameter therein ſuch, that the ſum of the ſquares of the parallel chords, drawn perpendicular to that diameter through the given points, ſhall be equal to a given ſpace.

Mr. Richard Nicholſon *is requeſted to ſend to Mr.* Glendinning's *for the Medal for ſolving the Prize Queſtion.*

ARTICLE LI.

MATHEMATICAL QUESTIONS.

To be anſwered in Number XIII.

I. QUESTION 241, *by Mr.* Tho. Bulmer, *Sunderland.*

TWO veſſels of equal dimenſions in all reſpects, and in form of the fruſtum of a cone, were placed perpendicularly to the

horizon, the one with its greater end up and the other with its leſs end up, and the liquor with which they were filled exactly even with the ſurface of the earth.—Now, although the diametrical depths of the two veſſels were exactly the ſame, yet, it was found that the veſſel which ſtood on its leſs end, would hold more than the other by 10 ale gallons. Required the diameters and content of the two veſſels, ſuppoſing the depth of each 12 feet, the greater diameter to the leſs as 4 to 1, and the earth a perfect ſphere whoſe radius is 4000 miles?

II. QUESTION 242, *by Mr.* William Marrat, *Boſton.*

From a cone, the length of which was 7½ feet and the diameter of its baſe 26 inches, I ordered 4 ſlices to be cut off, by ſections parallel to the axis, and at ſuch a diſtance as juſt to reduce the baſe to a ſquare. Query the ſolidity and ſuperficies of the part remaining?

III. QUESTION 243, *by Mr.* Wm. Peacock, *Birmingham.*

A land ſurveyor, meaſuring a triangular piece of ground, found the diſtances of the three points where perpendiculars from the angles met the oppoſite ſides to be 9, 10 and 11 chains. From theſe data to determine the area of the field by trigonometry.

IV. QUESTION 244, *by* Merones Minor.

To determine the thickneſs of a triangular wall at the bottom, neceſſary to ſupport a body of water; that ſide of the wall facing the water being upright, the depth of the water 10 feet, and the height of the wall 12 feet; the ſpecific gravity of the wall and water being as 11 to 7.

V. QUESTION 245, *by Mr.* Olinthus Gilbert Gregory, *Teacher of Mathematics, Cambridge.*

A carpenter has received orders to make an elliptical table the tranſverſe diameter of which is to be 12 feet; the length of the conjugate he has forgotten; but he knows the ellipſis was to be ſuch that the difference between the areas of the greateſt inſcribed triangle and of the leaſt circumſcribing triangle, was to be 70·148025 feet. From this he hopes ſome ingenious gentleman will tell him the length of the conjugate diameter.

VI.

VI. QUESTION 246, *by Mr.* Newton Bofworth, *Cambridge.*

There are two plane mirrors, each 3 feet 6 inches long, and, meeting each other at one end, form an angle of 15°. Now, if a ſmall object be placed 6 inches from the other extremity of one of theſe mirrors, it is required to determine how many images of the object will be formed upon the mirrors by reflection, and how far each image is behind the ſurface of the mirror upon which it is formed.

VII. QUESTION 247, *by Mr.* O. G. Gregory, *Cambridge.*

A gentleman who has a garden in form of a regular pentagon, each ſide of which meaſures 50 yards, is deſirous of knowing without actual admeaſurement the ſum of the lengths of walks which ſhall be made to connect each angle with its two oppoſite ones; and wiſhes alſo to know the expence of a pond which ſhall be dug in the form of the moſt regular curve that can be made to touch all the walks, reckoning the expence of digging the pond at 6d. per yard on the ſurface.

VIII. QUESTION 248, *by* Urſa Miner.

What is the nature of that curve, the diſtance of the ſub-tangent of which from its vertex is equal to the ordinate at the point of contact of the tangent?

IX. QUESTION 249, *by Mr.* I. T. M'Doneld.

A, B, and C are three given points in the circumference of a circle: it is required to draw a chord CD, in the circle, ſuch, that, being divided in a given ratio at P, the ſum or difference of the ſquares on AP and BP, may be equal to a given ſpace, and to ſhew the limits.

X. QUESTION 250, *by* Hypatia.

Given the area and the perimeter of a regular polygon to determine the number of its ſides.

XI. QUESTION 251, *by Mr.* T. S. Evans, *Mathematical Maſter at Chriſt's Hoſpital.*

At what height above the ſurface of a table muſt the flame of a candle be placed ſo as to illuminate, in the greateſt degree poſſible, a point of the table at a given diſtance from the point directly under the candle?

XII.

XII. QUESTION 252, *by Mr.* Henry Boiley.

Given the circumference of a cylinder ſtanding on its baſe perpendicular to the horizon $= a$, the diameter of a cord wound in a ſpiral upon it $= b$, and its length $= l$; alſo there is given the angle the rounds form with the horizon; to find the length of the curve made by the end of the cord in winding it all off, the cord being always ſtretched and its end continually depreſſing.

XIII. QUESTION 253, *by Mr.* Louis Hilk

There is a perfectly poliſhed circular table, whoſe diameter is 20 feet, placed ſo that its plane makes an angle of 10 degrees with the horizon. Now if a ball be projected from a point in the periphery, which is diſtant from the higheſt point 10 inches, and with a velocity of 8 feet in a ſecond, it is required to determine the direction of the projectile, and the point where the ball will quit the table, and the length of the track, ſuppoſing it to paſs through the centre of the table.

XIV. QUESTION 254, *by* Collator.

Two equal fires are placed in the foci of an ellipſis; a perſon walks from one apſis to the other along the periphery; where will he feel the leaſt heat?

XV. QUESTION 255, *by Mr.* J. H. Swale.

There are two given circles AEBR, CHDI, whoſe centres are C and O; the latter paſſing through the centre C of the former. It is required to draw the chord CP, ſuch that, demitting a perpendicular thereon from a given point G, the part EP intercepted between the circles ſhall be biſected by that perpendicular.

XVI. QUESTION 256, *by Mr.* James Cunliffe, *Bolton.*

Theorem. ABC is a plane Δ circumſcribed by a circle, the baſe AB being biſected in G by the diameter FE, and CH drawn parallel to AB meeting FE in H: if upon GH as a diameter another circle is deſcribed, and through O the centre of the circumſcribed circle a perpendicular to FE be drawn meeting the circumference of the circle, whoſe diameter is GH, in R, and the line FR be drawn from the point F, below the baſe AB: then will $4 \times FR^2 = AC^2 + BC^2$. Required the demonſtration?

XVI

XVII. QUESTION 257, *by* Amicus.

Suppoſe a ſtraight rod to be partly immerſed in a veſſel of water; to determine the angle at which it muſt be inclined to the ſurface that the apparent bending at the ſurface may be a maximum.

XVIII. QUESTION 258, *by* Amicus.

The latitude being given, to find at what time, on the longeſt day, the variation of the ſun's altitude is greateſt.

XIX. QUESTION 259, *by* A Miner.

In order to diſcover the poſition of a ſtratum of coal three points A, B, C were aſſumed on the earth's ſurface, the diſtances AB, AC, BC were found to be a, b, and c fathoms, and the line AB made with the meridian an angle of g degrees. After boring at the points A, B, C, the depth of the coal below the ſurface was found to be p, q and r fathoms reſpectively. From theſe data it is required to find the direction and dip of the ſtratum.

XX. QUESTION 260, *by* Collator.

The axis of a given cylinder paſſes through the centre of a ſyſtem whoſe weight is known; and the whole is made to revolve by a given weight P, ſuſpended by a ſtring wound round the cylinder; the ſpace, then, through which P deſcends in a given time, being known, it is propoſed to find the centre of gyration of the whole ſyſtem.

XXI. QUESTION 261, *by Mr.* James Cunliffe.

To determine the greateſt ordinate of a catenarian curve of a given length, when the ſolid formed by its revolution about that ordinate is a maximum.

XXII. QUESTION 262, *by the Rev.* L. Evans.

Suppoſe two weights connected by a ſtring, which paſſes over a pulley, and that one of the weights is placed upon an inclined plane, while the other hangs in the air by the connecting ſtring. It is required to determine the *locus* of the centre of gravity of the weights, ſuppoſing the relative poſition of the pulley and plane, the length of the ſtring, and the weights to be all given.

XXIII. QUESTION 263, *by* Collator.

Suppose a perfectly polished tube to pass through the centre of the earth in the plane of the equator; it is proposed to find the nature of the spiral described by a body descending in this tube by its own weight, while it is carried about at the same time by the earth's rotation.

XXIV. QUESTION 264, *by Mr.* J. H. Swale.

There are two given points in the periphery of a given circle. It is required to apply in the circle a chord of a given length such, that demitting perpendiculars thereon from the given points, the sum, sum, difference, rectangle, difference of the squares or sum of the squares of those perpendiculars shall be given.

XXV. QUESTION 265, *by Mr.* Cunliffe.

Let one end of a perfectly flexible heavy line or chain APB of a given length be fastened at the point A whilst the other end B is carried along the horizontal line AB. It is required to find the equation and quadrature of the curve which is the *locus* of a given point P in the chain.

XXVI. QUESTION 266, *by Mr.* John Johnson, *Birmingham*

There are two circles given in magnitude and position, and a point in the straight line joining their centres is also given. It is required to describe another circle, so that its circumference may pass through that point, and touch one of the given circles, and have its centre in the circumference of the other given circle.

XXVII. QUESTION 267, *by Mr.* John Lowry.

Given the base of a plane triangle to construct it when the sum of the squares of the sides has to the area of the triangle a given ratio, and when the greater side has to the less, the greatest ratio possible.

XXVIII. QUESTION 268, *by Mr.* Lowry.

[illegible] the vertical angle, and the difference between the square of the base and the rectangle under the sum of the sides and a given line, construct the Δ, when the perpendicular from one of the angles at the base upon the opposite side is a maximum.

XXIX

XXIX. QUESTION 269, *by Mr.* Lowry.

In a circle given in magnitude and position, it is required to inscribe a trapezium such that three of its sides shall pass through three given points in the same straight line, and that the fourth side shall touch a circle given in magnitude and position.

XXX. PRIZE QUESTION 270, *by* Philalethes Cantabrigiensis.

Given $\frac{\dot{x}}{(1+\sqrt{x}-2\sqrt[3]{x})^2} = \dot{y}$, in which equation x and y begin together, to find the value of y when $x = \frac{9}{10}$.

ARTICLE LII.

EIGHT PROPOSITIONS FROM LAWSON.

(To be answered in Number XIII.)

PROP. XLVII.

IF from one angle A of a rectangle ABCD a line be drawn to cut the two opposite sides BC, DC, the former in F, and the latter produced in E; then I say that the rectangle EAF is equal to the sum of the rectangles EDC, CBF.

PROP. XLVIII.

If a rectangle be inscribed in a right-angled triangle, so that one of its angles coincide with the angle of the triangle; then I say that the rectangle under the segments of the hypothenuse is equal to the sum of the rectangles under the segments of the sides about the right angle made by this inscription.

PORP. XLIX.

If from the same point C two tangents be drawn to a semicircle whose diameter is AB, and if the extremes of the diameter and the points of contact be joined, either cross-ways by two lines intersecting in F, or other-ways by two lines intersecting in H; then I say that CF or HC produced to meet the diameter AB will be perpendicular to the same.

PROP

PROP. L.

If in a ſemicircle whoſe diameter is AB the chord
equal to the radius be inſcribed, and from the centre E a pe
cular drawn thereto and produced to meet the circumferenc
then I ſay that AF, EF, BF are continual proportionals.

PROP. LI.

If a line be cut in extreme and mean proportion; the
that the ſquare of the whole, the rectangle under the wh
the greater ſegment, and the rectangle under the whole
leſſer ſegment are continual proportionals.

PROP. LII.

If three lines are continual proportionals; the ſum
ſquares of the mean and the greater extreme is to the re
contained under the ſame, as the ſum of the extremes is
mean.

PROP. LIII.

In every right-angled triangle, as the hypothenuſe is to t
of the ſides about the right angle, ſo is the ſaid ſum, to t
of the hypothenuſe and twice the perpendicular from the
angle.

PROP. LIV.

If a right line AD be any ways cut in B, and from thence
pendicular BE be erected equal to a mean proportional b
the whole AD and the part AB, and a circle be drawn t
the points A, D, E, and from A a perpendicular be erected t
the circumference in F; then I ſay AF, AB, BE, AD a
continual proportionals.

END OF THE SECOND VOLUME.

Glendinning, Printer, 25, Hatton Garden.

If in
equal to
cular dra
then I fa

If a l
that the
the great
leſſer ſeg

If thre
ſquares o
contained
mean.

In every
of the ſide
of the hyp
angle.

If a righ
pendicular
the whole
the points
the circumf
continual pr

THE NEW YORK PUBLIC LIBRARY
Aſtor, Lenox and Tilden Foundations.

THE
NEW YORK
PUBLIC LIBRARY
Astor, Lenox and Tilden
Foundations.

If in
equal to
cular dr
then I fi

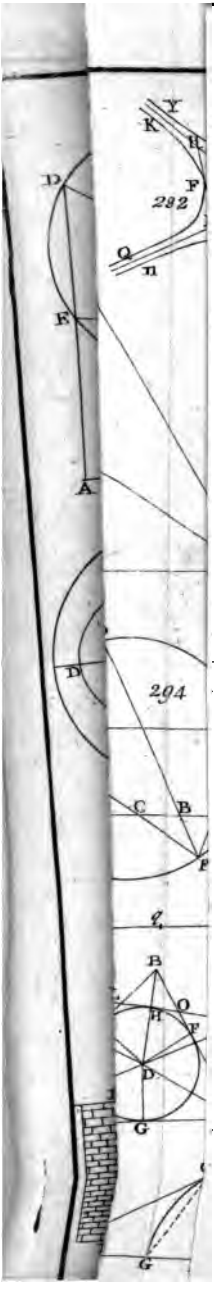
If a
that the
the grea
leſſer leg

If thr
ſquares
containe
mean.

THE
NEW YORK
PUBLIC LIBRARY
Aſtor, Lenox and Tilden
Foundations.

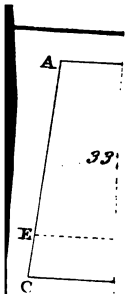
In eve
of the ſi
of the hy
angle.

If a ri
pendicula
the whole
the point
the circu
continual



THE
NEW YORK
PUBLIC LIBRARY
Astor, Lenox and Tilden
Foundations

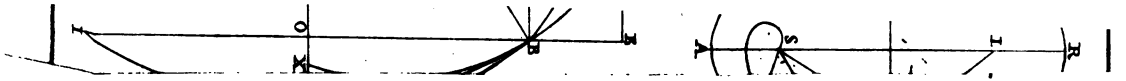
THE
NEW YORK
PUBLIC LIBRARY
Astor, Lenox and Tilden
Foundations.


A
E
C

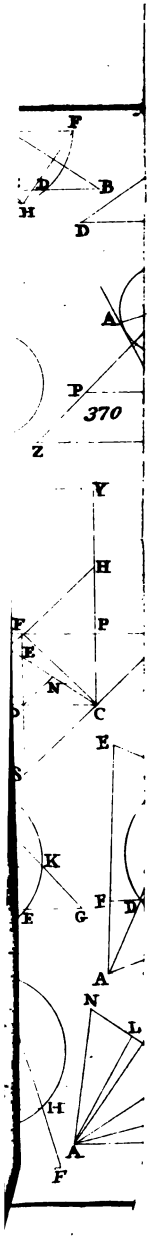
B
D

C
B
K
N
E
A
O
K

THE
NEW YORK
PUBLIC LIBRARY
Astor, Lenox and Tilden
Foundations.


I
O
E
V
S
L
R

THE
NEW YORK
PUBLIC LIBRARY
Astor, Lenox and Tilden
Foundations


F
D
B
H
D
A
P
370
Z
Y
H
F
P
E
N
C
E
S
K
E
G
F
D
A
N
L
H
A
F

If a
equal
cular
then I

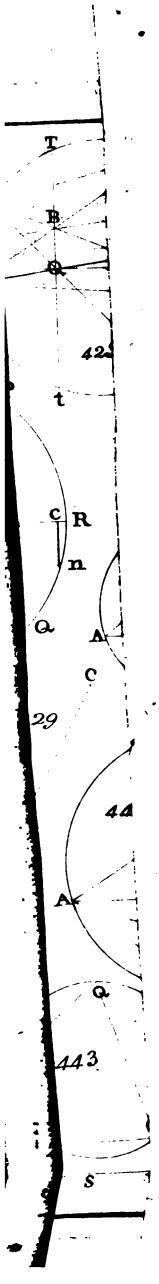
If a
that t
the gr
leſſer ſ

THE
NEW YORK
PUBLIC LISRARY
Astor, Lenox and Tilden
Foundations.

If th
ſquares
contain
mean.

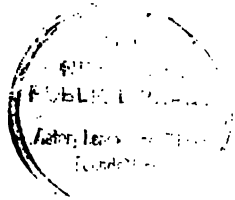
In ev
of the
of the
angle.

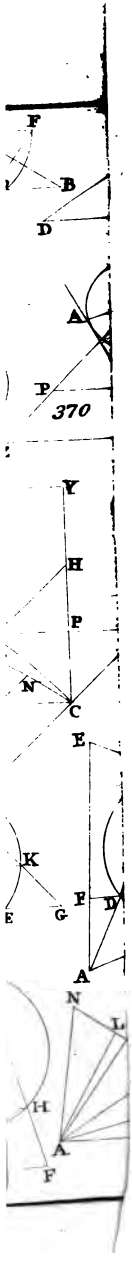
If a
pendicu
the who
the poi
the circ
continu


T
B
Q
42
t
C
R
n
Q
A
C
29
44
A
Q
443
S

THE
EW YORK
LIC LIBRARY
Lenox and Tilden
undations

PUBLIC LIBRARY
Astor, Lenox and Tilden
Foundations.




F
B
D
A
P
370
V
H
P
N
C
E
K
E
G
F
D
A
N
L
H
A
F

THE
NEW YO
PUBLIC LIB
Astor, Lenox
Foundation

THE
NEW YORK
PUBLIC LIBRARY
Astor, Lenox and Tilden
Foundations.

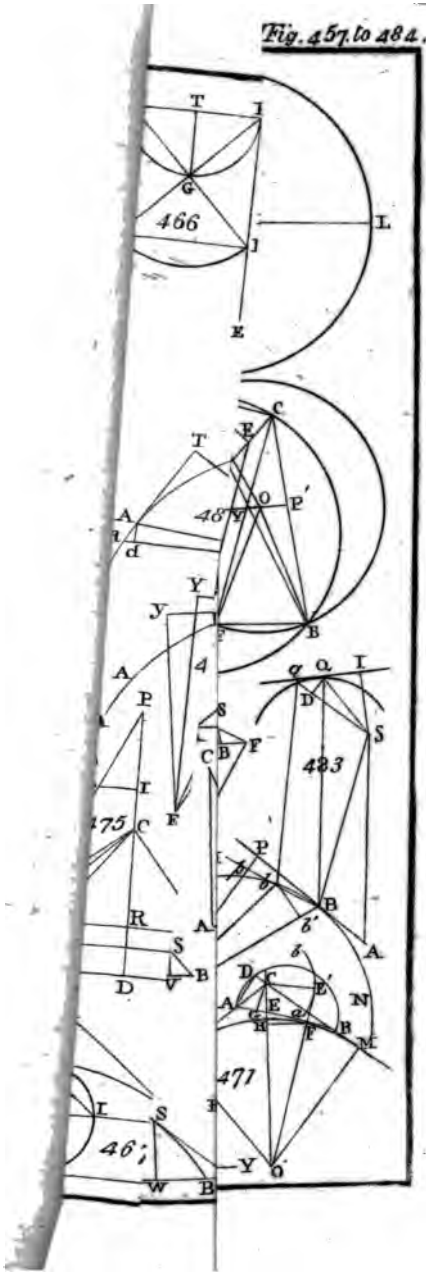


Fig. 457. to 484.
T
I
G
466
L
J
E
C
E
T
O
P'
A
d
Y
y
B
4
A
P
S
C
B
F
I
F
175
C
R
S
D
V
B
A
P
B
A
D
C
E'
A
E
N
B
M
471
I
S
46
W
B
Y
O
Q
I
D
S
423

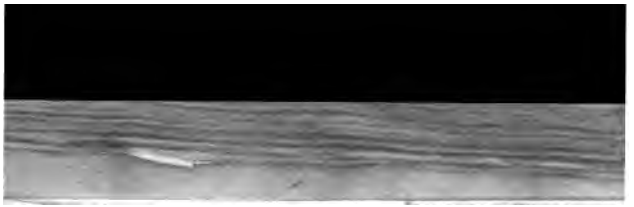
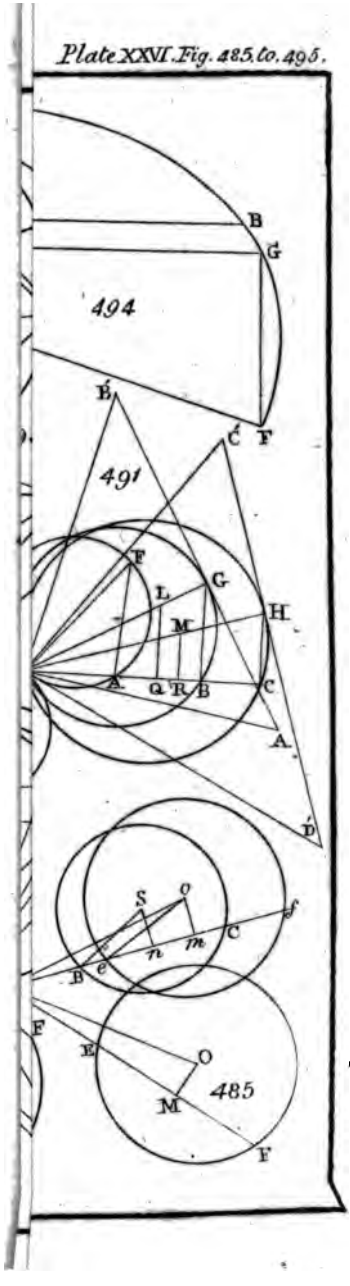
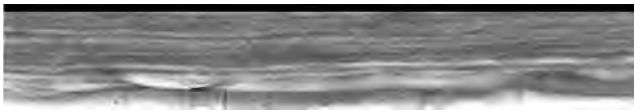
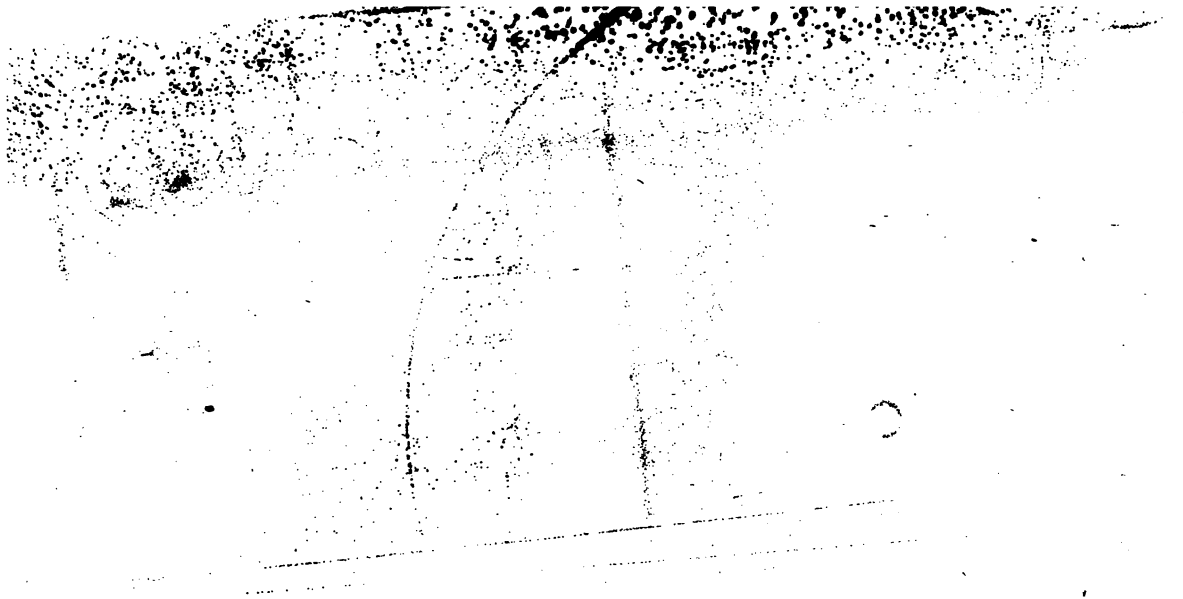
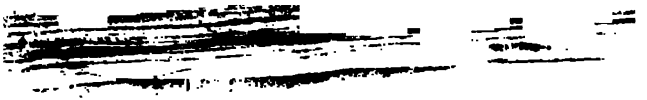



Plate XXVI. Fig. 485. to. 495.